

*Chemistry, The Central Science*, 11th edition Theodore L. Brown; H. Eugene LeMay, Jr.; Bruce E. Bursten; Catherine J. Murphy



## **Chapter 17** Additional Aspects of Aqueous Equilibria

Ahmad Aqel Ifseisi Assistant Professor of Analytical Chemistry College of Science, Department of Chemistry King Saud University P.O. Box 2455 Riyadh 11451 Saudi Arabia Building: 05, Office: AA53 Tel. 014674198, Fax: 014675992 Web site: http://fac.ksu.edu.sa/aifseisi E-mail: ahmad3qel@yahoo.com aifseisi@ksu.edu.sa



The various aqueous solutions encountered in nature typically contain many solutes. For example, the aqueous solutions in hot springs and oceans.

The idea in this chapter is to consider not only solutions in which there is a single solute but also those containing a mixture of solutes.

In this chapter we take a step toward understanding such complex solutions by looking first at further applications of acid-base equilibria.

## 17.1 The Common Ion Effect

In chapter 16 we examined the equilibrium concentrations of ions in solutions containing a weak acid or a weak base. We now consider solutions that contain a weak acid and a soluble salt of that acid.

Sodium acetate is a soluble ionic compound and is therefore a strong electrolyte. Consequently, it dissociates completely in aqueous solution to form Na<sup>+</sup> and  $CH_3COO^-$  ions:

$$CH_3COONa(aq) \longrightarrow Na^+(aq) + CH_3COO^-(aq)$$

In contrast, CH<sub>3</sub>COOH is a weak electrolyte that ionizes as follows:

 $CH_3COOH(aq) \Longrightarrow H^+(aq) + CH_3COO^-(aq)$ 

According to Le Chatelier's principle, the  $CH_3COO^-$  from  $CH_3COONa$  causes this equilibrium to shift to the left, thereby decreasing the equilibrium concentration of H<sup>+</sup>(aq).

Addition of CH<sub>3</sub>COO<sup>-</sup> shifts equilibrium, reducing [H<sup>+</sup>]

In other words, the presence of the added acetate ion causes the acetic acid to ionize less than it normally would.

Whenever a weak electrolyte and a strong electrolyte contain a common ion, the weak electrolyte ionizes less than it would if it were alone in solution. We call this observation the **common-ion effect**.

The extent of ionization of a weak electrolyte is decreased by adding to the solution a strong electrolyte that has an ion in common with the weak electrolyte.

The ionization of a weak base is also decreased by the addition of a common ion. For example, the addition of  $NH_4^+$  (as from the strong electrolyte  $NH_4CI$ ) causes the base-dissociation equilibrium of  $NH_3$  to shift to the left, decreasing the equilibrium concentration of OH<sup>-</sup> and lowering the pH:

$$NH_3(aq) + H_2O(l) \Longrightarrow NH_4^+(aq) + OH^-(aq)$$



#### Sample Exercise 17.1 Calculating the pH When a Common Ion is Involved

What is the pH of a solution made by adding 0.30 mol of acetic acid and 0.30 mol of sodium acetate to enough water to make 1.0 L of solution?

#### **Solution**

First, because  $CH_3COOH$  is a weak electrolyte and  $CH_3COONa$  is a strong electrolyte, the major species in the solution are  $CH_3COOH$  (a weak acid),  $Na^+$  (which is neither acidic nor basic and is therefore a spectator in the acid–base chemistry), and  $CH_3COO^-$  (which is the conjugate base of  $CH_3COOH$ ).

Second,  $[H^+]$  and, therefore, the pH are controlled by the dissociation equilibrium of CH<sub>3</sub>COOH: (We have written the equilibrium Using H<sup>+</sup>(*aq*) rather than H<sub>3</sub>O<sup>+</sup>(*aq*) but both representations of the hydrated hydrogen ion are equally valid).

Third, we tabulate the initial and equilibrium concentrations (as we did in solving other equilibrium problems):

$$CH_3COOH(aq) \implies H^+(aq) + CH_3COO^-(aq)$$

	$CH_3COOH(aq) \equiv$	$\Rightarrow$ H <sup>+</sup> (aq)	+ $CH_3COO^-(aq)$
Initial	0.30 M	0	0.30 M
Change	-x M	+x M	+x M
Equilibrium	(0.30 - x) M	x M	(0.30 + x) M

The equilibrium concentration of  $CH_3COO^-$  (the common ion) is the initial concentration that is due to  $CH_3COONa$  (0.30 *M*) plus the change in concentration (*x*) that is due to the ionization of  $CH_3COOH$ .

