ACUT 471 - Chapter 4: Options

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An option is a contract between two parties - **a buyer (holder)** and **a seller (writer)**, that gives the buyer the right (but not the obligation) to buy or sell underlying asset (e.g. stock), from the writer, at a later date, at a price agreed upon today.

The **holder** of the option must pay to its counterpart (the writer) for holding the option. The price of the option is called its **premium**.

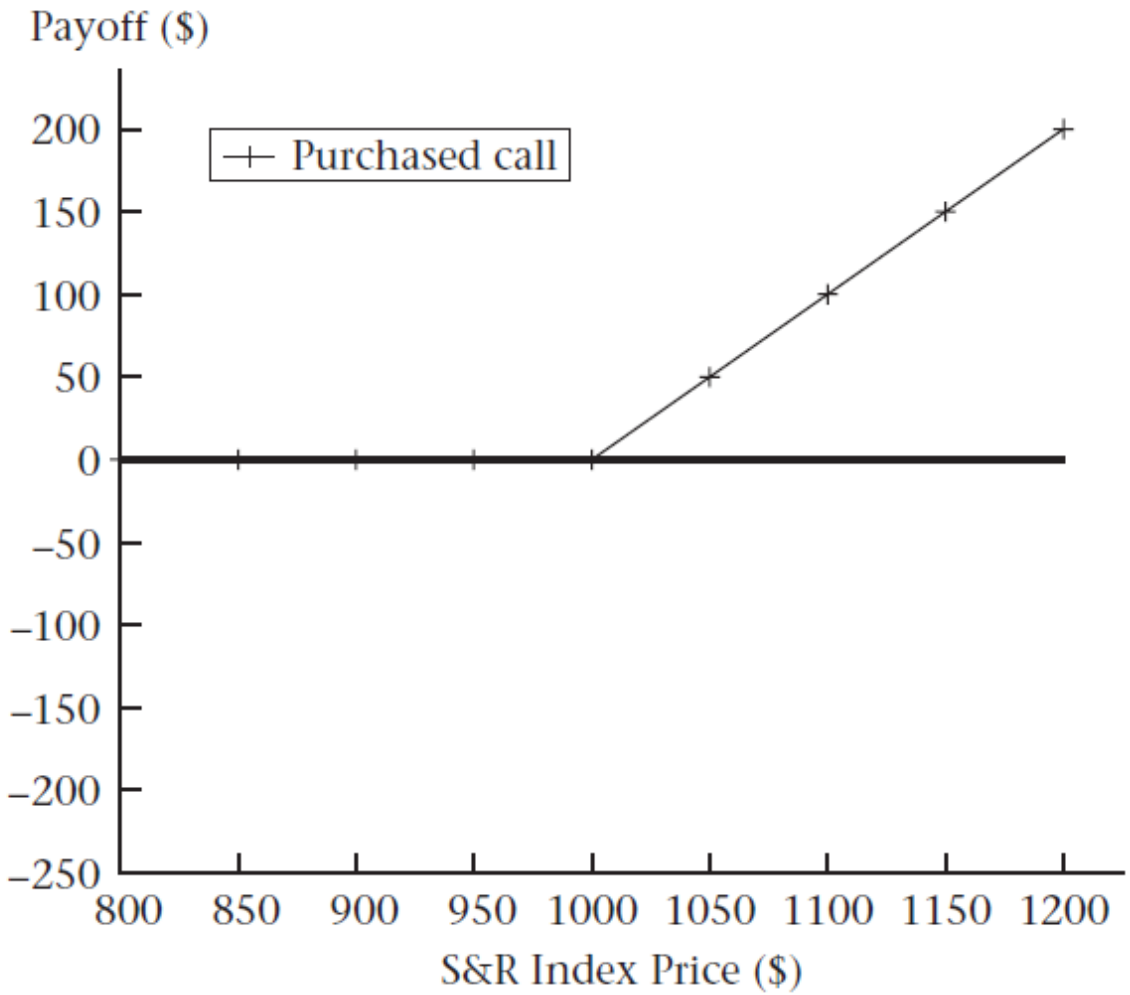
- **Strike price**: The strike price, or exercise price, is what the holder of the option agrees to buy (or sell) for the underlying asset.
- **Exercise**: the act of paying (or receiving) the strike price to buy (or sell) the asset.
- **Expiration**: The expiration of the option is the date by which the option must either be exercised or it becomes worthless.

