ACUT 471 - Chapter 6: Option Strategies and Risk Management

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- Cap and Floor.
- Option Spreads (bull, bear, ratio,butterfly).
- Collar and zero-cost collar.
- Straddle and Strangle.

- The purchase of a put option is also called a **floor**, because we are guaranteeing a minimum sale price for the value of the underlying asset.
- The purchase a call option is also called **cap**, because we are protecting against a higher price of repurchasing the underlying asset.

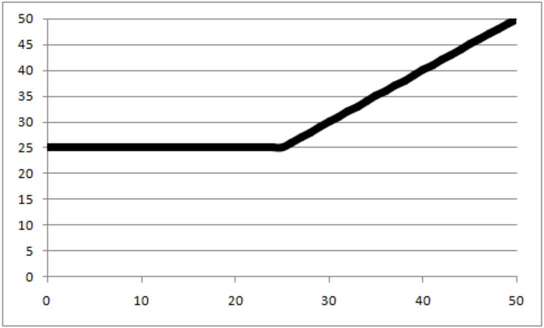

Figure: Hedged payoff (long put with $K = 25$ and holding underlying asset), $K = 25$ becomes a floor.

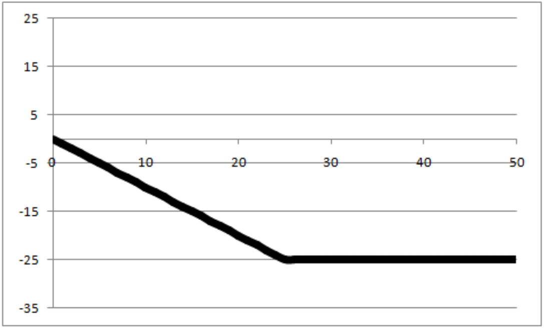

Figure: Hedged payoff (long call with $K = 25$ and short selling underlying asset), $-K = -25$ becomes a cap of our loss.

An option spread is a position consisting of only calls or only puts, in which some options are purchased and some written. Spreads are a common strategy. In this section we define some typical spread strategies.

- A **bull** spread consists of buying a K_1–strike and selling a K_2–strike, both with the same type (call or put) and expiration date T , and where $K_2 > K_1$.
- Now we will find the payoff of the call bull spread, which can be constructed by buying a K_1–strike call and selling a K_2–strike call.

Transaction	$S_T < K_1$	$K_1 < S_T < K_2$	$S_T > K_2$
Long call with K_1			
Short call with K_2			
Total			

- The payoff of the call bull spread can be written as

$$payoff = \begin{cases} 0 & S_T < K_1 \\ S_T - K_1 & K_1 < S_T < K_2 \\ K_2 - K_1 & S_T > K_2. \end{cases}$$

- The payoff of the put bull spread can be written as

$$payoff = \begin{cases} -(K_2 - K_1) & S_T < K_1 \\ S_T - K_2 & K_1 < S_T < K_2 \\ 0 & S_T > K_2. \end{cases}$$

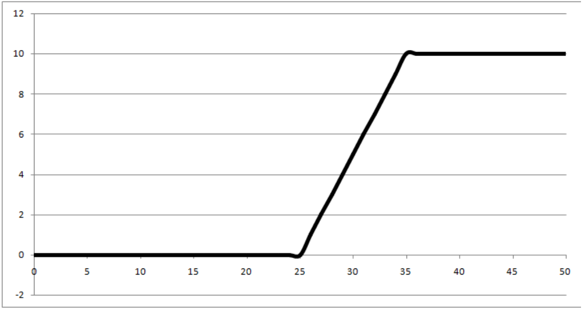

Figure: Call bull spread with $K_2 = 35$ and $K_1 = 25$.

Figure: Put bull spread with $K_2 = 35$ and $K_1 = 25$.

The prices of 1-year European put options on stock are:

- \$2.47 for the put with exercise price of \$40.
- \$7.42 for the put with exercise price of \$50.

Given that the annual continuously compounded risk-free interest rate is 5%, find the maximum and minimum profit for the put bull spread at the expiration.

