ACUT 471 - Chapter 2: Forward Contract

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A forward contract is an agreement to buy or sell an asset (called *underlying asset*) at a certain future time (called the *maturity date* or *expiration date*) for a certain price.

A forward contract does the following:

- Specifies the quantity and exact type of the asset or commodity the seller must deliver.
- Specifies delivery logistics, such as time, date, and place.
- Specifies the price the buyer will pay at the time of delivery.
- Obligates the seller to sell and the buyer to buy, subject to the above specifications.

Note that a forward contract:

- requires *no initial payment or premium*
- It is not normally traded on an exchange.

The party that agrees to *buy* the underlying asset on the future specified date holds a *long* position.

The party that agrees to *sell* the underlying asset on the future specified date holds a *short* position.

The payoff to a forward contract (with expiry T) is the value of the position at expiration.

Denoting a forward price as F and spot price at expiration as S_T.

- Payoff to long forward $= S_T - F$
- Payoff to short forward $= F - S_T$

consider a forward contract on a stock index, namely S&R ("Special and Rich") 500 index. This index has a current price of $1000 and the 6-month forward price is $1020.

S_T	payoff (long)	payoff (short)
900	...	...
950	...	...
1000	...	...
1020	...	...
1050	...	...
1100	...	...
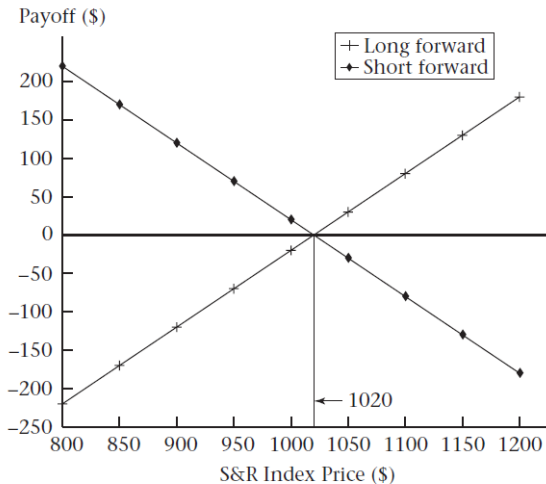
An alternative way to acquire the index is to purchase it *outright* at time 0, paying $1000.
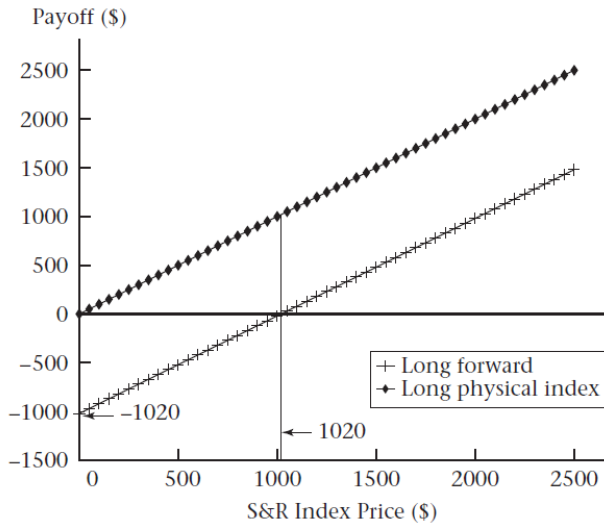
With both positions, we own the index after 6 months, but ...

Cash Vs. Delivery
Instead of requiring delivery of the actual underlying asset, the
forward contract settles financially. The two parties make a net
cash payment which yields the same cash flow as if delivery had
occurred.

In other words, each party receives (or pays) a cash payoff.

- Golddiggers is a gold-mining firm and will sell its production in one year from today.
- The total (fixed and variable) cost is $380/oz.
- The price of gold today is $405/oz.
- gold to be delivered in 1 year can be sold today for $420/oz.