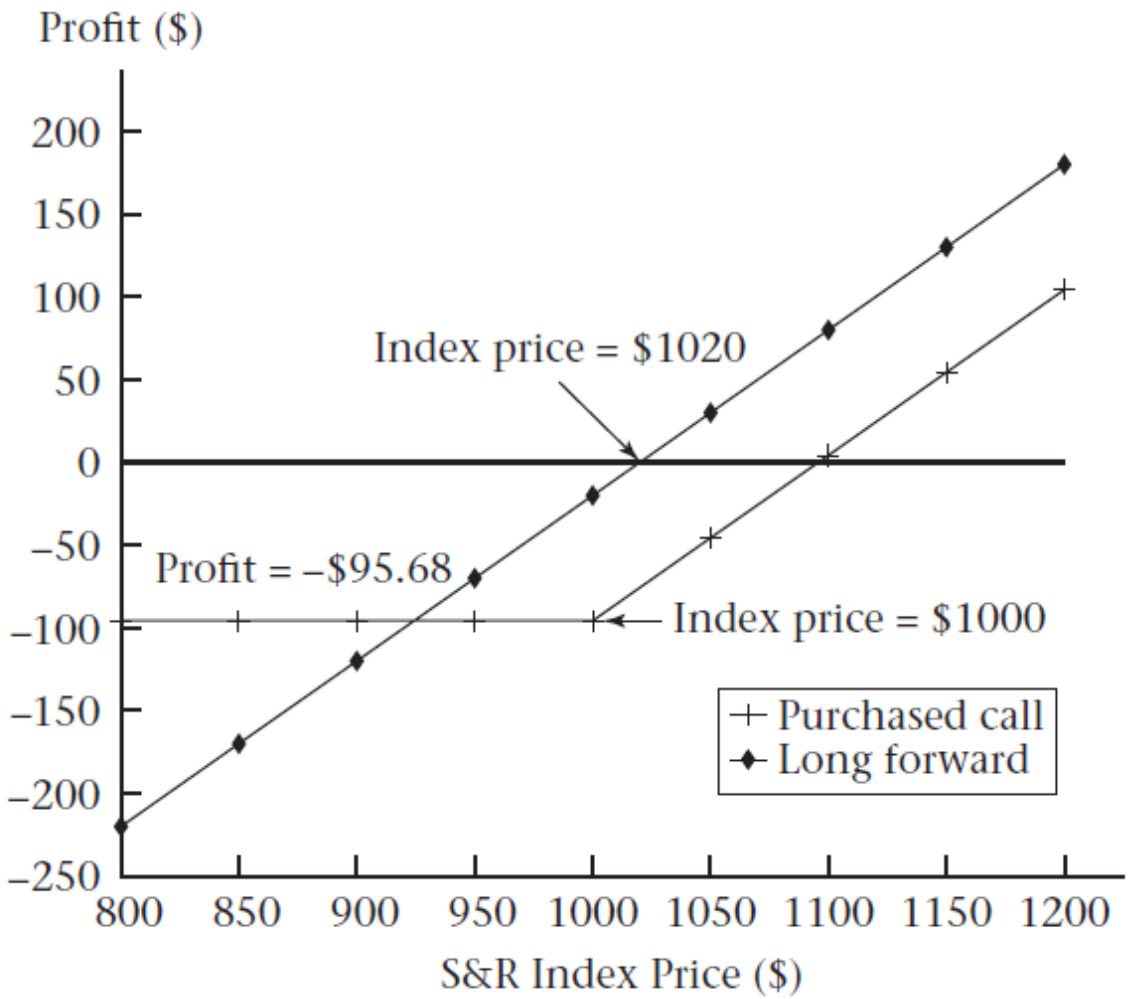
A European Call Option with strike or exercise price K and exercise date or maturity T on an asset is the right—but not the obligation—to buy the asset for K at time T.

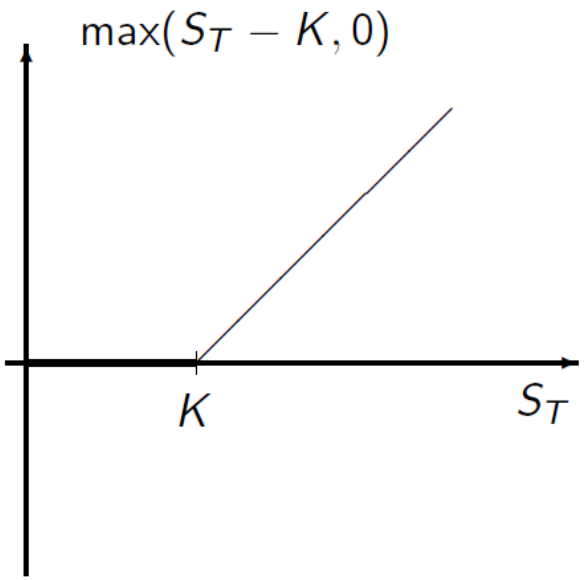
- Since we would only opt to pay K for an asset worth S_T if $S_T \geq K$, **the payoff** of a call option at time T is
  - $\max(S_T - K, 0)$ for the holder
  - $-\max(S_T - K, 0)$ for the writer.
- Note that the payoff on the option position is a function of the underlying price at time T (i.e. S_T).
- The holder's (writer's) **profit** can be found by subtracting (or adding) the future value of the option premium.

Example: consider a call option on the S&R index with 6 months to expiration and a strike price of \$1000. Let the option premium is \$93.81 and the effective interest rate is 2% over 6 months.

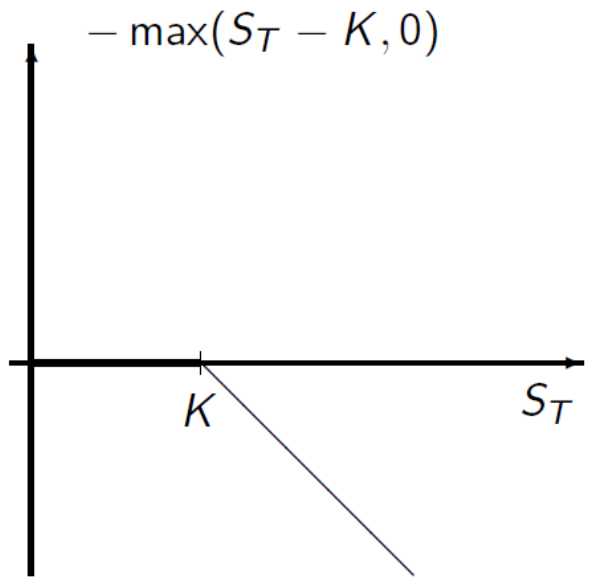
S_T	FV (premium)	call payoff (holder)	call profit (holder)
900	...	...	...
950	...	...	...
1000	...	...	...
1050	...	...	...
1100	...	...	...