Now we can use the equilibrium-constant expression:

(The dissociation constant for  $CH_3COOH$  at 25 °C is from Appendix D; addition of  $CH_3COONa$  does not change the value of this constant). Substituting the equilibrium-constant concentrations from our table into the equilibrium expression gives

$$K_a = 1.8 \times 10^{-5} = \frac{[H^+][CH_3COO^-]}{[CH_3COOH]}$$

$$K_a = 1.8 \times 10^{-5} = \frac{x(0.30 + x)}{0.30 - x}$$

Because  $K_a$  is small, we assume that x is small compared to the original concentrations of CH<sub>3</sub>COOH and CH<sub>3</sub>COO<sup>-</sup> (0.30 *M* each). Thus, we can ignore the very small x relative to 0.30 *M*, giving

The resulting value of x is indeed small relative to 0.30, justifying the approximation made in simplifying the problem.

The resulting value of x is indeed small relative to 0.30, justifying the approximation made in simplifying the problem.

Finally, we calculate the pH from the equilibrium concentration of  $H^+(aq)$ :

$$K_a = 1.8 \times 10^{-5} = \frac{x(0.30)}{0.30}$$
  
 $x = 1.8 \times 10^{-5} M = [H^+]$ 

$$pH = -log(1.8 \times 10^{-5}) = 4.74$$

**Comment:** In Section 16.6 we calculated that a 0.30 *M* solution of CH<sub>3</sub>COOH has a pH of 2.64, corresponding to  $H^+$ ] = 2.3 × 10<sup>-3</sup> *M*. Thus, the addition of CH<sub>3</sub>COONa has substantially decreased, [H<sup>+</sup>] as we would expect from Le Châtelier's principle.

#### **Sample Exercise 17.2** Calculating Ion Concentrations When a Common is Involved

Calculate the fluoride ion concentration and pH of a solution that is 0.20 *M* in HF and 0.10 *M* in HCl.

#### Solution

Because HF is a weak acid and HCl is a strong acid, the major species in solution  $HF(aq) \implies H^+(aq) + F^-(aq)$ are HF, H+, and Cl<sup>-</sup>. The Cl<sup>-</sup>, which is the conjugate base of a strong acid, is merely spectator ion in any acid-base chemistry. The problem asks for [F<sup>-</sup>], which is formed by ionization of HF. Thus, the important equilibrium is

The common ion in this problem is the hydrogen (or hydronium) ion. Now we can tabulate the initial and equilibrium concentrations of each species involved in this equilibrium:

The equilibrium constant for the ionization of HF, from Appendix D, is 6.8 10<sup>-4</sup>. Substituting the equilibrium-Х concentrations into the constant equilibrium expression gives

	HF(aq) =	$\Rightarrow$ H <sup>+</sup> (aq)	+ F <sup>-</sup> (aq)
Initial	0.20 M	0.10 M	0
Change	-x M	+x M	+x M
Equilibrium	(0.20 - x) M	(0.10 + x) M	x M

$$K_a = 6.8 \times 10^{-4} = \frac{[\text{H}^+][\text{F}^-]}{[\text{HF}]} = \frac{(0.10 + x)(x)}{0.20 - x}$$
$$\frac{(0.10)(x)}{0.20} = 6.8 \times 10^{-4}$$
$$x = \frac{0.20}{0.10} (6.8 \times 10^{-4}) = 1.4 \times 10^{-3} M = [\text{F}^-]$$

If we assume that x is small relative to 0.10 or 0.20 M, this expression simplifies to

This  $F^-$  concentration is substantially smaller than it would be in a 0.20 *M* solution of HF with no added HCl.

The common ion,  $H^+$ , suppresses the ionization of HF. The concentration of  $H^+(aq)$  is

 $[H^+] = (0.10 + x) M \simeq 0.10 M$ pH = 1.00

Thus,

**Comment:** Notice that for all practical purposes, [H<sup>+</sup>] is due entirely to the HCl; the HF makes a negligible contribution by comparison.

## 17.2 Buffered Solutions

Buffers are solutions of a weak conjugate acid-base pair.

They are particularly resistant to pH changes, even when strong acid or base is added.

Solutions, which contain a weak conjugate acid-base pair, can resist drastic changes in pH upon the addition of small amounts of strong acid or base. These solutions are called **buffered solutions** (or merely buffers).



Much of the chemical behavior of seawater is determined by its pH, buffered at about 8.1 to 8.3 near the surface. Buffered solutions find many important applications in the laboratory and in medicine.

Human blood is a complex aqueous mixture, it is slightly basic with a normal pH of 7.35 to 7.45. death may result if the blood pH falls below 6.8 or rises above 7.8. when the pH falls below 7.35, the condition is called acidosis; when it rises above 7.45, the condition is called alkalosis.

The major buffer system that used to control the pH of blood is the carbonic acid-bicarbonate buffer system. The important equilibria in this buffer system are:



A scanning electromicrograph of a group red blood cells

 $H^+(aq) + HCO_3^-(aq) \Longrightarrow H_2CO_3(aq) \Longrightarrow H_2O(l) + CO_2(g)$ 

## **Composition and Action of Buffered Solutions**

A buffer resists changes in pH because it contains both an acid to neutralize OH<sup>-</sup> ions and a base to neutralize H<sup>+</sup> ions.

