

# Chapter (9)

## Fundamentals of Hypothesis Testing: One-Sample Tests

# Lecture (1)

## Hypothesis Testing Steps

### Objectives

In this chapter, you learn: (Slide 9)

- The basic principles of hypothesis testing
- How to use hypothesis testing to test a mean or proportion
- The assumptions of each hypothesis-testing procedure, how to evaluate them, and the consequences if they are seriously violated
- Define Type I and Type II errors.

### What is a Hypothesis?

A hypothesis is a claim (assertion) about a population parameter (Slide 3)

### What is Hypothesis Testing?

Hypothesis testing is a procedure, based on sample evidence and probability theory, used to determine whether the hypothesis is a reasonable statement and should not be rejected, or is unreasonable and should be rejected.

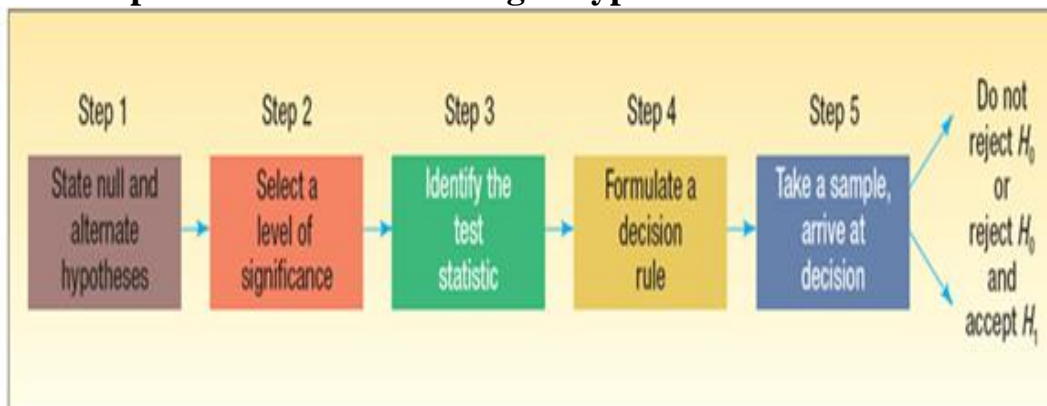
### Why we conduct the Hypothesis Testing? and How?

We conduct the hypothesis test to find out is the difference between sample statistic (which is used to estimate the population parameter) and the value of population parameter (hypothesized value) is difference due to:

- Sampling error (estimation error) which is occur when we draw a random sample from population
- Or is it significant difference? (Means actually there is difference which is has meaning in the study).

By using the sample information (mean, size, and variance)

### Five-Steps Procedure for Testing a Hypothesis



## Step (1): State the Null ( $H_0$ ) and alternate ( $H_1$ ) hypothesis

- **Null hypothesis ( $H_0$ ):**

Is the hypothesis being tested, here we write the hypothesized value which is called reference value (the value which we assume that equal to population parameter) and in null hypothesis only those operators are allowed (" $=$ ", " $\geq$ ", " $\leq$ ") because in this hypothesis always we assume that there is no difference at all (because we notice that the equal operator).

Similar to the notion of innocent until proven guilty (Slide 5)

- **Alternate hypothesis ( $H_1$ ):**

A statement that is accepted if the sample data provide sufficient evidence that the null hypothesis is false. (e.g. " $\neq$ ", " $<$ " and " $>$ ")

The alternative hypothesis which is called the researcher hypothesis it contains the opposite mathematical operation that are found in means always, we say that there is significant difference (means the population parameter will be  $<$ ,  $>$ ,  $\neq$ ) and in this hypothesis we write what the researcher hypothesis about population parameter.

When we reject the Null Hypothesis that means we accept the alternative hypothesis and Vice Versa (this will be our decision at the end of test)

(Textbook: Example 9.1 P308)

### Important Things to Remember about $H_0$ and $H_1$

- $H_0$ :  $H$  subzero or  $H$  not.
- $H_0$  and  $H_1$  are mutually exclusive and collectively exhaustive
- $H_0$  is always presumed to be true
- $H_1$  has the burden of proof
- If we conclude 'do not reject  $H_0$ ', this does not necessarily mean that the null hypothesis is true, it only suggests that there is not sufficient evidence to reject  $H_0$ ; rejecting the null hypothesis then, suggests that the alternative hypothesis may be true.
- Equality is always part of  $H_0$  (e.g. " $=$ ", " $\geq$ ", " $\leq$ ").
- " $\neq$ ", " $<$ " and " $>$ " always part of  $H_1$

### Types of Tests:

According to the alternative hypothesis the hypothesis tests can be one or two tailed.

#### Case1: One- tailed test (Right-tailed or an upper-tail test)

For Example:

$$\begin{array}{ll} H_0 : \pi \leq \pi_0 & \text{Or} & H_0 : \mu \leq \mu_0 \\ H_1 : \pi > \pi_0 & & H_1 : \mu > \mu_0 \end{array}$$

#### Case2: One- tailed test (left-tailed or a lower-tail test)

For Example

$$\begin{array}{ll} H_0 : \pi \geq \pi_0 & \text{Or} & H_0 : \mu \geq \mu_0 \\ H_1 : \pi < \pi_0 & & H_1 : \mu < \mu_0 \end{array}$$

#### Case3: Two-tailed test

For Example:

$$\begin{array}{ll} H_0 : \pi = \pi_0 & \text{Or} & H_0 : \mu = \mu_0 \\ H_1 : \pi \neq \pi_0 & & H_1 : \mu \neq \mu_0 \end{array}$$

### Step (2): Select a level of significance:

Level of significance ( $\alpha$ ) : The probability of rejecting the null hypothesis when it is true.

#### Risks in Decision Making Using Hypothesis Testing (Slide 12)

- **Type I Error**

- Reject a true null hypothesis
- A type I error is a “false alarm”
- The probability of a Type I Error is  $\alpha$ 
  - Called **level of significance** of the test
  - Set by researcher in advance

- **Type II Error**

- Failure to reject a false null hypothesis
- Type II error represents a “missed opportunity”
- The probability of a Type II Error is  $\beta$

**Possible Errors in Hypothesis Test Decision Making (Slide "13-14" and Textbook: Table 9.1 P"311")**

Possible Hypothesis Test Outcomes		
	Actual Situation	
Decision	$H_0$ True	$H_0$ False
Do Not Reject $H_0$	No Error Probability $1 - \alpha$	Type II Error Probability $\beta$
Reject $H_0$	Type I Error Probability $\alpha$	No Error Power $1 - \beta$

- The **confidence coefficient ( $1-\alpha$ )** is the probability of not rejecting  $H_0$  when it is true.
- The confidence level of a hypothesis test is  $(1-\alpha)*100\%$ .
- The **power of a statistical test ( $1-\beta$ )** is the probability of rejecting  $H_0$  when it is false.