Payoff ($)
200
150
100
50
0
–50
–100
–150
–200
–250
Purchased call
800
850
900
950
1000
1050
1100
1150
1200
S&R Index Price ($)


Profit ($)
200
150
100
50
0
–50
–100
–150
–200
–250
Index price = $1020
Profit = –$95.68
Index price = $1000
Purchased call
Long forward
800
850
900
950
1000
1050
1100
1150
1200
S&R Index Price ($)



Payoff for the call option holder



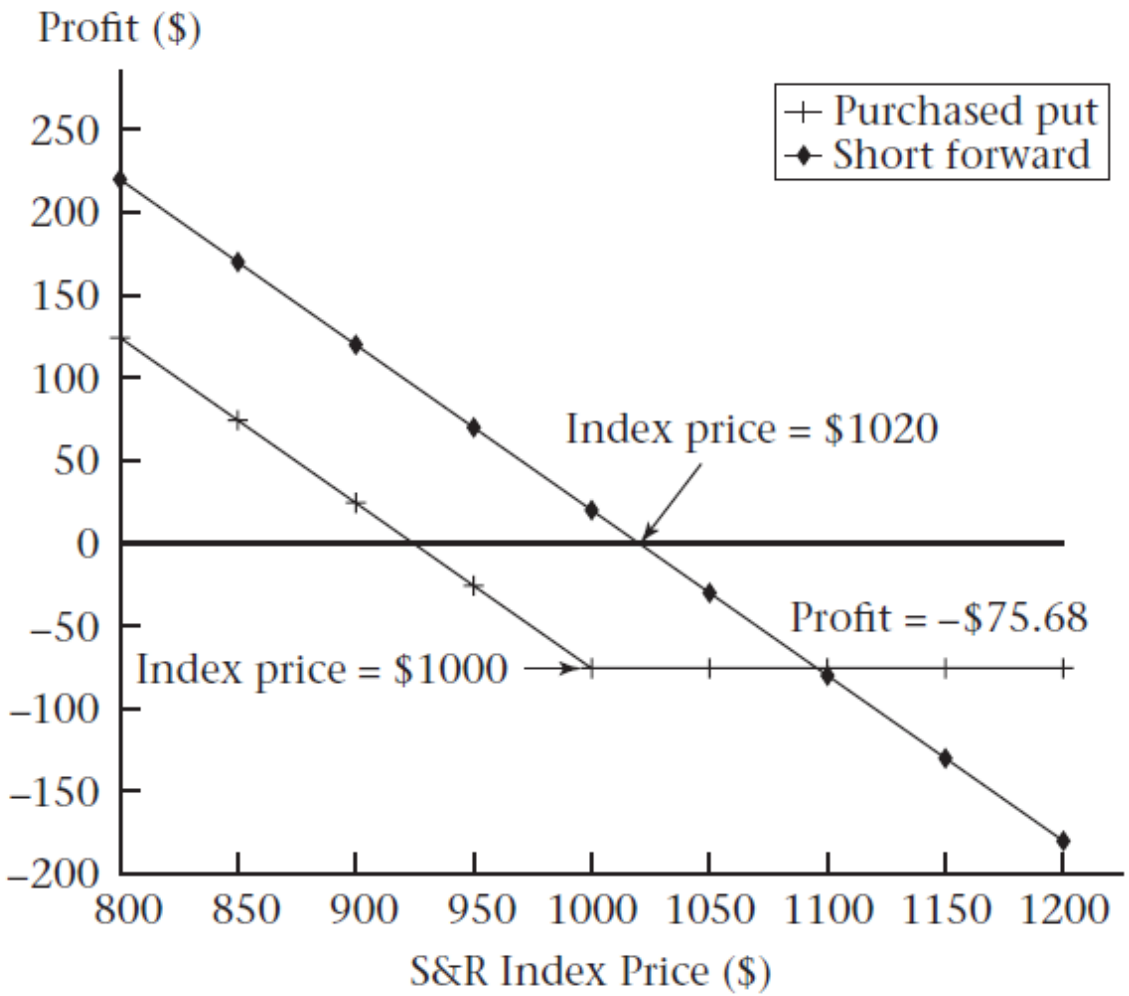
Payoff for the call option writer

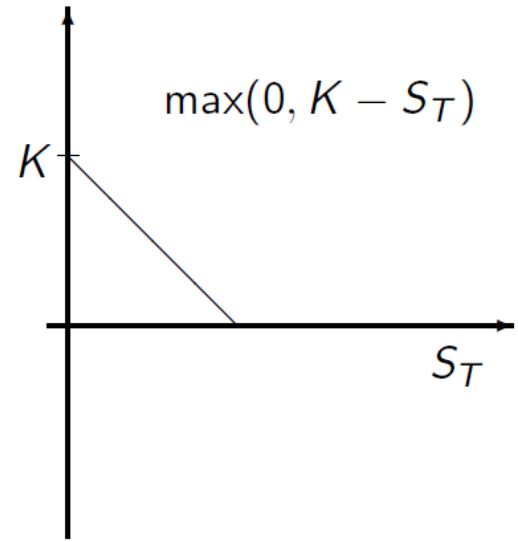
A European Put Option with strike or exercise price K and exercise date or maturity T on an asset is the right—but not the obligation—to sell the asset for K at time T.

