Basic statistics formulas

Sets

De Morgan's Law

$(A \cup B)^c = A^c \cap B^c$ & $(A \cap B)^c = A^c \cup B^c$

Commutativity

$A \cup B = B \cup A$ and $A \cap B = B \cap A$

Associativity

$(A \cup B) \cup C = A \cup (B \cup C)$
$(A \cap B) \cap C = A \cap (B \cap C)$

Distributivity

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

Probability

Let A, B, and C be three events. Then

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) \\ &- P(A \cap B) - P(A \cap C) - P(B \cap C) \\ &+ P(A \cap B \cap C) \end{aligned}$$

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"chapter 3"

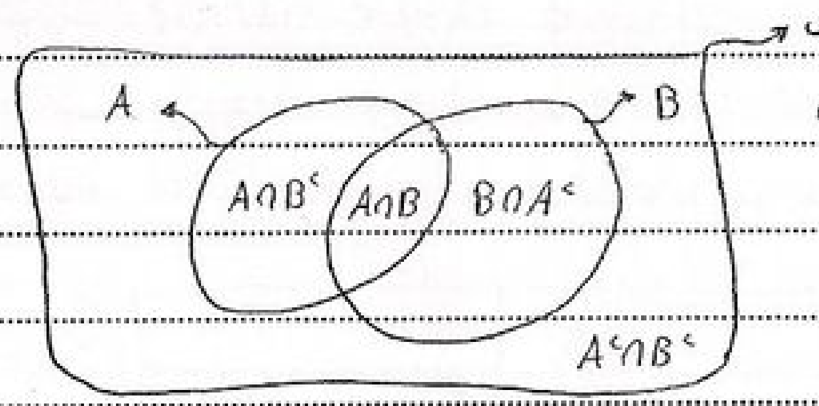
Summarize the concepts of chapter 3

1. see "the De Morgan law"
2. $\forall\ E \subseteq \Omega$ (where E event which is a subset and Ω is the universal set)

then

$P(E) = \frac{n(E)}{n(\Omega)}$, $P(\Omega) = \frac{n(\Omega)}{n(\Omega)} = 1$, $P(\emptyset) = \frac{n(\emptyset)}{n(\Omega)} = \frac{0}{n(\Omega)} = 0$

3. $\forall$ events $A, B \subseteq \Omega$ then :



"Venn diagram"

[1] $n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$\Rightarrow P(A\cup B) = P(A) + P(B) - P(A\cap B)$

or $= P(A^c \cap B^c)^c$ (using the de morgan law)

$= 1 - P(A^c \cap B^c)$

[2] $n(A^c) = n(\Omega) - n(A) \Rightarrow P(A^c) = 1 - P(A)$

[3] $A\cap A^c = \emptyset \Rightarrow n(A\cap A^c) = 0 \Rightarrow P(A\cap A^c) = 0$ and so for $P(B\cap B^c) = 0$

[4] $A = (A\cap B)\cup(A\cap B^c)$

$\Rightarrow n(A) = n[(A\cap B)\cup(A\cap B^c)]$

$= n(A\cap B) + n(A\cap B^c) - n[(A\cap B)\cap(A\cap B^c)]$ (using III)

$= n(A\cap B) + n(A\cap B^c) - n[\emptyset]$

$= n(A\cap B) + n(A\cap B^c) - 0$

$\Rightarrow P(A) = P(A\cap B) + P(A\cap B^c)$

and so for $P(B) = P(A\cap B) + P(A^c\cap B)$

[5] the events A and B are disjoint (mutually exclusive)

if $A\cap B = \emptyset \Rightarrow P(A\cap B) = 0$

[6] the events A and B are independent

if $P(A\cap B) = P(A).P(B)$

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7 the conditional probability of A given that (knowing that) B is $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{n(A\cap B)}{n(B)}$

other ideas for use of conditional probability :
if we have the following question "if we know B, what effect does this have on the probability that A?"
the answer to this will be by calculate $P(A|B)$ and $P(A)$ and we compare them as follows :

$P(A|B) < P(A) \Rightarrow P(A|B)$ decrease from $P(A)$

$P(A|B) > P(A) \Rightarrow P(A|B)$ increases from $P(A)$

$P(A|B) = P(A) \Rightarrow P(A|B)$ has no effect from $P(A)$

8 Conversion venn diagram to table is easy to deal :

	A	A^c	
B	$n(A\cap B)$	$n(A^c\cap B)$	$n(B)$
B^c	$n(A\cap B^c)$	$n(A^c\cap B^c)$	$n(B^c)$
	$n(A)$	$n(A^c)$	$n(\Omega)$

table 1 : using a number of elements in events . .