### Type I & II Error Relationship (Slide 15)

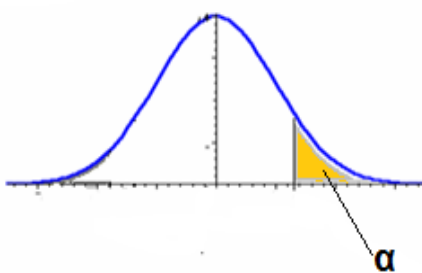
- Type I and Type II errors cannot happen at the same time
  - A Type I error can only occur if  $H_0$  is true
  - A Type II error can only occur if  $H_0$  is false

**Relationship between the alternative hypothesis and the level of significance ( $\alpha$ ):**

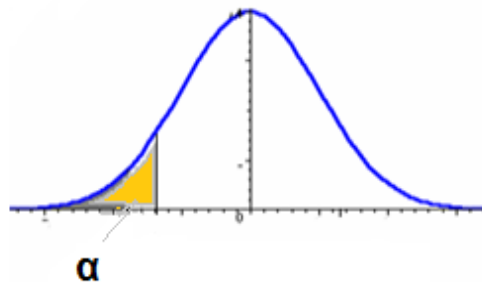
#### Case1 & 2:

If the alternative hypothesis is One- tailed test (Right or left-tailed); ( $\alpha$ ) appear in the one side of the curve.

Right tailed

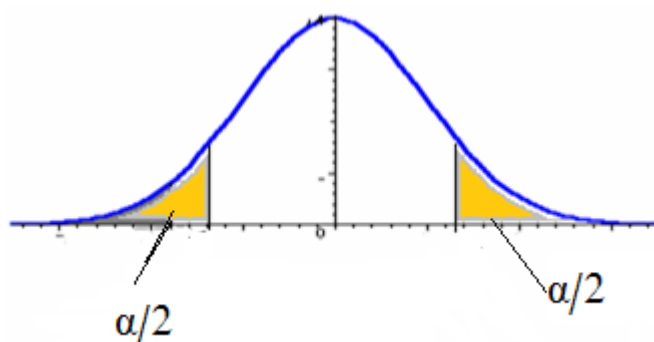


Left tailed



#### Case3:

If the alternative hypothesis is two- tailed test ( $\alpha$ ) appear in the left & right sides of the curve. We divide ( $\alpha$ ) by 2 ( $\alpha/2$ )



### Step3: Select the Test Statistic (computed value)

**The test Statistic:** A value, determined from sample information, used to determine whether to reject the null hypothesis.

There are many different statistical tests. The choice of which test to use depends on several factors:

- The type of data.
- The distribution of the data.
- The type of study design.

Example of test statistic used to test hypothesis: Z, t, F

### Step (4): Selected the Critical value

**Critical value:** The dividing point between the region where null hypothesis is rejected and the region where it is not rejected.

The critical value is read from the statistical tables and determined by:

- $\alpha$ .
- Type of test.
- Sometimes by the degrees of freedom.

### Step (5): Formulate the Decision Rule and Make a Decision

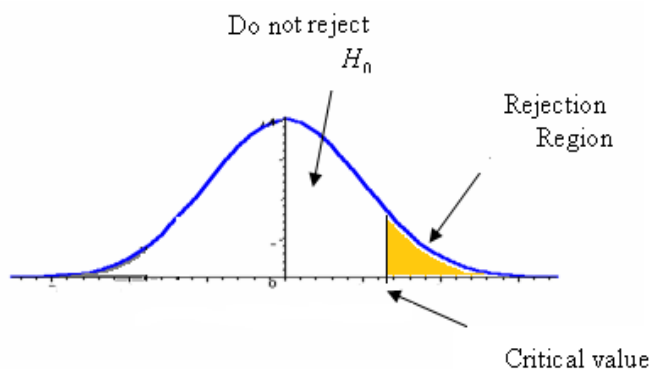
**Decision rule:**

We will compare the computed value with critical value (tabulated value) and make our decision:

If the test statistic falls in the rejection region, reject  $H_0$ ; otherwise do not reject  $H_0$  (Look the diagrams)

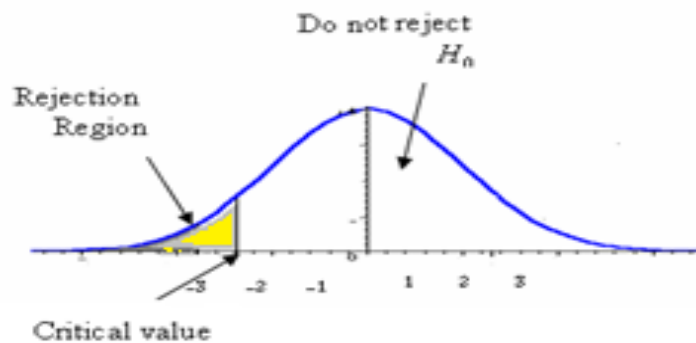
#### Case 1:

Reject  $H_0$  if T.S > (C.V)

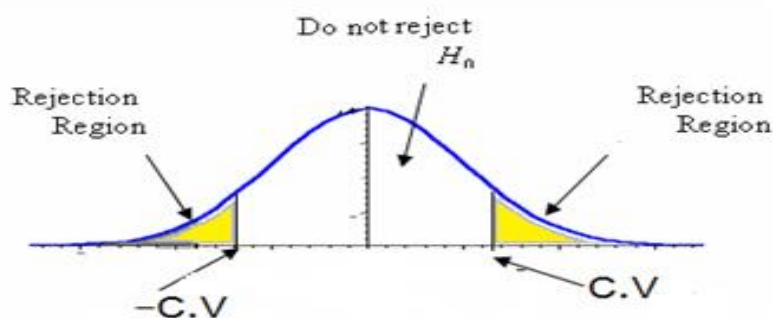


**Case 2:**

Reject  $H_0$  if  $T.S < - (C.V)$

**Case 3:**

Reject  $H_0$  if  $|T.S| > (C.V)$  which means  $T.S > (C.V)$  or  $T.S < - (C.V)$