- A **bear** spread consists of selling a K_1–strike and buying a K_2–strike, both with the same type (call or put) and expiration date T , and where $K_2 > K_1$.
- Now we will find the payoff of the put bear spread, which can be constructed by selling a K_1–strike put and buying a K_2–strike put.

Transaction	$S_T < K_1$	$K_1 < S_T < K_2$	$S_T > K_2$
Short put with K_1			
Long put with K_2			
Total			

- The payoff of the call bear spread can be written as

$$payoff = \begin{cases} 0 & S_T < K_1 \\ K_1 - S_T & K_1 < S_T < K_2 \\ K_1 - K_2 & S_T > K_2. \end{cases}$$

- The payoff of the put bear spread can be written as

$$payoff = \begin{cases} K_2 - K_1 & S_T < K_1 \\ K_2 - S_T & K_1 < S_T < K_2 \\ 0 & S_T > K_2. \end{cases}$$

Figure: Call bear spread with $K_2 = 35$ and $K_1 = 25$.

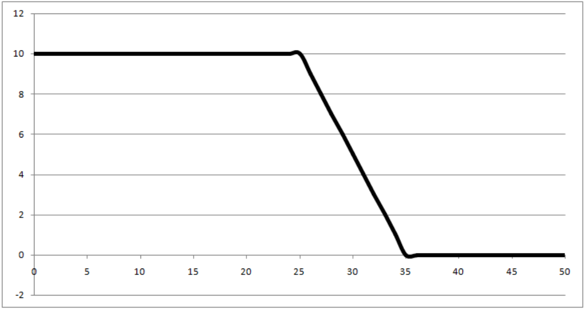

Figure: Put bear spread with $K_2 = 35$ and $K_1 = 25$.

The prices of 1-year European call options on stock are:

- \$5 for the call with exercise price of \$40.
- \$3 for the call with exercise price of \$50.

Given that the annual continuously compounded risk-free interest rate is 5%, find the maximum and minimum profit for the call bear spread at the expiration.

- A **ratio** spread consists of buying m options at one strike and selling n options at a different strike, with all options having the same type (call or put), same time to expiry T.
- Note that since ratio spreads involve buying and selling unequal numbers of options, it is possible to construct ratio spreads with zero premium.

The prices of 1-year European put options on stock are:

- \$2.47 for the put with exercise price of \$40.
- \$7.42 for the put with exercise price of \$50.

Given that the annual continuously compounded risk-free interest rate is 5%, find the maximum and minimum profit for the ratio spread formed a purchase position in a 50-strike put and a short position in 2 units of 40-strike puts.

- A **butterfly** spread consists of positions in options the same type (call or put) and same expiry T with three different strikes $(K_1 < K_2 < K_3)$.
- Define $\lambda = \frac{K_3 - K_2}{K_3 - K_1} \Rightarrow K_2 = \lambda K_1 + (1 - \lambda) K_3$.
- The value of λ can be interpreted as the relative distance of K_2 form K_3.
  - If $\lambda = 0.5$, then K_2 is the midway between K_1 and K_3 (symmetric spread).
  - If $0 < \lambda < 0.5$, then K_2 is closer to K_3 (asymmetric spread).
  - If $0.5 < \lambda < 1$, then K_2 is closer to K_1 (asymmetric spread).

Long call (put) position in a butterfly spread can be created by: buying λ calls (puts) with strike K_1 and $(1-\lambda)$ with strike K_3, and selling one call (put) with strike K_2.

Now we will find the payoff for the long call butterfly spread

Transaction	$S_T < K_1$	$K_1 < S_T < K_2$	$K_2 < S_T < K_3$	$S_T > K_3$
λ long calls with K_1				
one short call with K_2				
$(1-\lambda)$ long calls with K_3				
Total				

The payoff of the butterfly spread can be written as

$$payoff = \begin{cases} 0 & S_T < K_1 \\ \lambda(S_T - K_1) & K_1 < S_T < K_2 \\ (1-\lambda)(K_3 - S_T) & K_2 < S_T < K_3 \\ 0 & S_T > K_3. \end{cases}$$

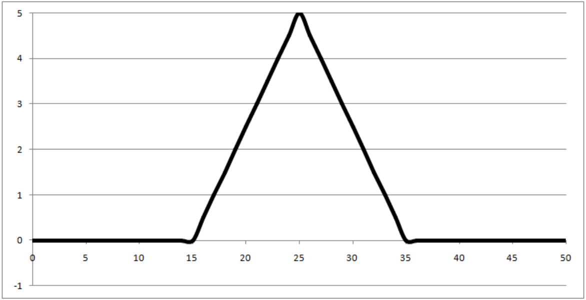

Figure: Long call butterfly spread with $K_1 = 15$, $K_2 = 25$ and $K_3 = 35$.