The acid and base that make up the buffer, however, must not consume each other through a neutralization reaction. These requirements are fulfilled by a weak acid-base conjugate pair such as

 $CH_3COOH-CH_3COO^-$  or  $NH_4^+-NH_3$ 

Thus, buffers are often prepared by mixing a weak acid or a weak base with a salt of that acid or base.

The  $CH_3COOH-CH_3COO^-$  buffer can be prepared, for example, by adding  $CH_3COONa$  to a solution of  $CH_3COOH$ . The  $NH_4^+-NH_3$  buffer can be prepared by adding  $NH_4CI$  to a solution of  $NH_3$ .

To understand better how a buffer works, let's consider a buffer composed of a weak acid (HX) and one of its salt (MX, where M<sup>+</sup> could be Na<sup>+</sup>, K<sup>+</sup>, or another cation). The acid-dissociation equilibrium in this buffered solution involves both the acid and its conjugate base:

 $HX(aq) \Longrightarrow H^+(aq) + X^-(aq)$ 

The corresponding acid-dissociation-constant expression is

$$K_a = \frac{[\mathrm{H}^+][\mathrm{X}^-]}{[\mathrm{H}\mathrm{X}]}$$

Solving this expression for [H<sup>+</sup>], we have

$$[\mathrm{H}^+] = K_a \frac{[\mathrm{HX}]}{[\mathrm{X}^-]}$$

We see from this exception that [H<sup>+</sup>], and thus the pH, is determined by two factors: the value of  $K_a$  for the weak-acid component of the buffer and the ratio of the concentrations of the conjugate acid-base pair, [HX] / [X<sup>-</sup>].



If OH<sup>-</sup> ions are added to the buffered solution, they react with the acid component of the buffer to produce water and X<sup>-</sup>:

 $OH^{-}(aq) + HX(aq) \longrightarrow H_2O(l) + X^{-}(aq)$ added base weak acid in buffer

This reaction causes [HX] to decrease and [X<sup>-</sup>] to increase. As long as the amounts of HX and X<sup>-</sup> in the buffer are large compared to the amount of OH<sup>-</sup> added, however, the ratio [HX] / [X<sup>-</sup>] does not change much, and thus the change in pH is small. If H<sup>+</sup> ions are added, they react with the base component of the buffer:

$H^+(aq)$	+	$X^{-}(aq)$	$\longrightarrow$	HX(aq)
added base		weak acid in buffer		

This reaction can also be represented using  $H_3O^+$ :

$$\mathrm{H}_{3}\mathrm{O}^{+}(aq) + \mathrm{X}^{-}(aq) \longrightarrow \mathrm{H}\mathrm{X}(aq) + \mathrm{H}_{2}\mathrm{O}(l)$$

Using either equation, we see that the reaction causes [X-] to decrease and [HX] to increase. As long as the change in the ratio [HX] / [X-] is small, the change in pH will be small.

## Calculating the pH of a Buffer

$$HX(aq) \Longrightarrow H^+(aq) + X^-(aq) \qquad K_a = \frac{[H^+][X^-]}{[HX]} \qquad [H^+] = K_a \frac{[HX]}{[X^-]}$$

Taking -log of both sides of equation, we have

$$-\log[\mathrm{H}^+] = -\log\left(K_a \frac{[\mathrm{HX}]}{[\mathrm{X}^-]}\right) = -\log K_a - \log \frac{[\mathrm{HX}]}{[\mathrm{X}^-]}$$

Because 
$$-\log[H^+] = pH$$
 and  $-\log K_a = pK_{a\nu}$  we have

$$pH = pK_a - \log \frac{[HX]}{[X^-]} = pK_a + \log \frac{[X^-]}{[HX]}$$

In general,

$$pH = pK_a + \log \frac{[base]}{[acid]}$$

$$pH = pK_a + \log \frac{[base]}{[acid]}$$

Where [acid] and [base] refer to the equilibrium concentrations of the conjugate acid-base pair. Note that when [base] = [acid],  $pH = pK_a$ .

Last Equation is known as the **Henderson-Hasselbalch equation**.

Biologists, biochemists, and others who work frequently with buffers often use this equation to calculate the pH of buffers.

In doing equilibrium calculations, we have seen that we can normally neglect the amounts of the acid and base of the buffer that ionize. Therefore, we can usually use the starting concentrations of the acid and base components of the buffer directly in the Henderson-Hasselbalch equation.

#### Sample Exercise 17.3 Calculating the pH of a Buffer

What is the pH of a buffer that is 0.12 *M* in lactic acid [CH<sub>3</sub>CH(OH)COOH, or HC<sub>3</sub>H<sub>5</sub>O<sub>3</sub>] and 0.10 *M* in sodium lactate [CH<sub>3</sub>CH(OH)COONa or NaC<sub>3</sub>H<sub>5</sub>O<sub>3</sub>]? For lactic acid,  $K_a = 1.4 \times 10^{-4}$ .