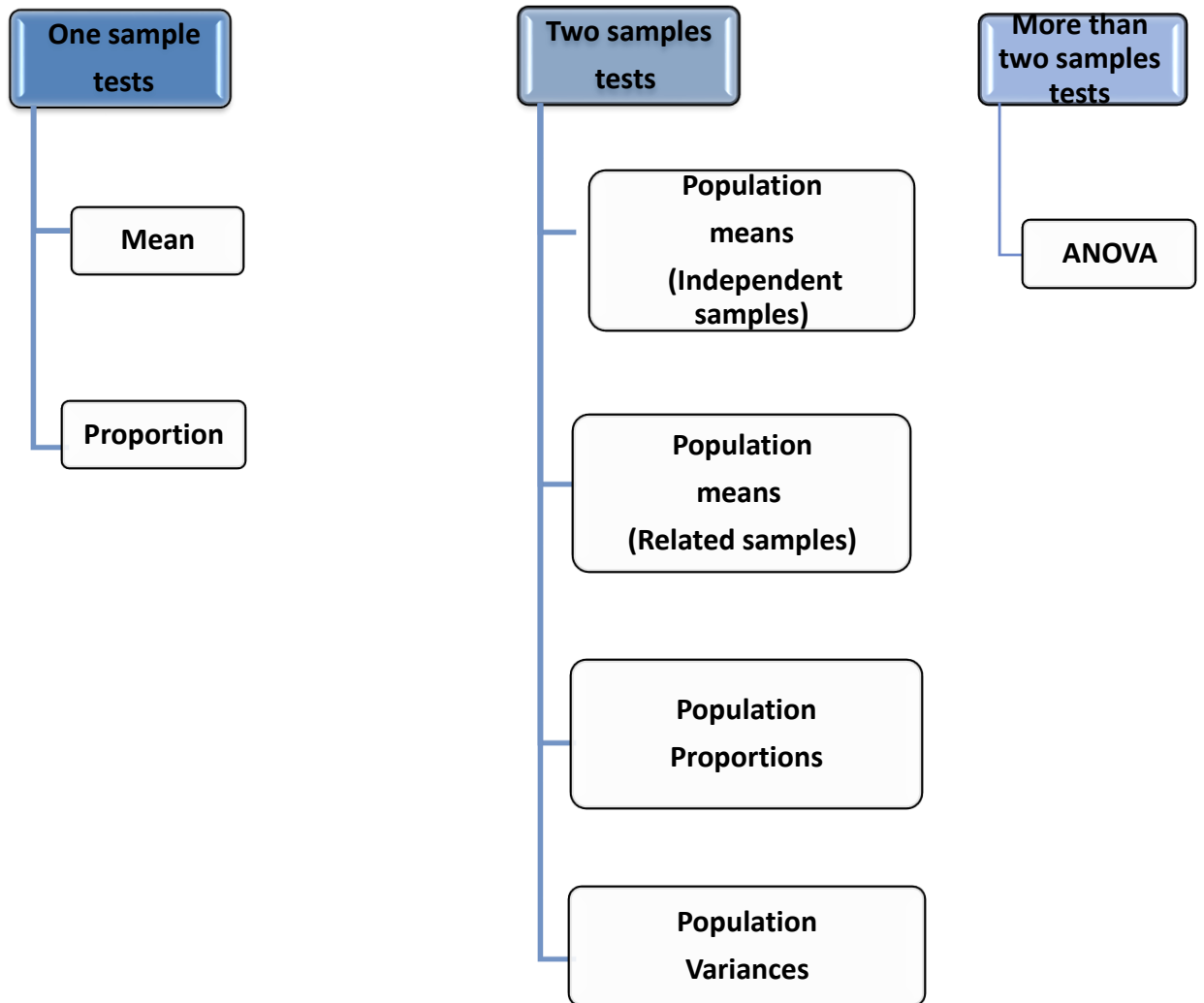
● Results of a Hypothesis test:

Either: Accept the null hypothesis ( $H_0$ ) as reasonable possibility	It is A weak conclusion; <b><u>not significant result</u></b> means the difference only cause of sampling error
Or: Reject the null hypothesis and Accept the researcher hypothesis ( $H_1$ )	It is A strong conclusion; <b><u>a significant result</u></b> (means there is difference)

**Note:**

Keywords	Inequality Symbol	Part of:
<i>Larger (or more) than</i>	$>$	$H_1$
<i>Smaller (or less)</i>	$<$	$H_1$
<i>No more than</i>	$\leq$	$H_0$
<i>At least</i>	$\geq$	$H_0$
<i>Has increased</i>	$>$	$H_1$
<i>Is there difference?</i>	$\neq$	$H_1$
<i>Has not changed</i>	$=$	$H_0$

**Types of Hypothesis Testing  
According to the number of  
samples**





## Lecture (2)

### Testing for a population mean

**Step (1):** State the null ( $H_0$ ) and alternate ( $H_1$ ) hypothesis

$$\begin{array}{lll} H_0 : \mu \leq \mu_0 & \text{or} & H_0 : \mu \geq \mu_0 & \text{or} & H_0 : \mu = \mu_0 \\ H_1 : \mu > \mu_0 & & H_1 : \mu < \mu_0 & & H_1 : \mu \neq \mu_0 \end{array}$$

**Step (2):** Select a level of significance.

**Step (3):** Select the Test Statistic (computed value)

- Known population Standard Deviation

$$Z_c = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

- unknown population Standard Deviation

$$t_c = \frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}}$$

**Note ( $\sigma$  Unknown)**

- If the population standard deviation is unknown, you instead use the sample standard deviation  $S$ .
- Because of this change, you use the  $t$  distribution instead of the  $Z$  distribution to test the null hypothesis about the mean.
- When using the  $t$  distribution you must assume the population you are sampling from follows a normal distribution.

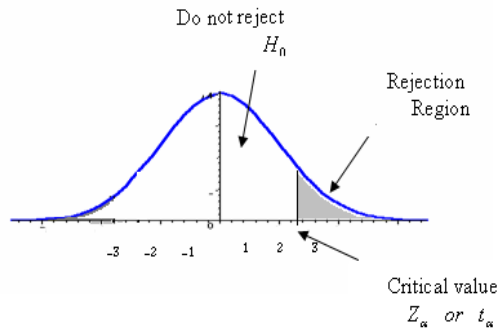
**Step (4):** Selected the Critical value

Types of test	<b>Z</b>	<b>t</b>
The one – tailed test (Right)	$Z_{\alpha}$	$t_{(\alpha, n-1)}$
The one – tailed test (left)	$- Z_{\alpha}$	$- t_{(\alpha, n-1)}$
The two – tailed test	$\pm Z_{\frac{\alpha}{2}}$	$\pm t_{(\frac{\alpha}{2}, n-1)}$

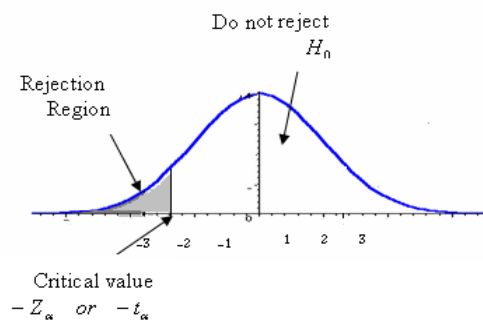
**Step (5):** Formulate the Decision Rule and Make a Decision