We will calculate the profit in three cases:

- Unhedged profit
- Profit on the short forward position
- hedged profit

| gold price in one year	total cost	profit		
		Unhedged	short forward	hedged
350	380	...	...	...
400	380	...	...	...
450	380	...	...	...
500	380	...	...	...
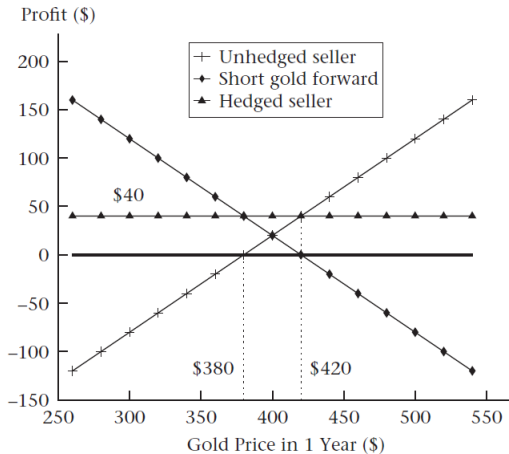
Auric Enterprises is a manufacturer of widgets (a product that uses gold as an input), and we suppose that the price of gold is the only uncertainty Auric faces.

- Auric sells each widget for a fixed price of $800
- The fixed cost per widget is $340.
- The manufacture of each widget requires 1 oz of gold as an input.
- The quantity of widgets to be sold is known in advance.
- gold to be delivered in 1 year can be sold today for $420/oz.

We will calculate the profit in three cases:

- Unhedged profit
- Profit on the long forward position
- hedged profit

gold price in one year	total cost	revenue per widget	profit		
			Unhedged	long forward	hedged
350	340	800	...	...	...
400	340	800	...	...	...
450	340	800	...	...	...
500	340	800	...	...	...
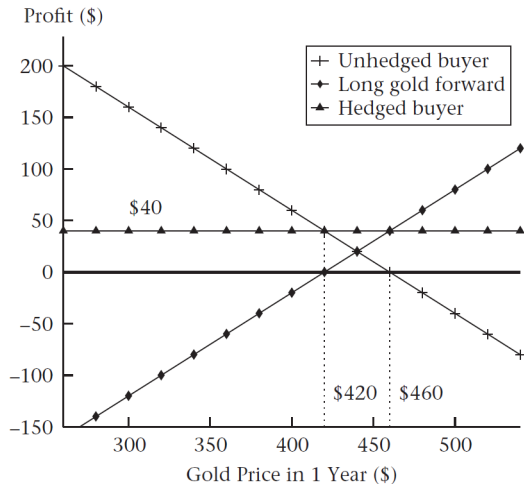
We have four different ways to buy a share of stock that has price S_0 at time 0.

Description	Pay at Time	Receive Security at Time	Payment
Outright	0	0	S_0
Fully leveraged	T	0	$S_0 e^{rT}$
Prepaid forward	0	T	?
Forward	T	T	$?e^{rT}$

- Pricing the Prepaid Forward
  1. No dividend
  2. Discrete Dividends
  3. Continuous Dividends

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Pricing the Prepaid Forward with no dividend

1 By Discounted Present Value

2 By Arbitrage

Pricing the Prepaid Forward with no dividend- By Discounted Present Value

Let $F_{0,T}^P$ a prepaid forward price for an asset bought at time 0 and delivered at time T.

We calculate the expected value of the stock at time T, and then discount that value at an appropriate rate of return.

If we assume that α is the expected return on the stock, then

$$F_{0,T}^P = ...$$

$$E_0(S_T) = ...$$

$$F_{0,T}^P = ...$$

For a non-dividend-paying stock, the prepaid forward price is the current stock price.

Pricing the Prepaid Forward with no dividend- By Arbitrage

Classical arbitrage describes a situation in which we can generate a positive cash flow either today or in the future by simultaneously buying and selling related assets, *with no net investment of funds and with no risk*. Arbitrage, in other words, is free money.

The price of a derivative should be such that no arbitrage is possible.

Case 1: if $F_{0,T}^P > S_0$

	Cash Flows	
Transaction	Time 0	Time T
sell prepaid forward @ $F_{0,T}^P$	...	...
buy stock @ S_0	...	...
total	...	...

We have earned positive profits today (the profit value=...) and offset all future risk.