- Since we would only opt to receive K for an asset worth S_T if $S_T \leq K$, **the payoff** of a put option at time T is
  - $\max(K - S_T, 0)$ for the holder
  - $-\max(K - S_T, 0)$ for the writer.
- Note that the payoff on the option position is a function of the underlying price at time T (i.e. S_T).
- The holder's (writer's) **profit** can be found by subtracting (or adding) the future value of the option premium.

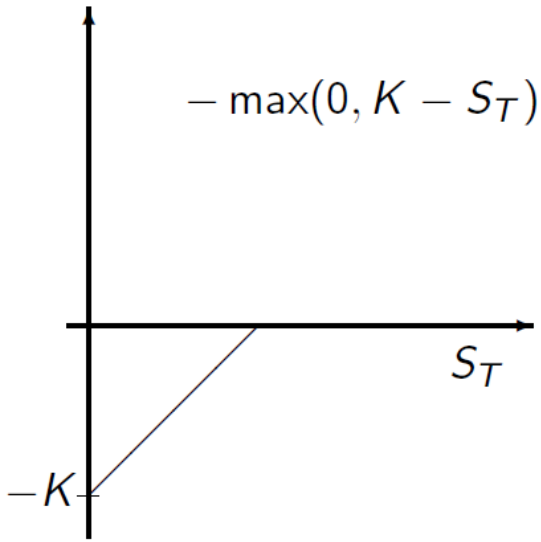
Example: consider a put option on the S&R index with 6 months to expiration and a strike price of \$1000. Let the option premium is \$74.20 and the effective interest rate is 2% over 6 months.

S_T	FV (premium)	put payoff (holder)	put profit (holder)
800	...	...	...
850	...	...	...
900	...	...	...
950	...	...	...
1000	...	...	...
1050	...	...	...


Profit ($)
Purchased put
Short forward
250
200
150
100
50
0
–50
–100
–150
–200
Index price = $1020
Profit = –$75.68
Index price = $1000
800
850
900
950
1000
1050
1100
1150
1200
S&R Index Price ($)



Payoff for the put option holder



Payoff for the put option writer

Exercise Style: the exercise style of the option governs the time at which exercise can occur.

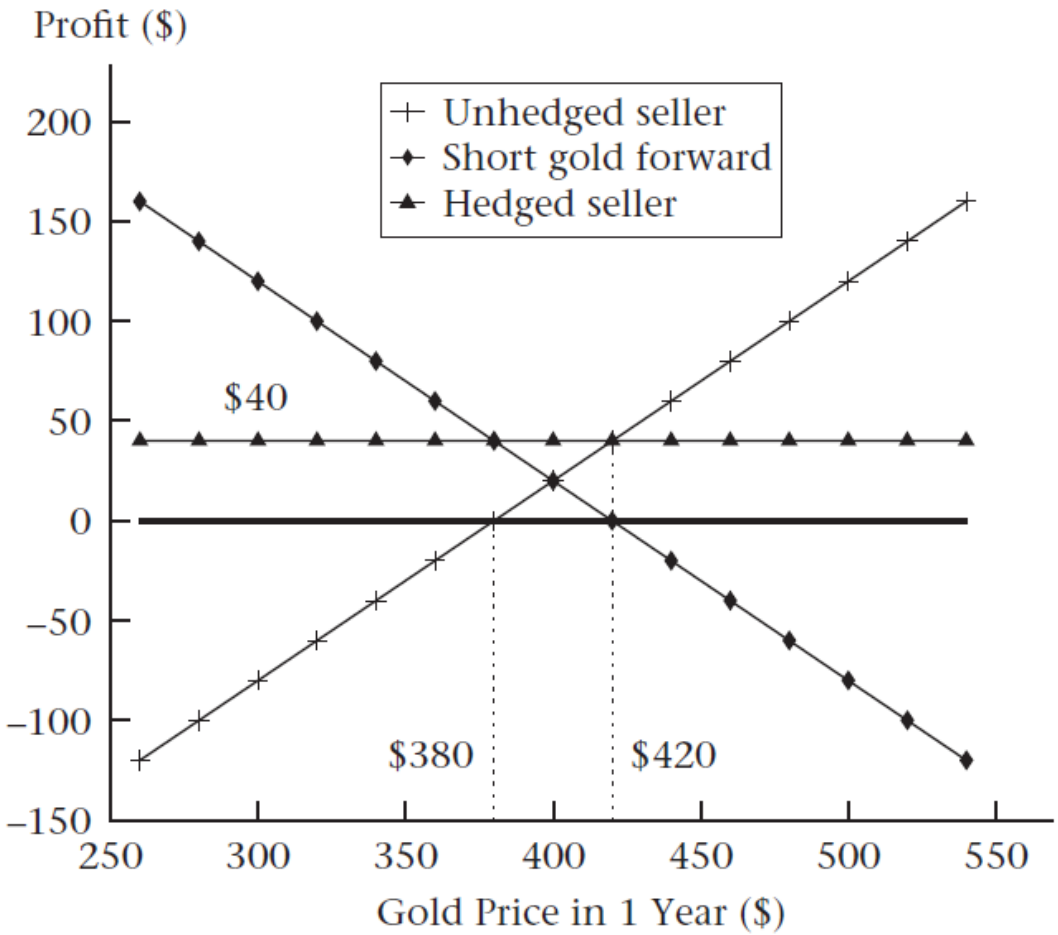
- *European-style option*: the exercise could occur only at expiration.
- *American-style option*: the exercise could occur at any time during the life of the option.
- *Bermudan-style option*: the exercise could occur during specified periods, but not for the entire life of the option

Options are often described by their degree of *moneyness*. This term describes whether the option payoff would be positive if the option were exercised *immediately*.