**Case1:** Reject  $H_0$  if  $|Z_c| > Z_{\alpha}$  ,  $|t_c| > t_{(v, \alpha)}$  ,  $v = n - 1$

This means that,  $Z_c > Z_{\alpha}$  ,  $t_c > t_{v, \alpha}$  ,



**Case2:** Reject  $H_0$  if  $|Z_c| > Z_\alpha$  ,  $|t_c| > t_{(v, \alpha)}$   
This means that,  $Z_c < -Z_\alpha$  ,  $t_c < -t_{v, \alpha}$

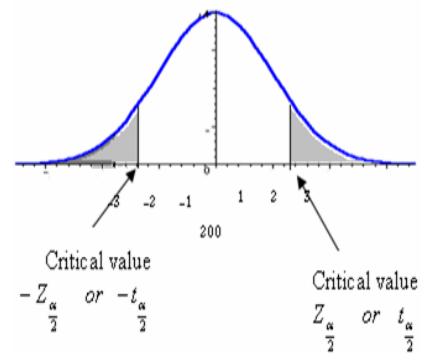


**Case3:** Reject  $H_0$  if  $|Z_c| > Z_{\frac{\alpha}{2}}$  ,  $|t_c| > t_{(v, \frac{\alpha}{2})}$  that is:

$$Z_c > Z_{\frac{\alpha}{2}} , t_c > t_{v, \frac{\alpha}{2}}$$

or

$$Z_c < -Z_{\frac{\alpha}{2}} , t_c < -t_{v, \frac{\alpha}{2}}$$



**Example (1)**

Jamestown Steel Company manufactures and assembles desks and other office equipment at several plants in western New York State. The weekly production of the Model A325 desk at the Fredonia Plant follows the normal probability distribution with a mean of 200 and a standard deviation of 16. Recently, because of market expansion, new production methods have been introduced and new employees hired. The vice president of manufacturing would like to investigate whether there has been a change in the weekly production of the Model A325 desk. Is the mean number of desks produced at Fredonia Plants **different** from 200 at the 0.01 significance level?

If the vice president takes sample of 50 weeks and he finds the mean is 203.5, use the statistical hypothesis testing procedure to investigate whether the production rate **has changed** from 200 per week.

**Solution:**

Step 1: State the null hypothesis and the alternate hypothesis.

$$H_0: \mu = 200$$

$$H_1: \mu \neq 200$$

This is Two-tailed test

(Note: keyword in the problem “has changed”, “different”)

Step 2: Select the level of significance.

$$\alpha = 0.01 \text{ as stated in the problem}$$

Step 3: Select the test statistic

Use Z-distribution since  $\sigma$  is known.

$$\mu_0 = 200, \quad \bar{X} = 203.5, \quad \sigma = 16, \quad n = 50$$

$$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{203.5 - 200}{\frac{16}{\sqrt{50}}} = \frac{3.5}{2.2627} = 1.55$$

Step 4: Formulate the decision rule (Critical Value)

$$\pm Z_{\frac{\alpha}{2}} = \pm Z_{\frac{0.01}{2}} = \pm Z_{0.005} = \pm 2.57$$

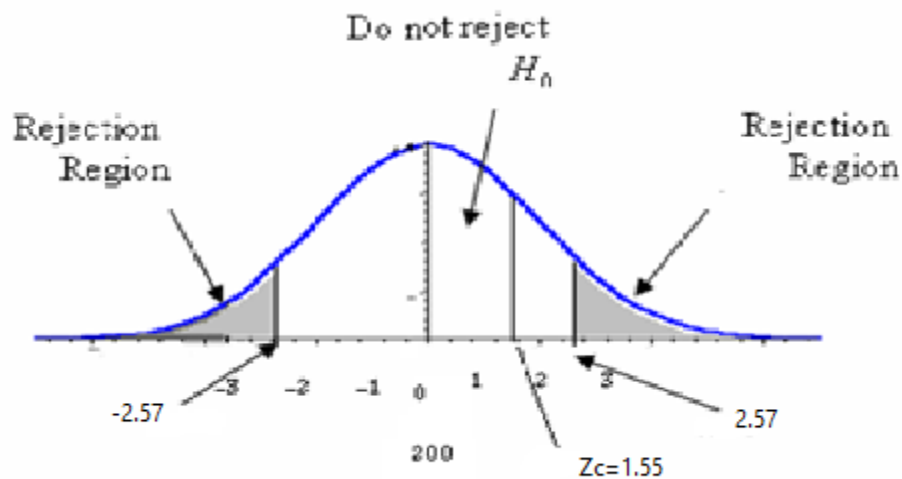
Reject  $H_0$  If  $Z_c < -Z_{\frac{\alpha}{2}}$  or  $Z_c > Z_{\frac{\alpha}{2}}$

$$(Z_c > 2.57 \text{ or } Z_c < -2.57)$$

Step 5: Make a decision and interpret the result.

$$\because Z_c(1.55) < Z_{0.005}(2.57)$$

Don't reject  $H_0$



Because 1.55 does not fall in the rejection region,  $H_0$  is not rejected at significance level 0.01. We conclude that the population mean is not different from 200. So, we would report to the vice president of manufacturing that the sample evidence does not show that the production rate at the Fredonia Plant has changed from 200 per week. The difference of 3.5 units between the historical weekly production rate and that last year can reasonably be attributed to sampling error.

### Example (2)

(Textbook: Example 9.3 – P 314)

### Example (3)

Suppose you are buyer of large supplies of light bulbs. You want to test at the 5% significant level, the manufacturer's claim that his bulbs last more than 800 hours, you test 36 bulbs and find that the sample mean is 816 hours, with standard deviations 70 hours. Should you accept the claim?

#### Solution:

Step 1: State the null hypothesis and the alternate hypothesis.

$$H_0 : \mu \leq 800$$

$$H_1 : \mu > 800$$

This is one-tailed test (right),

(Note: keyword in the problem "more than")

Step 2: Select the level of significance. ( $\alpha = 0.05$ )

Step 3: Select the test statistic.

Use t-distribution since  $\sigma$  is unknown

$$\mu_0 = 800, \quad \bar{X} = 816, \quad S = 70, \quad n = 36$$

$$t = \frac{\frac{\bar{X} - \mu_0}{S/\sqrt{n}}}{\frac{70}{\sqrt{36}}} = \frac{816 - 800}{11.6667} = 1.3714$$

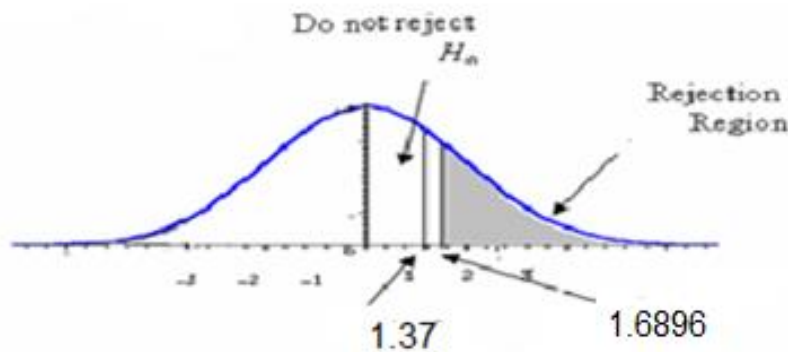
Step 4: Formulate the decision rule.