$\Rightarrow$ by dividing each of $n(\Omega)$ we get

	A	A^c	
B	$P(A\cap B)$	$P(A^c\cap B)$	$P(B)$
B^c	$P(A\cap B^c)$	$P(A^c\cap B^c)$	$P(B^c)$
	$P(A)$	$P(A^c)$	$1 \longrightarrow$ always

table 2 : using the probability in events

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(4) if we have A, B, C, D, E and F are events from Ω
where A, B and C in group 1
and D, E and F in group 2

	A	B	C	
D	$n(A \cap D)$	$n(B \cap D)$	$n(C \cap D)$	$n(D)$
E	$n(A \cap E)$	$n(B \cap E)$	$n(C \cap E)$	$n(E)$
F	$n(A \cap F)$	$n(B \cap F)$	$n(C \cap F)$	$n(F)$
	$n(A)$	$n(B)$	$n(C)$	$n(\Omega)$

table 3 : using a number of events in events

then :

[1] $A \cap B = \emptyset$, $A \cap C = \emptyset$, $B \cap C = \emptyset$, $A \cap B \cap C = \emptyset$

$A^c = B \cup C$, $B^c = A \cup C$, $C^c = A \cup B$

and so for group 2

[2] $n(A \cup B) = n(A) + n(B)$ where $A \cap B = \emptyset \Rightarrow P(A \cap B) = 0$

[3] for find $n(A \cap D^c)$ or $n(A \cup D^c)$:

if one of the two events is complement event we use the way
which is find the table between A and D where the code of complements
does not contain in table 3.

Now to find the table between A and D we do the following :

	A	A^c	
D			
D^c			

$\Rightarrow$

	A	A^c	
D	* $n(A \cap D)$		* $n(D)$
D^c			
	* $n(A)$		* $n(\Omega)$

which * is only the given by using the table 3

$\Rightarrow$ now we will supplement this table as follows

1. $n(A^c \cap D) = n(D) - n(A \cap D)$

2. $n(D^c) = n(\Omega) - n(D)$

	A	A^c	
D	$n(A \cap D)$		$n(D)$
D^c			
	$n(A)$		$n(\Omega)$

table 4

5. $n(A^c \cap D^c) = n(A^c) - n(A^c \cap D)$ or $= n(D^c) - n(A \cap D^c)$

3. $n(A \cap D^c) = n(A) - n(A \cap D)$

4. $n(A^c) = n(\Omega) - n(A)$

and now we can find $n(A \cap D^c)$ by using the table 4 directly

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and $n(A \cup D^c) = n(A) + n(D^c) - n(A \cap D^c)$ (which $n(A \cap D^c)$ find it by table 4.)

[4] for find $n(A^c \cap D^c)$ or $n(A^c \cup D^c)$: if both events are complements events, the solution here by using the law of de morgan, and after that we use the table 3 and : is as follows :

$$n(A^c \cap D^c) = n(A \cup D)^c$$
$$= n(\Omega) - n(A \cup D)$$
$$= n(\Omega) - [n(A) + n(D) - n(A \cap D)]$$
$$\Rightarrow p(A^c \cap D^c) = 1 - [p(A) + p(D) - p(A \cap D)]$$

and

$$n(A^c \cup D^c) = n(A \cap D)^c$$
$$= n(\Omega) - n(A \cap D)$$
$$\Rightarrow p(A^c \cup D^c) = 1 - p(A \cap D)$$

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"Q #19"

1.)

we know that

$* P(A) = P(A \cap B) + P(A \cap B^c) \Rightarrow P(A \cap B) = P(A) - P(A \cap B^c) \longmapsto (1)$

$* P(B) = P(A \cap B) + P(A^c \cap B) \Rightarrow P(A \cap B) = P(B) - P(A^c \cap B) \longmapsto (2)$

$* P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A|B)\, P(B) \longmapsto (3)$

$* P(B|A) = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A \cap B) = P(B|A)\, P(A) \longmapsto (4)$

so the closest answer of between (1), (2), (3) and (4) from among the options given is (2) $\therefore$ b

2)

we know that

$* P(A \cup B) = P(A) + P(B) - P(A \cap B) \longrightarrow (1)$

$* P(A \cup B) = P(A^c \cap B^c)^c$

$= 1 - P(A^c \cap B^c) \longrightarrow (2)$

so the closest answer of between (1) and (2) from among the options given is (2) $\therefore$ b