- An option is **in–the–money** option if it would have a **positive** payoff if exercised immediately.
- An option is **out–of-the–money** option if it would have a **negative** payoff if exercised immediately.
- An option is **at–the–money** option if it would have a **zero** payoff if exercised immediately.

In Chapter 2 the forward contract was used to hedge Golddiggers' risk.

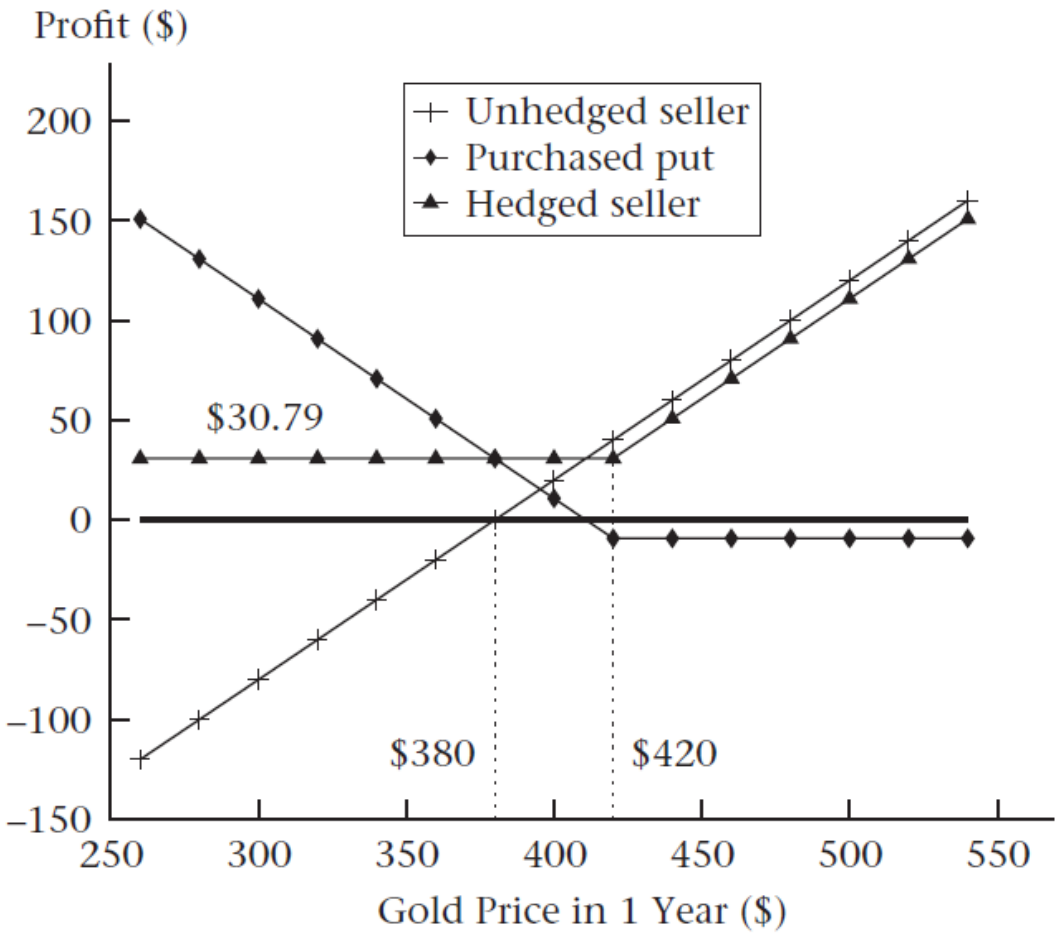
- Golddiggers is a gold-mining firm and will sell its production in one year from today.
- The total (fixed and variable) cost is \$380/oz.
- The price of gold today is \$405/oz.
- Gold to be delivered in 1 year can be sold today for \$420/oz.