Reject  $H_0$  if  $t_c > t_\alpha$  ,  $t_{(n-1, \alpha)} = t_{(36-1, 0.05)} = t_{(35, 0.05)} = 1.6896$

Step 5: Make a decision and interpret the result.

Reject  $H_0$  if  $t_{stat} = 1.37 < 1.6896$

$H_0$  is not rejected



Because 1.37 does not fall in the rejection region,  **$H_0$  is not rejected**. We conclude that the population mean is less than or equal 800. The decision that we don't reject the null hypothesis at significant level 0.05, but we reject the alternative hypothesis the manufacturer's claim that  $\mu > 800$ .

The difference of 16 hours between sample and production can reasonably be attributed to sampling error.

#### Example (4)

The McFarland Insurance Company Claims Department reports the mean cost to process a claim is \$60. An industry comparison showed this amount to be larger than most other insurance companies, so the company instituted cost-cutting measures. To evaluate the effect of the cost-cutting measures, the Supervisor of the Claims Department selected a random sample of 26 claims processed last month. The sample information is reported below.

$n = 26$  ,  $\bar{X} = 56.42$  ,  $S = 10.04$

At the .01 significance level is it reasonable a claim is now **less than** \$60?

#### Solution:

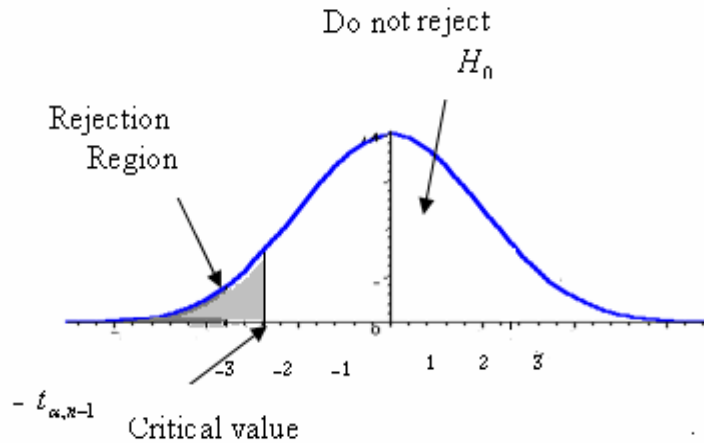
Step 1: State the null hypothesis and the alternate hypothesis.

$H_0 : \mu \geq \$60$

$H_1 : \mu < \$60$

This is one-tailed test (Left)

(Note: keyword in the problem "now less than")



Step 2: Select the level of significance. ( $\alpha = 0.01$ )

Step 3: Select the test statistic.

Use t-distribution since  $\sigma$  is unknown

$$\mu_0 = 60, \quad \bar{X} = 56.42, \quad S = 10.04, \quad n = 26$$

$$t_c = \frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}} = \frac{56.42 - 60}{\frac{10.04}{\sqrt{26}}} = \frac{-3.58}{1.9690} = -1.82$$

Step 4: Formulate the decision rule.

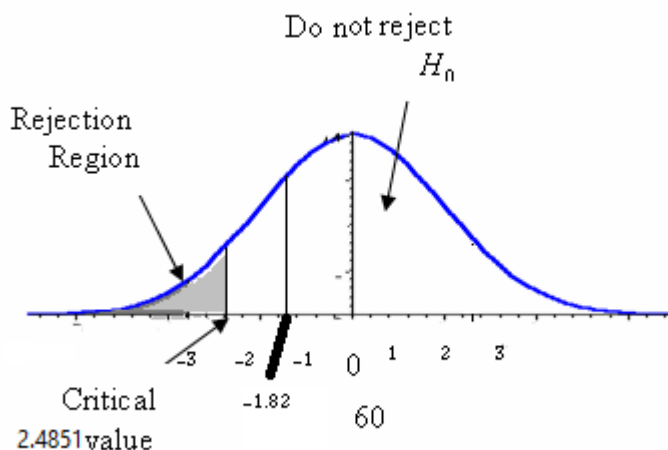
$$\text{Reject } H_0 \text{ if } t_c < -t_{\alpha, n-1}$$

$$-t_{\alpha, n-1} = -t_{0.01, 25} = 2.4851$$

Step 5: Make a decision and interpret the result.

$$\text{Reject } H_0 \text{ if } t_c < -t_{0.01, 25} \because t_c(-1.82) > -2.4851$$

$H_0$  is not rejected



Because -1.82 does not fall in the rejection region,  **$H_0$  is not rejected at the .01 significance level.** We have not demonstrated that the cost-cutting measures reduced the mean cost per claim to less than \$60. The difference of \$3.58 (\$56.42 - \$60) between the sample mean and the population mean could be due to sampling error.

**Example (5) (Slide 48-52)**

A phone industry manager **thinks** that customer monthly cell phone bills have **increased**, and now average over \$52 per month.

Suppose a sample is taken with the following results:

$$n = 25 \quad \bar{X} = 53.1 \quad \text{and} \quad S = 10$$

The company wishes to test this claim. (Assume a normal population and suppose that  $\alpha = 0.10$  is chosen for this test)

**Solution:**

Step 1: State the null hypothesis and the alternate hypothesis.

$H_0: \mu \leq 52$  the mean is not over \$52 per month

$H_1: \mu > 52$  the mean **is** greater than \$52 per month  
(i.e., sufficient evidence exists to support the manager's claim)

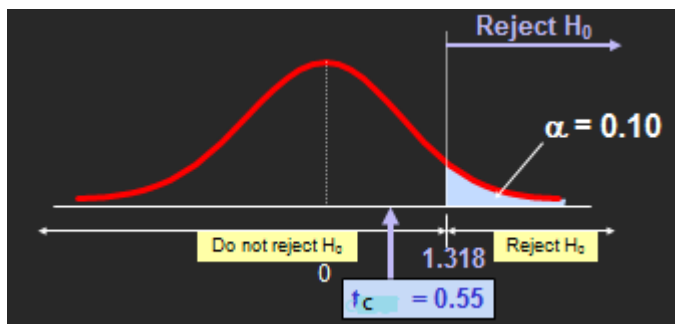
Step 2: Select the level of significance. ( $\alpha = 0.10$ )

Step 3: Select the test statistic.

$$t_c = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{25}}} = 0.55$$

Step 4: The Critical Value:  $t_{(24,0.10)} = 1.318$

Step 5: Make a decision and interpret the result.