#### **Solution**

The initial and equilibrium concentrations of the species involved in this equilibrium are

	$HC_3H_5O_3(aq)$	$\rightleftharpoons$ H <sup>+</sup> (aq)	) + $C_3H_5O_3^{-}(aq)$
Initial	0.12 M	0	0.10 M
Change	-x M	+ <i>x M</i>	+x M
Equilibrium	(0.12 - x) M	x M	(0.10 + x) M

The equilibrium concentrations are governed by the equilibrium expression:

Because  $K_a$  is small and a common ion is present, we expect x to be small relative to either 0.12 or 0.10 *M*. Thus, our equation can be simplified to give

Solving for *x* gives a value that justifies our approximation:

Alternatively, we could have used the Henderson–Hasselbalch equation to calculate pH directly:

$$K_a = 1.4 \times 10^{-4} = \frac{[\mathrm{H}^+][\mathrm{C}_3\mathrm{H}_5\mathrm{O}_3]}{[\mathrm{H}\mathrm{C}_3\mathrm{H}_5\mathrm{O}_3]} = \frac{x(0.10 + x)}{(0.12 - x)}$$

$$K_a = 1.4 \times 10^{-4} = \frac{x(0.10)}{0.12}$$

$$[H^+] = x = \left(\frac{0.12}{0.10}\right)(1.4 \times 10^{-4}) = 1.7 \times 10^{-4} M$$
  
pH = -log(1.7 × 10<sup>-4</sup>) = 3.77

$$pH = pK_a + \log\left(\frac{[base]}{[acid]}\right) = 3.85 + \log\left(\frac{0.10}{0.12}\right)$$
$$= 3.85 + (-0.08) = 3.77$$

#### Sample Exercise 17.4 Preparing a Buffer

How many moles of  $NH_4Cl$  must be added to 2.0 L of 0.10  $MNH_3$  to form a buffer whose pH is 9.00? (Assume that the addition of  $NH_4Cl$  does not change the volume of the solution.)

#### **Solution**

The key to this exercise is to use this  $K_b$  expression to calculate [NH<sub>4</sub><sup>+</sup>].

NH

We obtain  $[OH^-]$  from the given pH: and so

Because  $K_b$  is small and the common ion NH<sub>4</sub><sup>+</sup> is present, the equilibrium concentration of NH<sub>3</sub> will essentially equal its initial concentration:

We now use the expression for  $K_b$  to calculate [NH<sub>4</sub><sup>+</sup>]:

Thus, for the solution to have pH = 9.00,  $[NH_4^+]$  must equal 0.18 *M*. The number of moles of NH<sub>4</sub>Cl needed to produce this concentration is given by the product of the volume of the solution and its molarity:

$$g_{3}(aq) + H_{2}O(l) \Longrightarrow NH_{4}^{+}(aq) + OH^{-}(aq) \qquad K_{b} = \frac{[NH_{4}^{+}][OH^{-}]}{[NH_{3}]} = 1.8 \times 10^{-5}$$

$$pOH = 14.00 - pH = 14.00 - 9.00 = 5.00$$

$$[OH^{-}] = 1.0 \times 10^{-5} M$$

$$[NH_{3}] = 0.10 M$$

$$[NH_{4}^{+}] = K_{b} \frac{[NH_{3}]}{[OH^{-}]} = (1.8 \times 10^{-5}) \frac{(0.10 M)}{(1.0 \times 10^{-5} M)} = 0.18 M$$

$$(2.0 L)(0.18 mol NH_{4}Cl/L) = 0.36 mol NH_{4}Cl$$

## **Buffer Capacity and pH Range**

**Buffer capacity** is the amount of acid or base the buffer can neutralize before the pH begins to change to an appreciable degree. The buffer capacity depends on the amount of acid and base from which the buffer is made.

The pH of the buffer depends on the  $K_a$  for the acid and on the relative concentrations of the acid and base that comprise the buffer.

According to the Equation below, [H<sup>+</sup>] for a 1-L solution that is 1 M in  $CH_3COOH$  and 1 M in  $CH_3COONa$  will be the same as for a 1-L solution that is 0.1 M in  $CH_3COOH$  and 1.0 M in  $CH_3COONa$ . The first solution has a greater buffering capacity, however, because it contains more  $CH_3COOH$  and  $CH_3COO^-$ .

The greater the amounts of the conjugate acid-base pair, the more resistant is the ratio of their concentrations, and hence the pH, is to change.

$$[\mathrm{H}^+] = K_a \frac{[\mathrm{HX}]}{[\mathrm{X}^-]}$$

The **pH range** of any buffer is the pH range over which the buffer system works or acts effectively.

Buffers most effectively resist a change in pH in either direction when the concentrations of weak acid and conjugate base are about the same.

From Equation below, when the concentrations of weak acid and conjugate base are equal,  $pH = pK_a$ . This relationship gives the optimal pH of any buffer. Thus, we usually try to select a buffer whose acid form has a  $pK_a$  close to the desired pH.