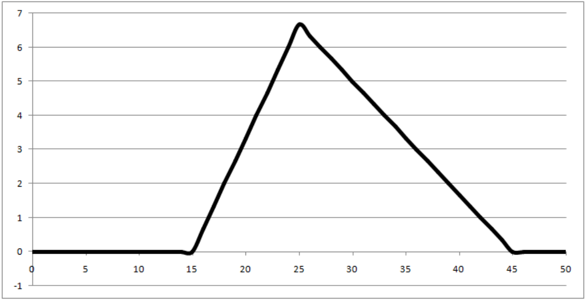

Figure: Long put butterfly spread with $K_1 = 15$, $K_2 = 25$ and $K_3 = 45$.

The prices of 1-year European call options on stock are:

- \$14.63 for the call with exercise price of \$90.
- \$6.80 for the call with exercise price of \$100.
- \$2.17 for the call with exercise price of \$110.

Given that the annual continuously compounded risk-free interest rate is 5%, find the maximum and minimum profit for the long call butterfly.

- A **long collar** consists of buying a put and selling a call with higher strike, both options have the same time to expiry T.
- A **short collar** consists of selling a put and buying a call with higher strike, both options have the same time to expiry T.
- The difference between the strike prices of the two options is called the collar width.
- The payoff of the long (short) collar looks like a short (long) forward contract.
- If the collar width is zero, then the collar becomes forward contract.
- K_1 and K_2 can be selected so that the prices of the call and put options are equal, which implies that the collar costs zero (known as **zero-cost collar**).

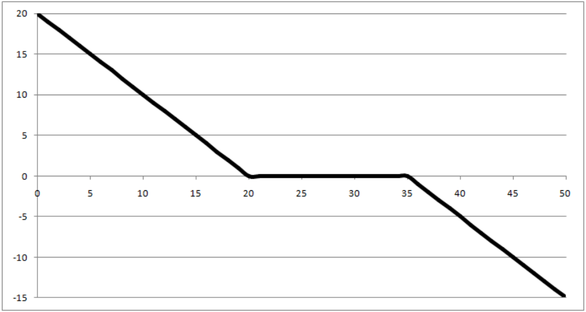

Figure: Long collar with $K_1 = 20$, $K_2 = 35$.

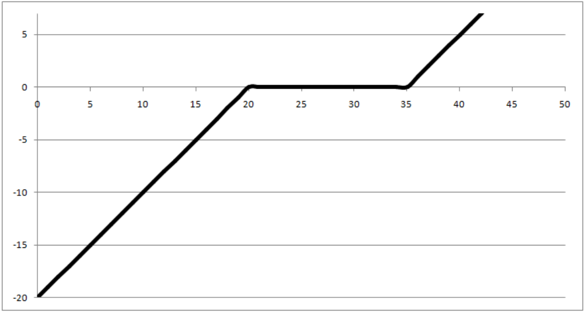

Figure: Short collar with $K_1 = 20$, $K_2 = 35$.

The prices of 1-year European options on stock are:

- \$6 for the put with exercise price of \$30.
- \$4 for the call with exercise price of \$50.

Given that the annual continuously compounded risk-free interest rate is 5%, find the maximum and minimum profit for the short collar.

- A **long (short) straddle** involves buying (selling) a call and a put with same strike K, time to expiry T.
- The payoff for the long straddle is $|S_T - K|$, and for the short straddle is $-|S_T - K|$.
- Long (short) straddle is called bottom (top) straddle.
- A bottom straddle is usually not cheap, because two options have to bought.
- A top straddle is a risky position, because the loss is unlimited (when S_T takes high value).