3) as the events A and B are independent so $P(A \cap B) = P(A)\, P(B)$

$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)\, P(B)}{P(A)} = P(B)$ $\therefore$ a

4) C

5) b

6

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"Q # 20"

$n(\Omega) = 4$ where Ω is the universal set
as the probability of each element in Ω
are equally likely so :
$p(A) = p(B) = p(C) = p(D) = \frac{1}{n(\Omega)} = \frac{1}{4}$

1) $p(A) = \frac{1}{4}$ $\therefore$ C

2) $2 p(D) = 2(\frac{1}{4}) = \frac{2}{2 \times 2} = \frac{1}{2}$

Now we have to examine all the options given to see any of them is equal to $\frac{1}{2}$:

a) $p(A) - p(B) = \frac{1}{4} - \frac{1}{4} = 0 \neq \frac{1}{2}$

b) $\frac{p(A)}{2} = \frac{1}{4} \div 2 = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \neq \frac{1}{2}$

C) $p(C) = \frac{1}{4} \neq \frac{1}{2}$

d) $p(A) + p(B) = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{2}{2 \times 2} = \frac{1}{2}$

so the correct answer is d

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"Q # 21"

as given $P(A^c) = .6$, $P(B) = .5$, $P(A \cap B) = .1$

where we have two events A and B

we will put the given in the table of probability

	A	A^c	
B	* $P(A \cap B) = .1$	$P(A^c \cap B) = .5 - .1 = .4$ ①	* $P(B) = .5$
B^c	$P(A \cap B^c) = .5 - .2 = .4 - .1 = .3$ ⑤	$P(A^c \cap B^c) = .6 - .4 = .2$ ③	$P(B^c) = 1 - .5 = .5$ ②
	$P(A) = 1 - .6 = .4$ ④	* $P(A^c) = .6$	* $P(\Omega) = 1$

where:

* : is the given

①, ②, ③, ④ and ⑤ : steps to fill in the table

i) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .4 + .5 - .1 = .8$

or

$= P(A^c \cap B^c)^c$ (using the de morgan law)

$= 1 - P(A^c \cap B^c) = 1 - .2 = .8$

ii) $P(A \cap B^c) = .3$

iii) $P(A^c \cap B) = .4$

iv) $P(A^c \cap B^c) = .2$

⑧

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"Q # 22"

as given $p(A \cap B^c) = .3$, $p(A \cap B) = .2$, $p(A^c \cap B^c) = .1$

where we have two events A and B

we will put the given in the table of probability

	A	A^c	
B	* $P(A \cap B) = .2$	$P(A^c \cap B) = .5 - .1 = .6 - .2 = .4$ ⑤	$P(B) = 1 - .4 = .6$ ③
B^c	* $P(A \cap B^c) = .3$	* $P(A^c \cap B^c) = .1$	$P(B^c) = .3 + .1 = .4$ ②
	$P(A) = .2 + .3 = .5$ ①	$P(\acute{A}) = 1 - .5 = .5$ ④	* $P(\Omega) = 1$

where :

*: is the given

①,②,③,④ and ⑤ : steps to fill in the table

i) $p(A) = .5$

ii) $p(A^c \cap B) = .4$

iii) $p(A \cup B) = p(A) + p(B) - p(A \cap B) = .6 + .5 - .2 = .9$

or

$= p(A^c \cap B^c)^c$ (using the de morgan law)

$= 1 - p(A^c \cap B^c) = 1 - .1 = .9$

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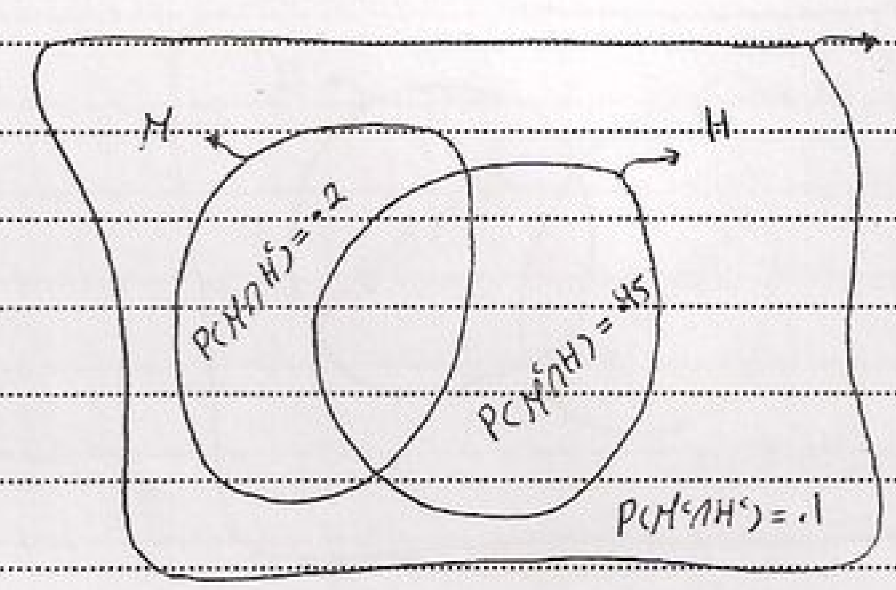
"Q # 23"