In practice, we find that if the concentration of one component of the buffer is more than 10 times the concentration of the other component, the buffering action is poor. Because log 10 = 1, buffers usually have a usable range within ±1 pH unit of pK<sub>a</sub> (that is, a range of pH = pK<sub>a</sub> ±1).

$$pH = pK_a + \log \frac{[base]}{[acid]}$$

## **Addition of Strong Acids or Bases to Buffers**



When a strong acid is added to the buffer, the added  $H^+$  is consumed by X<sup>-</sup> to produce HX; thus, [HX] increases and [X<sup>-</sup>] decreases.

When a strong base is added to the buffer, the added OH<sup>-</sup> is consumed by HX to produce X<sup>-</sup>; thus, [HX] decreases and [X<sup>-</sup>] increases.

#### Sample Exercise 17.5 Calculating pH Changes in Buffers

A buffer is made by adding 0.300 mol  $CH_3COOH$  and 0.300 mol  $CH_3COONa$  to enough water to make 1.00 L of solution. The pH of the buffer is 4.74 (Sample Exercise 17.1). (a) Calculate the pH of this solution after 0.020 mol of NaOH is added. (b) For comparison, calculate the pH that would result if 0.020 mol of NaOH were added to 1.00 L of pure water (neglect any volume changes).

#### **Solution**



Stoichiometry Calculation: The OH<sup>-</sup> provided by NaOH reacts with CH<sub>3</sub>COOH, the weak acid component of the buffer. Prior to this neutralization reaction, there are 0.300 mol each of CH<sub>3</sub>COOH and CH<sub>3</sub>COO<sup>-</sup>. Neutralizing the 0.020 mol OH<sup>-</sup> requires 0.020 mol of CH<sub>3</sub>COOH. Consequently, the amount of CH<sub>3</sub>COOH *decreases* by 0.020 mol, and the amount of the product of the neutralization, CH<sub>3</sub>COO<sup>-</sup>, *increases* by 0.020 mol. We can create a table to see how the composition of the buffer changes as a result of its reaction with OH<sup>-</sup>:

 $CH_3COOH(aq) + OH^-(aq) \longrightarrow H_2O(l) + CH_3COO^-(aq)$ 

Buffer before addition	0.300 mol	0	7 <u></u> 68	0.300 mol
Addition	3 <del>1</del> 20	0.020 mol		_
Buffer after addition	0.280 mol	0		0.320 mol

*Equilibrium Calculation:* We now turn our attention to the equilibrium that will determine the pH of the buffer, namely the ionization of acetic acid.

 $CH_3COOH(aq) \implies H^+(aq) + CH_3COO^-(aq)$ 

Using the quantities of  $CH_3COOH$  and  $CH_3COO^-$  remaining in the buffer, we can determine the pH using the Henderson–Hasselbalch equation.

$$pH = 4.74 + \log \frac{0.320 \text{ mol}/1.00 \text{ L}}{0.280 \text{ mol}/1.00 \text{ L}} = 4.80$$

pH = 4.68

0.320 M CH<sub>3</sub>COOH 0.280 M CH<sub>3</sub>COONa **Comment** Notice that we could have used mole amounts in place of concentrations in the Henderson– Hasselbalch equation and gotten the same result. The volumes of the acid and base are equal and cancel. If 0.020 mol of H<sup>+</sup> was added to the buffer, we would proceed in a similar way to calculate the resulting pH of the buffer. In this case the pH decreases by 0.06 units, giving pH = 4.68, as shown in the figure in the margin.

(b) To determine the pH of a solution made by adding 0.020 mol of NaOH to 1.00 L of pure water, we can first determine pOH using Equation 16.18 and subtracting from 14.

$$pH = 14 - (-log 0.020) = 12.30$$

Note that although the small amount of NaOH changes the pH of water significantly, the pH of the buffer changes very little.

## **17.3 Solubility Equilibria**

The equilibria that we have considered thus far in this chapter have involved acids and bases. Furthermore, they have been homogeneous; that is, all the species have been in the same phase.

Through the rest of this chapter we will consider the equilibria involved in the dissolution or precipitation of ionic compounds. These reactions are heterogeneous.

## The Solubility Product Constant, K<sub>sp</sub>

A saturated solution is one in which the solution is in contact with undissolved solute. Consider, for example, a saturated aqueous solution of  $BaSO_4$  that is in contact with solid  $BaSO_4$ . Because the solid is an ionic compound, it is a strong electrolyte and yields  $Ba^{2+}(aq)$  and  $SO_4^{2-}(aq)$  ions upon dissolving.

The following equilibrium is readily established between the undissolved solid and hydrated ions in solution:

$$BaSO_4(s) \Longrightarrow Ba^{2+}(aq) + SO_4^{2-}(aq)$$

Because this equilibrium equation describes the dissolution of a solid, the equilibrium constant indicates how soluble the solid is in water and is referred to as the **solubility-product constant** (or the **solubility product**). It is denoted  $K_{sp}$ , where *sp* stand for solubility product.

The equilibrium constant expression for this equilibrium is

 $K_{sp} = [Ba^{2+}] [SO_4^{2-}]$ 

In general, the solubility product of a compound equals the product of the concentration of the ions involved in the equilibrium, each raised to the power of its coefficient in the equilibrium equation. The coefficient for each ion in the equilibrium equation also equals its subscript in the compound's chemical formula.