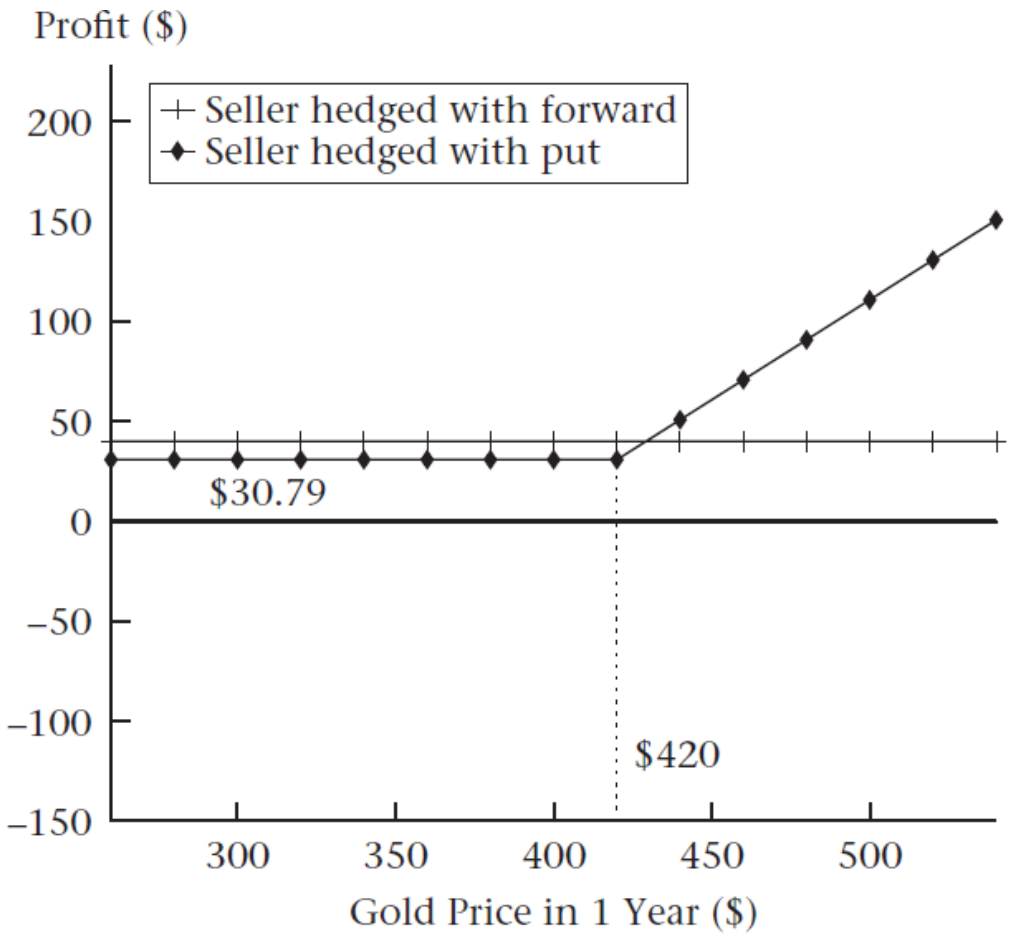

Profit ($)
Unhedged seller
Short gold forward
Hedged seller
200
150
100
50
0
–50
–100
–150
$40
$380
$420
250
300
350
400
450
500
550
Gold Price in 1 Year ($)

Now suppose that the market price for a 420-strike put is \$8.77/oz, and let the continuous compounded risk-free interest rate is 4.879%.
We will calculate the profit in three cases:

- Unhedged profit
- Profit on holding (buying) put with $K = 420$
- hedged profit

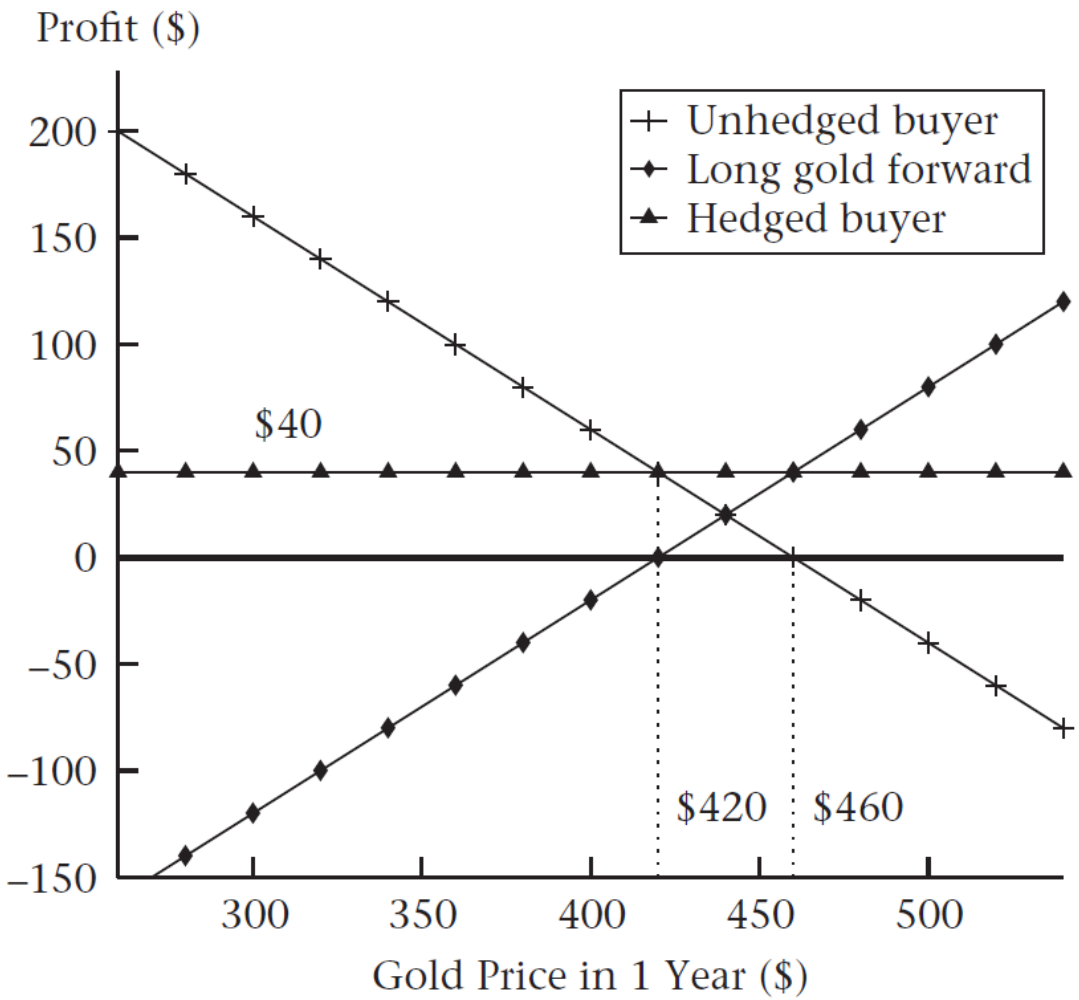
gold price in one year	total cost	profit		
		Unhedged	holding put	hedged
350	380	...	...	...
400	380	...	...	...
450	380	...	...	...
500	380	...	...	...