M : is a man , H : has a heart disease



where we have two events M and H

⇒

	M	M^c	
H	$P(M\cap H) = .7 - .45 = .45 - .2 = .25$ ⑤	* $P(M^c \cap H) = .45$	$P(H) = 1 - .3 = .7$ ③
H^c	* $P(M \cap H^c) = .2$	* $P(M^c \cap H^c) = .1$	$P(H^c) = .1 + .2 = .3$ ①
	$P(M) = 1 - .55 = .45$ ④	$P(M^c) = .1 + .45 = .55$ ②	* $P(\Omega) = 1$

where :

* : is the given

①, ②, ③, ④ and ⑤ : steps to fill in the table

1) $P(M \cap H) = .25$ ∴ c

2) $P(M^c) = .55$ ∴ a

3) $P(H^c) = .3$ ∴ d

4) $P(M \cup H) = P(M) + P(H) - P(M \cap H) = .45 + .7 - .25 = .9$

or

$= P(M^c \cap H^c)^c$ (using the de morgan law)

$= 1 - P(M^c \cap H^c) = 1 - .1 = .9$ ∴ e

5) $P(M^c \cap H) = .45$ ∴ a

6) $P(M) = .45$

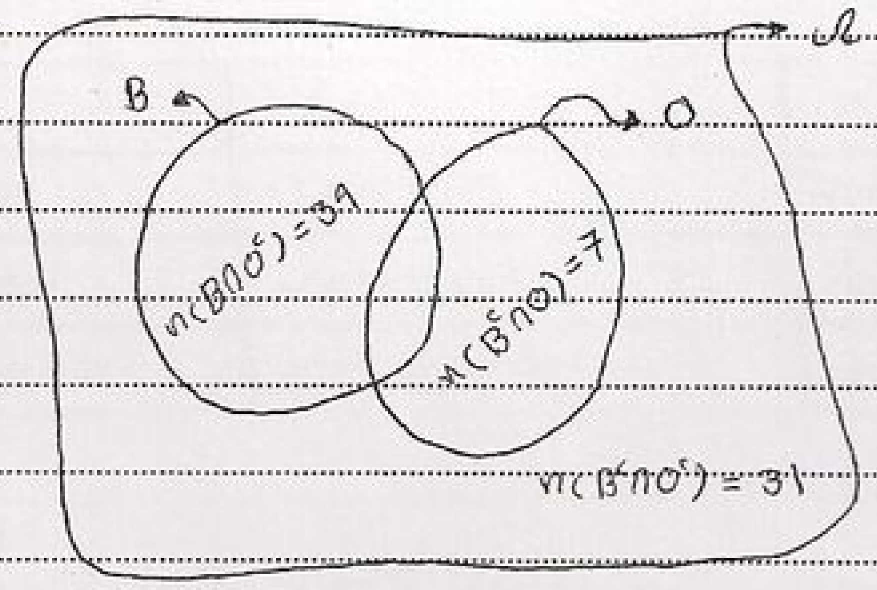
$$P(M \mid H) = \frac{P(M \cap H)}{P(H)} = \frac{.25}{.7} = .357$$

∴ $P(M \mid H) = .357 < P(M) = .45$

so $P(M \mid H)$ is decreases than $P(M)$ ∴ a

" Q # 24 "

B : is a boy , O : is over weight , $n(\Omega) = 80$



where we have two events B and O

⇒

	B	B^c	
O	$n(B\cap O) = 10-7 = 42-39 = 3$ ⑤	* $n(B^c\cap O) = 7$	$n(O) = 80-70 = 10$ ④
O^c	* $n(B\cap O^c) = 39$	* $n(B^c\cap O^c) = 31$	$n(O^c) = 39+31 = 70$ ②
	$n(B) = 80-38 = 42$ ③	$n(B^c) = 31+7 = 38$ ①	* $n(\Omega) = 80$

where :