The value of  $K_{sp}$  for BaSO<sub>4</sub> is 1.1 x 10<sup>-10</sup>, a very small number, indicating that only a very small amount of the solid will dissolve in water.

**Sample Exercise 17.9** Writing Solubility-Product ( $K_{sp}$ ) Expressions

Write the expression for the solubility-product constant  $K_{sp}$  for CaF<sub>2</sub>.

#### **Solution**

$$CaF_2(s) \Longrightarrow Ca^{2+}(aq) + 2 F^{-}(aq)$$

Following the italicized rule stated previously, the expression for is

$$K_{sp} = [Ca^{2+}][F^{-}]^2$$

In Appendix D we see that this  $K_{sp}$  has a value of 3.9  $\times$  10<sup>-11</sup>.

## **Solubility and K**<sub>sp</sub>

What is the difference between solubility and solubility-product constant  $K_{sp}$ 

- The solubility of a substance is the quantity that dissolves to form a saturated solution. Solubility is often expressed as grams of solute that dissolve in forming a liter of saturated solution of the solute (g/L).

- The solubility-product constant is the equilibrium constant for the equilibrium between an ionic solid and its saturated solution and is a unitless number. Thus, the magnitude of  $K_{sp}$  is a measure of how much of the solid dissolves to form a saturated solution.

#### Relationships between solubility and $K_{sp}$



*K<sub>sp</sub>* is *not* the same as solubility.

The solubility of a substance can change considerably as the concentrations of other solutes change.

The solubility of Mg(OH)<sub>2</sub>, for example, depends highly on pH, and also affected by the concentrations of other ions in solution. In contrast, the solubility-product constant,  $K_{sp}$ , has only one value for a given solute at any specific temperature.

The concentrations of ions calculated from  $K_{sp}$  sometimes deviate appreciably from those found experimentally. In part, these deviations are due to electrostatic interactions between ions in solution which can lead to ion pairs.

#### **Sample Exercise 17.10** Calculating $K_{sp}$ from Solubility

Solid silver chromate is added to pure water at 25 °C. Some of the solid remains undissolved at the bottom of the flask. The mixture is stirred for several days to ensure that equilibrium is achieved between the undissolved Ag<sub>2</sub>CrO<sub>4</sub>(*s*) and the solution. Analysis of the equilibrated solution shows that its silver ion concentration is  $1.3 \times 10^{-4} M$ . Assuming that Ag<sub>2</sub>CrO<sub>4</sub> dissociates completely in water and that there are no other important equilibria involving the Ag<sup>+</sup> or CrO4<sup>2-</sup> ions in the solution, calculate  $K_{sp}$  for this compound.

#### **Solution**

The equilibrium equation and the expression for  $K_{sp}$  are

$$\operatorname{Ag_2CrO_4(s)} \Longrightarrow 2\operatorname{Ag^+(aq)} + \operatorname{CrO_4^{2-}(aq)} \qquad K_{sp} = [\operatorname{Ag^+}]^2[\operatorname{CrO_4^{2-}}]$$

To calculate  $K_{sp}$ , we need the equilibrium concentrations of Ag<sup>+</sup> and CrO<sub>4</sub><sup>2-</sup>. We know that at equilibrium [Ag<sup>+</sup>] = 1.3 × 10<sup>-4</sup> *M*. All the Ag<sup>+</sup> and CrO<sub>4</sub><sup>2-</sup> ions in the solution come from the Ag<sub>2</sub>CrO<sub>4</sub> that dissolves. Thus, we can use [Ag<sup>+</sup>] to calculate [CrO<sub>4</sub><sup>2-</sup>].

From the chemical formula of silver chromate, we know that there must be 2 Ag<sup>+</sup> ions in solution for each  $\text{CrO}_4^{2-}$  ion in solution. Consequently, the concentration Of  $\text{CrO}_4^{2-}$  is half the concentration of Ag<sup>+</sup>:

$$[\mathrm{CrO_4}^{2^-}] = \left(\frac{1.3 \times 10^{-4} \operatorname{mol} \mathrm{Ag}^+}{\mathrm{L}}\right) \left(\frac{1 \operatorname{mol} \mathrm{CrO_4}^{2^-}}{2 \operatorname{mol} \mathrm{Ag}^+}\right) = 6.5 \times 10^{-5} \, M$$

We can now calculate the value of  $K_{sp}$ .

 $K_{sp} = [Ag^+]^2 [CrO_4^{2-}] = (1.3 \times 10^{-4})^2 (6.5 \times 10^{-5}) = 1.1 \times 10^{-12}$ 

#### **Sample Exercise 17.11** Calculating Solubility from $K_{sp}$

The  $K_{sp}$  for CaF<sub>2</sub> is 3.9 × 10<sup>-11</sup> at 25 °C. Assuming that CaF<sub>2</sub> dissociates completely upon dissolving and that there are no other important equilibria affecting its solubility, calculate the solubility of CaF<sub>2</sub> in grams per liter.