**Do not reject  $H_0$  since  $t_c = 0.55 < 1.318$**

There is no sufficient evidence that the mean bill is over \$52

**Example (6): (Textbook: Example 9.5 - P"328-329")**

**Example (7) (Slide 39-40)**

The average cost of a hotel room in New York is said to be \$168 per night. To determine if this is true, a random sample of 25 hotels is taken and resulted in an  $\bar{X}$  of \$172.50 and an S of \$15.40. Test the appropriate hypotheses at  $\alpha = 0.05$ . (Assume the population distribution is normal)

**Solution:**

Step 1: State the null hypothesis and the alternate hypothesis.

$$H_0: \mu = 168 \quad H_1: \mu \neq 168$$

Step 2: Select the level of significance. ( $\alpha = 0.05$ .)

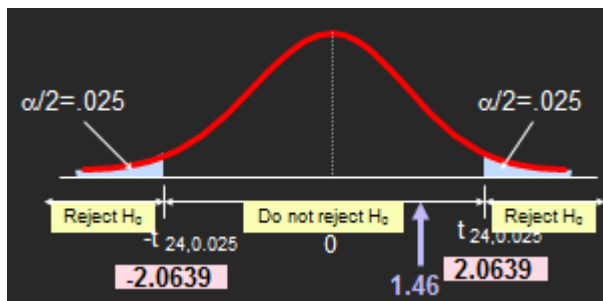
Step 3: Select the test statistic.

- $\sigma$  is unknown, so use a  $t$  statistic

$$t_c = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.4}{\sqrt{25}}} = 1.46$$

Step 4: The Critical Value:  $\pm t_{(24,0.025)} = \pm 2.0639$

Step 5: Make a decision and interpret the result.

**Do not reject  $H_0$ :**

Insufficient evidence that true mean cost is different from \$168.

**Connection of Two Tail Tests to Confidence Intervals**

For  $X = 172.5$ ,  $S = 15.40$  and  $n = 25$ , the 95% confidence interval for  $\mu$  is:

$$172.5 \pm (2.0639) \frac{15.4}{\sqrt{25}}$$

$$166.14 \leq \mu \leq 178.86$$

Since this interval contains the Hypothesized mean (168), we do not reject the null hypothesis at  $\alpha = 0.05$



## Lecture (3)

### P-Value Approach to Testing

**P-value:** Probability of obtaining a test statistic equal to or more extreme than the observed sample value given  $H_0$  is true

The p-value is also called the observed level of significance

It is the smallest value of  $\alpha$  for which  $H_0$  can be rejected (Slide 28)

In testing a hypothesis, we can also compare the  $p$ -value to with the significance level ( $\alpha$ ).

- If the  $p$ -value  $<$  significance level ( $\alpha$ ) ,  $H_0$  is rejected (means significant result)
- If the  $p$ -value  $\geq$  significance level ( $\alpha$ ),  $H_0$  is not rejected. (means not significant result).

Compute the P- value (Only we used the Z distribution)

**Case 1** : If the one – tailed test (Right)

$$P - value = P(Z > z_c)$$

**Case 2**: If the one – tailed test (left)

$$P - value = P(Z < -z_c)$$

**Case3**: If the two – tailed test

$$P - value = P(Z > z_c) + P(Z < -z_c)$$

**Note:**

The p-value is considered as a way to express the likelihood that  $H_0$  is false.

If the p-value is very large, then it is likely that  $H_0$  is true.

If the p-value is small, then it is likely that  $H_0$  is not true.

**Example (8)**

Refer to example (1)

$$H_0 : \mu = 200$$

$$H_1 : \mu \neq 200$$

$$\mu_0 = 200 \quad , \quad \bar{X} = 203.5 \quad , \quad \sigma = 16 \quad , \quad n = 50 \quad , \quad \alpha = 0.01$$

$$P - value = P(Z > z_c) + P(Z < -z_c)$$

$$P(Z > z_c) = P(Z > 1.55) = 1 - 0.9394 = 0.0606$$

$$P(Z < -z_c) = P(Z < -1.55) = 0.0606$$

$$P - value = 0.0606 + 0.0606 = 0.1212$$

$$P - value(0.1212) > \alpha(0.01)$$

Do not reject  $H_0$

**Example (9) Slide (31-33)**

Test the claim that the true mean diameter of a manufactured bolt is 30 mm. Used the **p-Value Approach** and connection between two tail tests and confidence intervals.

Suppose the sample results are

$$n = 100 \quad \bar{X} = 29.84 \quad (\text{Assume } \sigma = 0.8 \quad \alpha = 0.05)$$

**Solution:**

Step (1) : State the null hypothesis and the alternate hypothesis.

$$H_0: \mu = 30$$

$$H_1: \mu \neq 30$$

Step (2) : The level of significance. ( $\alpha = 0.05$ .)

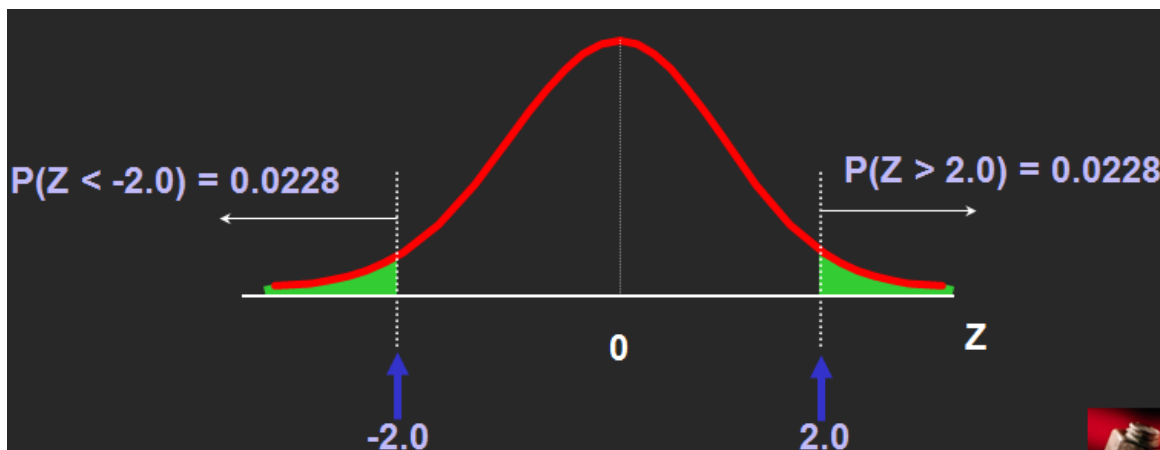
Step (3) : Select the test statistic and compute the P-value.