Profit ($)
Unhedged seller
Purchased put
Hedged seller
200
150
100
50
0
–50
–100
–150
$30.79
$380
$420
250
300
350
400
450
500
550
Gold Price in 1 Year ($)


Profit ($)
Seller hedged with forward
Seller hedged with put
200
150
100
50
0
–50
–100
–150
$30.79
$420
300
350
400
450
500
Gold Price in 1 Year ($)

In Chapter 2 the forward contract was used to hedge Auric's risk.

- Auric sells each widget for a fixed price of \$800
- The fixed cost per widget is \$340.
- The manufacture of each widget requires 1 oz of gold as an input.
- The quantity of widgets to be sold is known in advance.
- Gold to be delivered in 1 year can be sold today for \$420/oz.

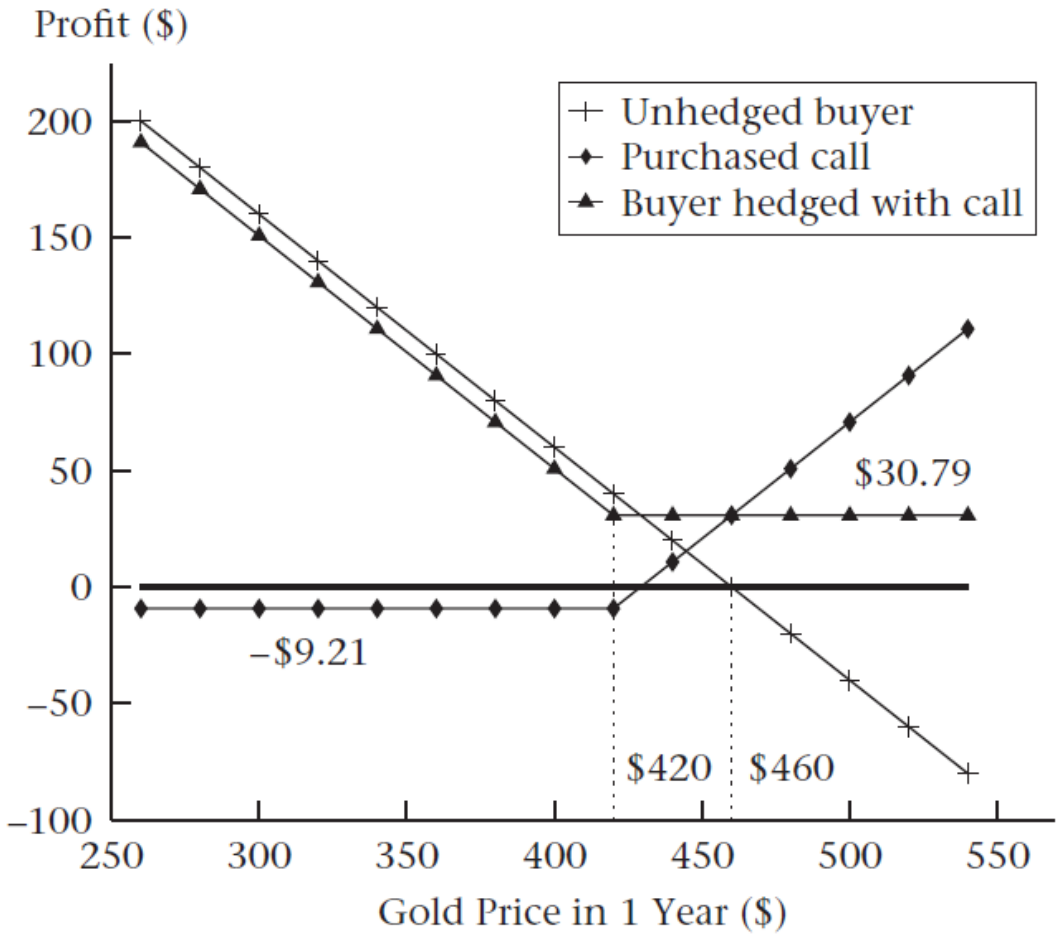

Profit ($)
Unhedged buyer
Long gold forward
Hedged buyer
200
150
100
50
0
−50
−100
−150
$40
$420
$460
300
350
400
450
500
Gold Price in 1 Year ($)

Now suppose that the market price for a 420-strike call is \$8.77/oz, and let the continuous compounded risk-free interest rate is 4.879%