#### Solution

Assume initially that none of the salt has dissolved, and then allow x moles/liter of CaF<sub>2</sub> to dissociate completely when equilibrium is achieved.

The stoichiometry of the equilibrium dictates that 2x moles/liter of F<sup>-</sup> are produced for each x moles/liter of CaF<sub>2</sub> that dissolve. We now use the expression for  $K_{sp}$  and substitute the equilibrium concentrations to solve for the value of x:

(Remember that  $\sqrt[3]{y} = y^{1/3}$  to calculate the cube root of a number, you can use the *yx* function on your calculator. Thus, the molar solubility of CaF<sub>2</sub> is 2.1 × 10<sup>-4</sup> mol/L. The mass of CaF<sub>2</sub> that dissolves in water to form a liter of solution is

	$CaF_2(s) \implies$	Ca <sup>2+</sup>	+ $2 F^{-}(aq)$
Initial		0	0
Change	<u></u>	+x M	+2x M
Equilibrium		x M	2 <i>x</i> M

$$K_{sp} = [Ca^{2+}][F^{-}]^{2} = (x)(2x)^{2} = 4x^{3} = 3.9 \times 10^{-11}$$
$$x = \sqrt[3]{\frac{3.9 \times 10^{-11}}{4}} = 2.1 \times 10^{-4} M$$

$$\left(\frac{2.1\times10^{-4}\,\mathrm{mol}\,\mathrm{CaF}_2}{1\,\mathrm{L}\,\mathrm{soln}}\right)\left(\frac{78.1\,\mathrm{g}\,\mathrm{CaF}_2}{1\,\mathrm{mol}\,\mathrm{CaF}_2}\right) = 1.6\times10^{-2}\,\mathrm{g}\,\mathrm{CaF}_2/\mathrm{L}\,\mathrm{soln}$$

**Comment:** Because F<sup>-</sup> is the anion of a weak acid, you might expect that the hydrolysis of the ion would affect the solubility of CaF<sub>2</sub>. The basicity of F<sup>-</sup> is so small ( $K_b = 1.5 \times 10^{-11}$ ), however, that the hydrolysis occurs to only a slight extent and does not significantly influence the solubility. The reported solubility is 0.017 g/L at 25 °C, in good agreement with our calculation









The addition of bromide ion will decrease the water solubility of which of the following salts?

- a.  $BaSO_4$ b.  $Li_2CO_3$ c. PbS
- d. AgBr

Which pair of compounds will form a buffer solution when dissolved in water in equimolar amounts?

- a. HCI and KCI
- b. HNO<sub>3</sub> and NaNO<sub>3</sub>
- c. HCI and NH<sub>4</sub>CI
- d.  $NH_3$  and  $NH_4CI$

## The $K_a$ of HCN is 4.9 x 10<sup>-10</sup>. What is the pH of a buffer solution that is 0.100 *M* in both HCN and KCN?

a. 4.7
b. 7.0
c. 9.3
d. 14.0

The  $K_a$  of HCN is 4.9 x 10<sup>-10</sup>. What is the pH of a buffer solution that is 0.100 *M* in HCN and 0.200 *M* in KCN?

- a. 7.0
- b. 9.0
- c. 9.3
- d. 9.6

## The $K_a$ of HCN is 4.9 x 10<sup>-10</sup>. What is the pH of a buffer solution that is 1.00 *M* in HCN and 0.100 *M* in KCN?

- a. 7.0
- b. 8.3
- c. 9.0
- d. 9.3

In titrating a weak base with a strong acid, the best indicator to use would be:

- a. methyl red (changes color at pH = 5)
- b. bromothymol blue (changes at pH = 7)
- c. phenolphthalein (changes at pH = 9)
- d. none of the above

In titrating a weak acid with a strong base, the best indicator to use would be:

- a. methyl red (changes color at pH = 5)
- b. bromothymol blue (changes at pH = 7)
- c. phenolphthalein (changes at pH = 9)
- d. none of the above

## The $K_{sp}$ of BaCO<sub>3</sub> is 5.0 x 10<sup>-9</sup>. What is the concentration of barium ion in a saturated aqueous solution of BaCO<sub>3</sub>?

- a. 7.1 x 10<sup>-5</sup> M
- b. 2.5 x 10<sup>-9</sup> *M*
- c.  $5.0 \times 10^{-9} M$
- d. 1.0 x 10<sup>-8</sup> M

## The $K_{sp}$ of BaF<sub>2</sub> is 1.7 x 10<sup>-6</sup>. What is the concentration of barium ion in a saturated aqueous solution of BaF<sub>2</sub>?

- a. 1.7 x 10<sup>-6</sup> M
- b. 3.4 x 10<sup>-6</sup> M
- c. 7.6 x 10<sup>-3</sup> M
- d. 1.5 x 10<sup>-2</sup> M

## The $K_{sp}$ of BaF<sub>2</sub> is 1.7 x 10<sup>-6</sup>. What is the concentration of fluoride ion in a saturated aqueous solution of BaF<sub>2</sub>?