$\sigma$  is assumed known so this is a Z test.

$$Z_c = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{29.84 - 30}{\frac{0.8}{\sqrt{100}}} = -\frac{0.16}{0.08} = -2.0$$

$$\begin{aligned} P\text{-value} &= P(Z > z_c) + P(Z < -z_c) \\ &= P(Z > 2) + P(Z < -2) = 2(0.0228) = 0.0456 \end{aligned}$$

Step (4) : Make a decision and interpret the result.



Since  $p\text{-value} = 0.0456 < \alpha = 0.05$  Reject  $H_0$

There is sufficient evidence to conclude the mean diameter of a manufactured bolt is not equal to 30mm.

### Connection Between Two Tail Tests and Confidence Intervals

For  $\bar{X} = 29.84$ ,  $\sigma = 0.8$  and  $n = 100$ , the 95% confidence interval for  $\mu$  is:

$$\begin{aligned} &29.84 \pm (1.96) \frac{0.8}{\sqrt{100}} \\ &29.6832 \leq \mu \leq 29.9968 \end{aligned}$$

Since this interval does not contain the hypothesized mean (30), we reject the null hypothesis at  $\alpha = 0.05$

### Example (10): (Textbook: Example 9.4 - P"317")

## Lecture (4)

### Hypothesis Tests for Proportions

- Involves categorical variables
- Two possible outcomes
  - Possesses characteristic of interest
  - Does not possess characteristic of interest

Fraction or proportion of the population in the category of interest is denoted by  $\pi$  (Slide 55)

Sample proportion in the category of interest is denoted by  $p$

$$p = \frac{X}{n} = \frac{\text{number in category of interest in sample}}{\text{sample size}}$$

When both  $n\pi$  and  $n(1-\pi)$  are at least 5 ( $n\pi \geq 5$  and  $n(1-\pi) \geq 5$ )

$p$  can be approximated by a normal distribution with mean and standard deviation (Slide 56)

$$\mu_p = \pi$$

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

- The sampling distribution of  $p$  is approximately normal, so the test statistic is a  $Z_c$  value:

$$Z_c = \frac{P - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$$

Step (1): State the Null( $H_0$ ) and alternate( $H_1$ ) hypothesis

$$\begin{array}{lll} H_0 : \pi \leq \pi_0 & \text{or} & H_0 : \pi \geq \pi_0 & \text{or} & H_0 : \pi = \pi_0 \\ H_1 : \pi > \pi_0 & & H_1 : \pi < \pi_0 & & H_1 : \pi \neq \pi_0 \end{array}$$

Step (2): Select a level of significance:

Step (3): Select the Test Statistic (computed value)

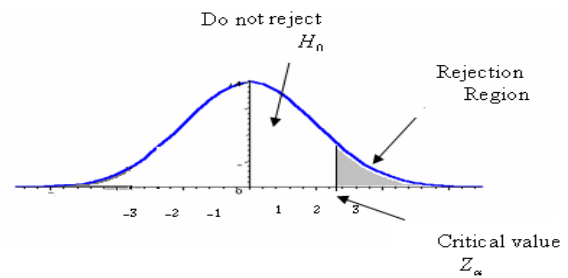
$$Z_c = \frac{P - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$$

Step (4): Selected the Critical value

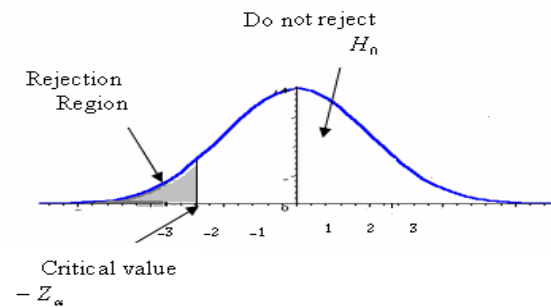
The one – tailed test (Right)	$Z_{\alpha}$
The one – tailed test (left)	$-Z_{\alpha}$
The two – tailed test	$\pm Z_{\alpha/2}$

Step (5): Formulate the Decision Rule and Make a Decision

Case1: Reject  $H_0$  if  $Z_c > Z_{\alpha}$

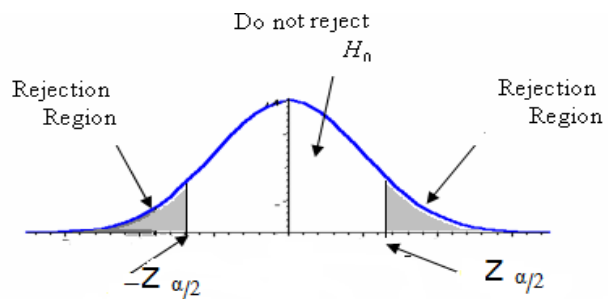


Case2: Reject  $H_0$  if  $|Z_c| > Z_{\alpha}$  or  $Z_c < -Z_{\alpha}$



Case3: Reject  $H_0$  if,

$$Z_c > Z_{\frac{\alpha}{2}} \quad \text{or} \quad Z_c < -Z_{\frac{\alpha}{2}}$$



**Example (11)**

Suppose prior elections in a certain state indicated it is necessary for a candidate for governor to receive **at least 80** percent of the vote in the northern section of the state to be elected. The incumbent governor is interested in assessing his chances of returning to office. A sample survey of 2,000 registered voters in the northern section of the state revealed that 1550 planned to vote for the incumbent governor. Using the hypothesis-testing procedure, assess the governor's chances of reelection (**the level of significance is 0.05**).

**Solution:**

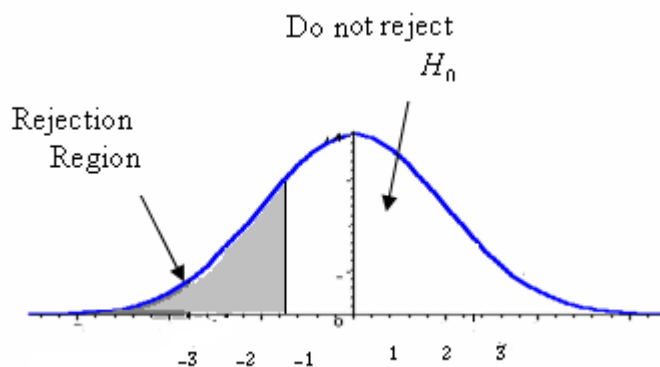
Step 1: State the null hypothesis and the alternate hypothesis.

$$H_0 : \pi \geq 0.80$$

$$H_1 : \pi < 0.80$$

This is one-tailed test (Left)

(Note: keyword in the problem “at least”)



Step 2: Select the level of significance.

$$\alpha = 0.05 \text{ as stated in the problem}$$

Step 3: Select the test statistic.

$$\pi = 0.80 \quad P = \frac{1550}{2000} = 0.78 \quad n = 2000$$

Use Z-distribution since the assumptions are met

and  $n\pi$  and  $n(1-\pi) \geq 5$

$$n\pi = 2000 * 0.80 = 1600 \text{ \& } n(1 - \pi) = 2000 * 0.20 = 400$$

$$Z_c = \frac{0.78 - 0.80}{\sqrt{\frac{0.8 * 0.2}{2000}}} = -2.24$$

Step 4: Formulate the decision rule.