* : is the given

①, ②, ③, ④ and ⑤ : steps to fill in the table

1) $P(B) = \frac{n(B)}{n(\Omega)} = \frac{42}{80} = .525$ ∴ b

2) $P(O^c) = \frac{n(O^c)}{n(\Omega)} = \frac{70}{80} = .875$ ∴ a

3) $P(B^c \cup O) = P(B^c) + P(O) - P(B^c\cap O) = \frac{n(B^c)}{n(\Omega)} + \frac{n(O)}{n(\Omega)} - \frac{n(B^c\cap O)}{n(\Omega)}$

$= \frac{n(B^c) + n(O) - n(B^c\cap O)}{n(\Omega)} = \frac{38+10-7}{80} = .5125$ ∴ d

or $P(B^c\cup O) = P(B\cap O^c)^c$ (using the de morgan law)

$= 1 - P(B\cap O^c) = 1 - \frac{n(B\cap O^c)}{n(\Omega)} = 1 - \frac{39}{80} = .5125$

4) $P(O|B) = \frac{P(O\cap B)}{P(B)} = \frac{n(O\cap B)}{n(B)} = \frac{3}{42} = 0.714$ ∴ a

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"Q #25"

A: is a smoker , C: is has a cancer , $n(\Omega) = 400$

	A	A^c	
C	$n(A\cap C) = 200$	$n(A^c \cap C) = 50$	$n(C) = 200 + 50 = 250$
C^c	$n(A\cap C^c) = 50$	$n(A^c\cap C^c) = 100$	$n(C^c) = 50 + 100 = 150$
	$n(A) = 200 + 50 = 250$	$n(A^c) = 100 + 50 = 150$	$n(\Omega) = 400$

a) $P(A\cap C) = \frac{n(A\cap C)}{n(\Omega)} = \frac{200}{400} = .5$

b) $P(A\cup C) = P(A) + P(C) - P(A\cap C) = \frac{n(A) + n(C) - n(A\cap C)}{n(\Omega)} = \frac{250 + 250 - 200}{400} = .75$

or

$P(A\cup C) = P(A^c \cap C^c)^c$ (using the de morgan law)

$= 1 - P(A^c\cap C^c) = 1 - \frac{n(A^c\cap C^c)}{n(\Omega)} = 1 - \frac{100}{400} = .75$

c) $P(A^c\cup C) = P(A^c) + P(C) - P(A^c\cap C) = \frac{n(A^c) + n(C) - n(A^c\cap C)}{n(\Omega)} = \frac{150 + 250 - 50}{400} = .8$

or

$P(A^c\cup C) = P(A\cap C^c)^c$ (using the de morgan law)

$= 1 - P(A\cap C^c) = 1 - \frac{n(A\cap C^c)}{n(\Omega)} = 1 - \frac{50}{400} = .8$

d) i) $P(C|A) = \frac{P(C\cap A)}{P(A)} = \frac{n(C\cap A)}{n(A)} = \frac{200}{250} = .8$

ii) $P(A^c|C^c) = \frac{P(A^c\cap C^c)}{P(C^c)} = \frac{n(A^c\cap C^c)}{n(C^c)} = \frac{100}{150} = .66$

e) we want to see that $P(A\cap C) \overset{?}{=} P(A) \cdot P(C)$

$L.H.S = P(A\cap C) = \frac{n(A\cap C)}{n(\Omega)} = \frac{200}{400} = .5$

$R.H.S = P(A) \cdot P(C) = \frac{n(A)}{n(\Omega)} \cdot \frac{n(C)}{n(\Omega)} = \frac{250}{400} \cdot \frac{250}{400} = .3906$

$\therefore L.H.S \neq R.H.S \Rightarrow$ A and C are not independent

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"Q # 26"

H: home, W: work, S: school } in group 1 "place where injury"
A: 17 or less, B: 18 to 26, C: 27 to 40 } in group 2 "age"

	H	W	S	
A	$n(H\cap A)=40$	$n(W\cap A)=15$	$n(S\cap A)=80$	$n(A)=185$
B	$n(H\cap B)=30$	$n(W\cap B)=100$	$n(S\cap B)=35$	$n(B)=165$
C	$n(H\cap C)=5$	$n(W\cap C)=140$	$n(S\cap C)=5$	$n(C)=150$
	$n(H)=125$	$n(W)=255$	$n(S)=120$	$n(\Omega)=500$