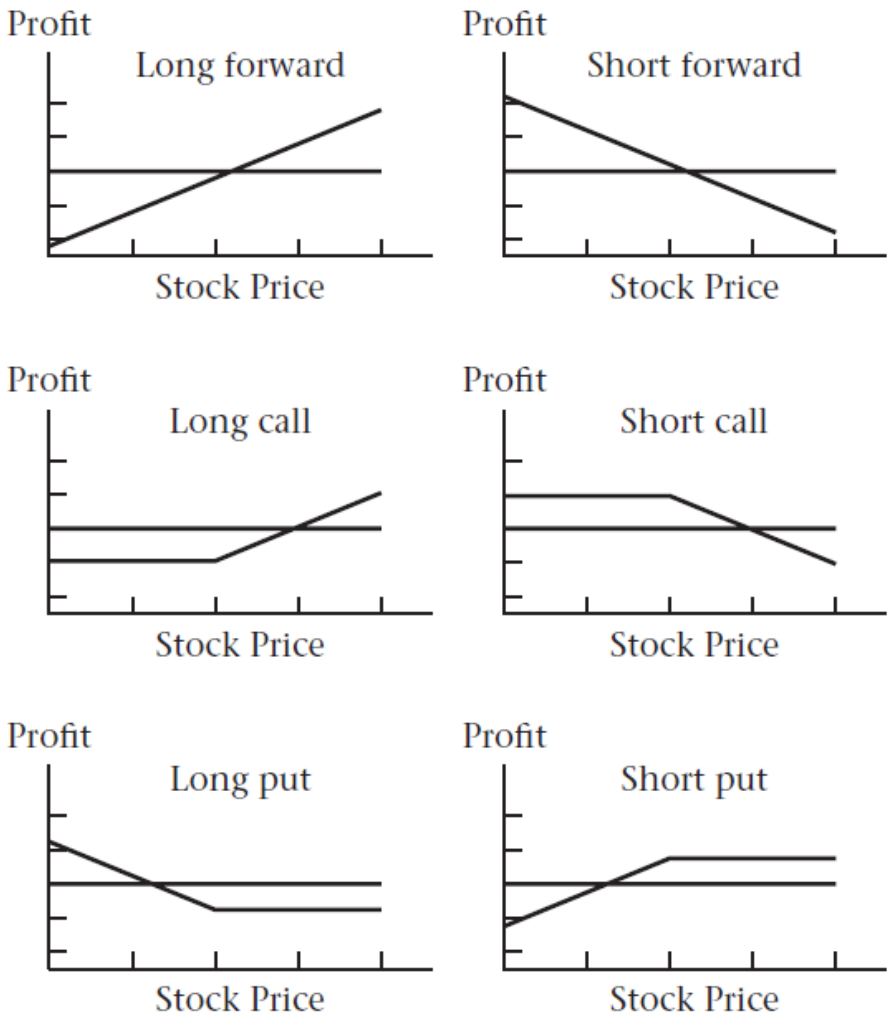
We will calculate the profit in three cases:

- Unhedged profit
- Profit on holding (buying) call with $K = 420$
- hedged profit

gold price in one year	total cost	revenue per widget	profit		
			Unhedged	holding call	hedged
350	340	800	...	...	...
400	340	800	...	...	...
450	340	800	...	...	...
500	340	800	...	...	...


Profit ($)
Unhedged buyer
Purchased call
Buyer hedged with call
200
150
100
50
0
–50
–100
$30.79
–$9.21
$420
$460
250
300
350
400
450
500
550
Gold Price in 1 Year ($)

Derivative Position	Position with Respect to Underlying Asset	Asset Price Contingency	Strategy
Long forward	Long (buy)	Always	Guaranteed purchase price
Short forward	Short (sell)	Always	Guaranteed sale price
Long call	Long (buy)	$>$ Strike	Insures against high price
Short call	Short (sell)	$>$ Strike	Sells insurance against high price
Long put	Short (sell)	$<$ Strike	Insures against low price
Short put	Long (buy)	$<$ Strike	Sells insurance against low price


Profit
Long forward
Stock Price
Profit
Short forward
Stock Price
Profit
Long call
Stock Price
Profit
Short call
Stock Price
Profit
Long put
Stock Price
Profit
Short put
Stock Price

Suppose the stock price is \$40 and the effective annual interest rate is 8%.
Draw on a single graph payoff and profit diagrams for the following options:

- 35-strike call with a premium of \$9.12.
- 45-strike put with a premium of \$5.75.

Consider 6-months 500-strike European put on stock. The continuously compounded risk-free interest rate is 8% and the put costs 41.9 today.
Calculate the stock price after 6 months so that being short in the put would produce 10 dollars more profit as being long in the put.

A producer of gold has expenses of 800 per ounce of gold produced. Assume that the cost of all production-related expenses is negligible and the producer will be able to sell all gold produced at the market price. In one year, the market price of gold will be one of the three possible prices, corresponding to the following probability table:

Gold price in one year	750	850	950
Probability	0.2	0.5	0.3

The producer hedges the price of gold by buying a one-year put option with an exercise price of 900 per ounce the option costs 100 per ounce now, and the continuously compounded annual risk-free interest rate is 6%.
Calculate the expected one-year profit per ounce of gold produced.

An investor purchased Option A and Option B for a certain stock today, with a strike prices 70 and 80 respectively. Both options are European one-year put options.
Determine which statements about the moneyness of these options based on a stock price.

1. If Option A is in-the-money, then Option B is in-the-money.
2. If Option A is at-the-money, then Option B is out-of-the-money.
3. If Option A is in-the-money, then Option B is out-of-the-money.
4. If Option A is out-of-the-money, then Option B is in-the-money.
5. If Option A is out-of-the-money, then Option B is out-of-the-money.