- a. 1.7 x 10<sup>-6</sup> M
- b. 5.7 x 10<sup>-5</sup> M
- c. 7.6 x 10<sup>-3</sup> M
- d. 1.5 x 10<sup>-2</sup> M

### Which of the following will produce a buffer solution?

- HCI/NaCI
- $HC_2H_3O_2/NH_3$
- NaH<sub>2</sub>PO<sub>4</sub>/Na<sub>2</sub>HPO<sub>4</sub>
- $HNO_3/Ca(OH)_2$
- KNO<sub>3</sub>/ NaOH



Copyright © 2009 Pearson Prentice Hall, Inc.

## In an $NH_4^+/NH_3$ buffer, what concentration changes will occur if a small amount of $OH^-$ is added?

	[NH <sub>4</sub> +]	[NH <sub>3</sub> ]	рН
1.	Increase	Increase	Increase
2.	Decrease	Increase	Increase
3.	Increase	Decrease	Increase
4.	Increase	Decrease	Decrease
5.	Decrease	Increase	Decrease

# Which indicator would best determine the endpoint of the titration of $NaC_2H_3O_2$ with $HNO_3$ ?

- Methyl violet
- Methyl orange
- Bromthymol blue
- Phenolphthalein
- Alizarin yellow R



Copyright © 2006 Pearson Prentice Hall, Inc.

This titration curve was obtained in the titration of an unknown acid with 0.10 M NaOH. What is the p $K_a$  of the acid?



## Addition of \_\_\_\_\_ will increase the solubility of $MgCO_3$ .

 $MgCO_{3}(s) \iff Mg^{2+}(aq) + CO_{3}^{2-}(aq)$ 

- MgCl<sub>2</sub>
- $Na_2CO_3$
- NaOH
- HCI
- KHCO<sub>3</sub>

Which of the following conjugate acid-base pairs will not function as a buffer:  $C_2H_5COOH$  and  $C_2H_5COO^ HCO_3^-$  and  $CO_3^{2^-}$  $HNO_3$  and  $NO_3^-$ 

**Answer:** The  $HNO_3$  will not work as buffer because it is strong acid

Predict which of the following compounds will have the greatest molar solubility in water: AgCl ( $K_{sp} = 1.8 \times 10^{-10}$ ), AgBr ( $K_{sp} = 5.0 \times 10^{-13}$ ), AgI ( $K_{sp} = 8.3 \times 10^{-17}$ ).

Answer. AgCl > AgBr > Agl

Calculate the pH of a solution containing 0.085 *M* nitrous acid (HNO<sub>2</sub>;  $K_a = 4.5 \times 10^{-4}$ ) and 0.10 *M* potassium nitrite (KNO<sub>2</sub>).

*Answer:* 3.42

Calculate the formate ion concentration and pH of a solution that is 0.050 *M* in formic acid (HCOOH;  $K_a = 1.8 \times 10^{-4}$ ) and 0.10 *M* in HNO<sub>3</sub>.

*Answer:* [HCOO<sup>-</sup>] = 9.0 × 10<sup>-5</sup>; pH = 1.00

Calculate the pH of a buffer composed of 0.12 *M* benzoic acid and 0.20 *M* sodium benzoate. (Refer to Appendix D.)

**Answer:** 4.42

Calculate the concentration of sodium benzoate that must be present in a 0.20 M solution of benzoic acid (C<sub>6</sub>H<sub>5</sub>COOH) to produce a pH of 4.00.

**Answer:** 0.13 M

Determine (a) the pH of the original buffer described in Sample Exercise 17.5 after the addition of 0.020 mol HCl and (b) the pH of the solution that would result from the addition of 0.020 mol HCl to 1.00 L of pure water.

**Answers:** (a) 4.68, (b) 1.70

Give the solubility-product-constant expressions and the values of the solubility-product constants (from Appendix D) for the following compounds: (a) barium carbonate, (b) silver sulfate.

*Answers:* (a) 
$$K_{sp} = [Ba^{2+}][CO_3^{2-}] = 5.0 \times 10^{-9}$$
; (b)  $K_{sp} = [Ag^+]^2[SO_4^{2-}] = 1.5 \times 10^{-5}$ 

A saturated solution of Mg(OH)<sub>2</sub> in contact with undissolved solid is prepared at 25 °C. The pH of the solution is found to be 10.17. Assuming that Mg(OH)<sub>2</sub> dissociates completely in water and that there are no other simultaneous equilibria involving the Mg<sup>2+</sup> or OH<sup>-</sup> ions in the solution, calculate  $K_{sp}$  for this compound.

### *Answer:* 1.6 × 10<sup>-12</sup>

The  $K_{sp}$  for LaF<sub>3</sub> is 2 × 10<sup>-19</sup>. What is the solubility of LaF<sub>3</sub> in water in moles per liter?

## *Answer:* 9 × 10<sup>-6</sup> mol/L