1) "injured at work or at school and the age is between 18 to 26"

$\Rightarrow (W \cup S) \cap B$ $\therefore b$

2) $P(B\cup W) = P(B) + P(W) - P(B\cap W) = \dfrac{n(B)+n(W)-n(B\cap W)}{n(\Omega)}$

$= \dfrac{165+255-100}{500} = .64$ $\therefore c$

3) $n(A\cap W) = 15$ $\therefore b$

4) $P(B^c) = 1 - P(B) = 1 - \dfrac{n(B)}{n(\Omega)} = 1 - \dfrac{165}{500} = .67$ $\therefore a$

5) $P(S^c \cup C^c) = P(S\cap C)^c$ (using the de morgan law)

$= 1 - P(S\cap C) = 1 - \dfrac{n(S\cap C)}{n(\Omega)} = 1 - \dfrac{5}{500} = .99$ $\therefore e$

6) $P(S|B) = \dfrac{P(S\cap B)}{P(B)} = \dfrac{n(S\cap B)}{n(B)} = \dfrac{35}{165} = .2121$ $\therefore b$

7) $n(A\cup C) = n(A) + n(C) - n(A\cap C)$

$= 185 + 125 - 0$ → because they are from the same group which is group 2 as we can see

$= 335$ $\therefore b$

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"Q #27"

B: below needed, E: enough, A: above needed } in group 1 "Calcium intake"
M: Man, W: Woman, C: Child } in group 2

	B	E	A	
M	$n(B\cap M)=72$	$n(E\cap M)=200$	$n(A\cap M)=48$	$n(M)=320$
W	$n(B\cap W)=104$	$n(E\cap W)=160$	$n(A\cap W)=16$	$n(W)=280$
C	$n(B\cap C)=64$	$n(E\cap C)=106$	$n(A\cap C)=30$	$n(C)=200$
	$n(B)=240$	$n(E)=466$	$n(A)=94$	$n(\Omega)=800$

1) $B^c \cap C$

$\Rightarrow$ "the calcium intake is not below needed and a child"

2) "above needed calcium given a woman."

$\Rightarrow A \mid W$

3) $P(B)=\dfrac{n(B)}{n(\Omega)}=\dfrac{240}{800}=.3$ $\therefore$ c

4) $P(W^c)=1-P(W)=1-\dfrac{n(W)}{n(\Omega)}=1-\dfrac{280}{800}=.65$ $\therefore$ d

5) $P(W\cup B)=P(W)+P(B)-P(W\cap B)=\dfrac{n(W)+n(B)-n(W\cap B)}{n(\Omega)}=\dfrac{280+240-104}{800}=.52$ $\therefore$ c

6) $P(M\cap C)=0$ because M and C from the same group which they are from group 2 so $M\cap C=\emptyset \Rightarrow P(M\cap C)=\dfrac{n(M\cap C)}{n(\Omega)}=\dfrac{0}{n(\Omega)}=0$ $\therefore$ a

7) $P(W\cap B^c)$ here we need to configure a special table between W and B

	W	W^c	
B	* $n(W\cap B)=104$	$n(W^c\cap B)=240-104=136$ (2)	* $n(B)=240$
B^c	$n(W\cap B^c)=280-104=176$ (1)	$n(W^c\cap B^c)=520-136=560-176=384$ (5)	$n(B^c)=800-240=560$ (4)
	* $n(W)=280$	$n(W^c)=800-280=520$ (3)	* $n(\Omega)=800$

*: is the given by take it from the large table
(1),(2),(3),(4) and (5): steps to fill in the table

$P(W\cap B^c)=\dfrac{n(W\cap B^c)}{n(\Omega)}=\dfrac{176}{800}=.22$ $\therefore$ b

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8) $P(A \cup E) = P(A) + P(E) - P(A \cap E)$

$= \frac{n(A)}{n(\Omega)} + \frac{n(E)}{n(\Omega)} - \frac{n(A \cap E)}{n(\Omega)}$

$= \frac{n(A) + n(E) - n(A \cap E)}{n(\Omega)}$

$= \frac{44 + 466 - 0}{800}$

as A and E are from the same group which is from group 1
so $A \cap E = \emptyset \Rightarrow n(A \cap E) = 0$

$= .7$

$\therefore d$

9) $P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{n(B \cap C)}{n(C)} = \frac{64}{200} = .32 \quad \therefore C$