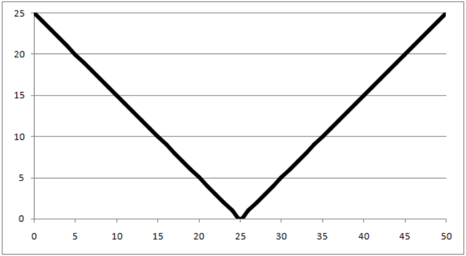

Figure: Long (bottom) straddle with $K = 25$.

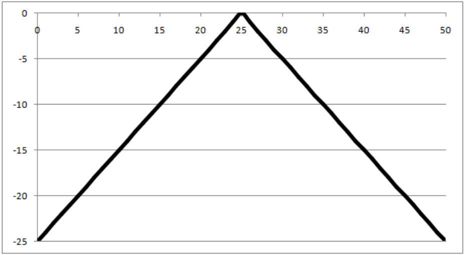

Figure: Short (top) straddle with $K = 25$.

- A **long (short) strangle** involves buying (selling) a put with strike K_1 and a call with strike K_2 where $K_1 < K < K_2$, and both options have the same time to expiry T.
- The cost of long strangle is low compared with long (bottom) straddle. (why?)
- The risk with short strangle is low compared with short (top) straddle.

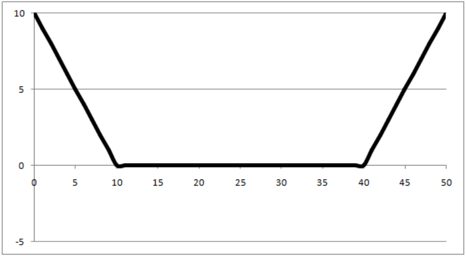

Figure: Long strangle with $K_1 = 10$ and $K_2 = 40$.

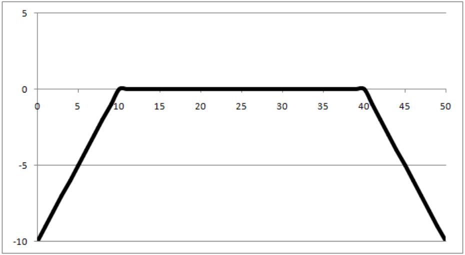

Figure: Short strangle with $K_1 = 10$ and $K_2 = 40$.

The prices of 1-year European options on stock are:

strike price	call premium	put premium
40	4	2
50	2.5	4
60	1	6

Given that the annual continuously compounded risk-free interest rate is 5%.

- Find the maximum and minimum profit for the long straddle with strike \$50.
- Find the maximum and minimum profit for the long strangle with strikes \$40 and \$60.

The current price of a nondividend-paying stock is 40 and the annual effective risk-free interest rate is 8%. You enter into a short position of 5 call options each with one year to maturity, a strike price of 35 and a premium of 9.21. Simultaneously, you enter into a long position on 3 call options, each with one year to maturity, a strike price of 42 and premium of 4.29.
Calculate the possible maxismum and minmum profit for the entire option portfolio.

An investor bought a 70-strike European put option on an index with six months to expiration. The premium for this option was 1. The investor also wrote an 80-strike European put option on the same index with six months to expiration. The premium for this option was 8. The six-month interest rate is 0
Calculate the index price at expiration that will allow the investor to break even.

Stock ABC has the following characteristics:

- The current price is 30.
- The annual effective risk-free interest rate is 10%.
- The following prices of 6-months European call option on ABC.

strike price	call premium
27	4.7806
30	2.8362
34	1.1965

Dertmine the range of the stock price at expiry such that the 34-strike call produces a higher profit than the 30-strike call, but a lower profit than the 27-strike call.

The current price of a non-dividend paying stock is 40 and the continuously compounded risk-free interest rate is 8%. The following table shows call and put option premiums for three-month European of various exercise prices:

strike price	call premium	put premium
35	6.13	0.44
40	2.78	1.99
45	0.97	5.08

A trader interested in speculating on volatility in the stock price is considering two investment strategies. The first is a 40-strike straddle. The second is a strangle consisting of a 35-strike put and a 45-strike call. Determine the range of stock prices in 3 months for which the strangle outperforms the straddle.