Reject  $H_0$  if  $Z_c < -Z_\alpha$

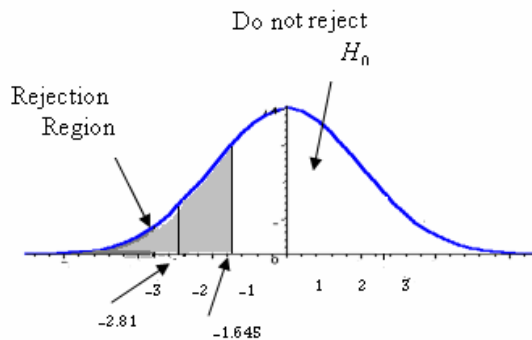
$$-Z_\alpha = -Z_{0.05} = -1.65$$

Step 5: Make a decision and interpret the result.

Reject  $H_0$  if

$$Z_c < -Z_{0.05}(-1.65) \quad \because Z_c(-2.24) < -1.65$$

We reject  $H_0$



Reject  $H_0$

The computed value of  $(-2.24)$  is in the rejection region, so the null hypothesis is rejected at the .05 level. The difference of 2 percentage points between the sample percent (78 percent) and the hypothesized population percent (80) is statistically significant. The evidence at this point does not support the claim that the incumbent governor will return to the governor's mansion for another four years.

$$P - value = P(Z < -z_c)$$

$$P - value = P(Z < -2.24) = 0.0125$$

$$P - value(0.0125) < \alpha(0.05)$$

We reject  $H_0$

### Example (12) (Slide 59-61)

A marketing company claims that it receives 8% responses from its mailing. To test this claim, a random sample of 500 were surveyed with 25 responses. Test at the  $\alpha = 0.05$  significance level.

#### Solution:

Step (1): State the null hypothesis and the alternate hypothesis.

$$H_0: \pi = 0.08$$

$$H_1: \pi \neq 0.08$$

Step (2): The level of significance: ( $\alpha = 0.05$ )

Step (3): The test statistic:

Check:

$$n\pi = (500)(0.08) = 40, \quad n(1-\pi) = (500)(.92) = 460$$

$$n = 500, \quad p = 25/500 = 0.05$$

$$Z_c = \frac{P - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.05 - 0.08}{\sqrt{\frac{0.08(1-0.08)}{500}}} = -2.47$$

Step (4): The Critical Values:  $Z_{0.025} = \pm 1.96$

Step (5): Make a decision and interpret the result. Reject  $H_0$  at  $\alpha = 0.05$   
There is sufficient evidence to reject the company's claim of 8% response rate.

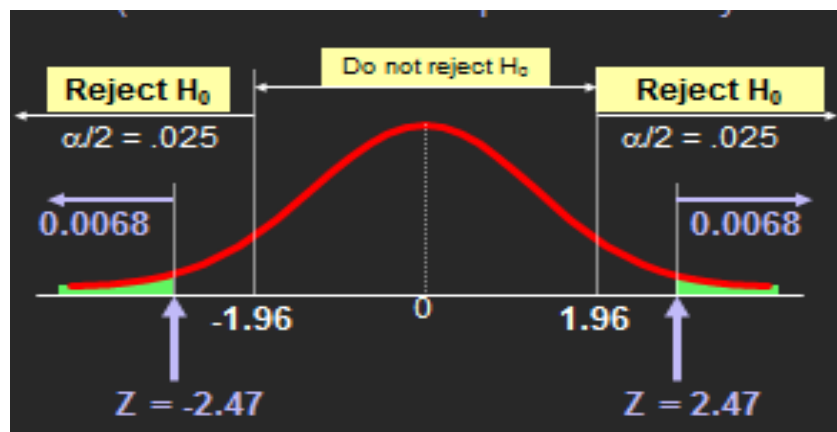


### p-Value Solution

Calculate the p-value and compare to  $\alpha$  (For a two-tail test the p-value is always two-tail)

$$\begin{aligned} P\text{-value} &= P(Z > z_c) + P(Z < -z_c) \\ &= P(Z > 2.47) + P(Z < -2.47) = 2(0.0068) = 0.0136 \end{aligned}$$

**Reject  $H_0$  since p-value = 0.0136 <  $\alpha = 0.05$**



**Example (13): (Textbook: Example 9.6 - P"332")**

Example (14)

Null Hypothesis	Does Not Reject $H_0$	Rejects $H_0$
$H_0$ is true	<p>Do not rejecting The null hypothesis, <math>H_0</math>, when It is true(<math>1 - \alpha</math>) Example  <math>\checkmark H_0 : \mu \geq 60</math>  <math>\times H_1 : \mu &lt; 60</math>                      If the decision is Do not reject <math>H_0</math>                      Let <math>\mu_1 = 80</math>  <math>\therefore</math> The decision is correct</p>	<p>rejecting The null hypothesis, <math>H_0</math>, when It is true(<math>\alpha</math>) Example  <math>\times H_0 : \mu \geq 60</math>  <math>\checkmark H_1 : \mu &lt; 60</math>                      If the decision is reject <math>H_0</math>                      Let <math>\mu_1 = 80</math>  <math>\therefore</math> The decision is incorrect rejecting</p>
$H_0$ is false	<p>Do not rejecting The null hypothesis, <math>H_0</math>, when It is false(<math>\beta</math>) Example  <math>\checkmark H_0 : \mu \geq 60</math>  <math>\times H_1 : \mu &lt; 60</math>                      If the decision is Do not reject <math>H_0</math>                      Let <math>\mu_1 = 50</math>  <math>\therefore</math> The decision is incorrect</p>	<p>rejecting The null hypothesis, <math>H_0</math>, when It is false (Power)(<math>1 - \beta</math>) Example  <math>\times H_0 : \mu \geq 60</math>  <math>\checkmark H_1 : \mu &lt; 60</math>                      If the decision is reject <math>H_0</math>                      Let <math>\mu_1 = 50</math>  <math>\therefore</math> The decision is correct rejecting</p>

References

Text Books:

- 1- David M Levine / Kathryn A. Szabat / David F. Stephan / **Business Statistics**, Pearson, Seventh Edition. PP(306-344)
- 2- Lind, Marchal and Wathen, **Statistical Techniques in Business and Economics**, McGraw Hill International, The Fourteenth Edition.