Case 2: if $F_{0,T}^P < S_0$

Transaction	Cash Flows	
	Time 0	Time T
short sell stock @ S_0	...	...
buy prepaid forward @ $F_{0,T}^P$	...	...
total	...	...

We have earned positive profits today (the profit value=...) and offset all future risk.

Therefore, we must have $F_{0,T}^P = S_0$ to prevent arbitrage.

Pricing Prepaid Forwards with Dividends

In general, the price for a prepaid forward contract will be the
stock price less the present value of dividends to be paid over the
life of the contract.

Pricing Prepaid Forwards with Discrete Dividends

A stock is expected to make dividend payments of D_{t_i} at times $t_i, \ i = 1, ..., n$. Then the prepaid forward price is given by:

$$F_{0,T}^{P} = S_0 - \sum_{i=1}^{n} PV_{0,t_i}(D_{t_i})$$

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Example

Suppose XYZ stock costs $100 today and is expected to pay a
$1.25 quarterly dividend, with the first coming 3 months from
today and the last just prior to the delivery of the stock.
Suppose the annual continuously compounded risk-free rate is 10%.
Calculate the 1-year prepaid forward.

Pricing Prepaid Forwards with Continuous Dividends

Suppose the stock price is S_0 and the annualized daily compounded dividend yield is δ. Then the prepaid forward price is given by:

$$F_{0,T}^P = S_0 e^{-\delta T}$$

Example

Suppose that the index is $125 and the annualized daily compounded dividend yield is 3%.
Calculate the 1-year prepaid forward.

Pricing Forwards

The forward price $F_{0,T}$ is just the future value of the prepaid forward price:

$$F_{0,T} = FV(F_{0,T}^P)$$

Given that the annual continuously compounded risk-free rate is r

	$F_{0,T}^P$	$F_{0,T}$
No Dividend	S_0	$S_0 e^{rT}$
Discrete Dividends	$S_0 - \sum_{i=1}^{n} PV_{0,t_i}(D_{t_i})$	$S_0 e^{rT} - \sum_{i=1}^{n} FV_{t_i,T}(D_{t_i})$
Continuous Dividends	$S_0 e^{-\delta T}$	$S_0 e^{(r-\delta)T}$

Note

If your investment is on the time period $[t, T]$, then:

r	Discount Factor	Accumulation Factor
annual continuously compounded	$e^{-r(T-t)}$	$e^{r(T-t)}$
the effective annual	$\frac{1}{(1+r)^{(T-t)}}$	$(1+r)^{(T-t)}$

A stock has a current price of \$50. The continuous compounded risk-free interest rate is %8, and the dividend yield on the index is x.

1. Find the values of x that make the fair price of a 9-month forward contract grater than 52.

2. If $x = \%6$, find the time to delivery of the forward contract with fair price of 55.

A stock index stands at 100. The continuous compounded risk-free interest rate is %8.

1. What should the prepaid forward price of a 6-months prepaid forward contract on the index be?

2. If the market prepaid forward price of the contract is 98.8 how can you make an arbitrage profit?

A stock index stands at 100. The continuous compounded risk-free interest rate is %8 and the dividend yield on the index is %3.

1. What should the prepaid forward price of a 6-months prepaid forward contract on the index be?

2. If the market prepaid forward price of the contract is 98.8 how can you make an arbitrage profit?

A non dividend-paying stock has a current price of 100, and the effective annual interest rate is %5.

1. Calculate the one-year forward price on one stock.
2. If the market forward price of the contract is 104 how can you make an arbitrage profit?

A stock has a current price of \$50. The continuous compounded risk-free interest rate is %8. The stock is going to pay a dividend of \$0.5 one month from now and another dividend of \$1 five months from now.

1. What should the prepaid forward price of a 6-months prepaid forward contract on the stock be?

2. What should the forward price of a 6-months forward contract on the stock be?

3. If the market forward price of the contract is 50.2, construct an arbitrage portfolio?

You are giving the following information about stock XYZ

- The current price of the stock is 35 per share
- The expected continuously compounded annual rate of return is 8%
- The stock pays semi-annual dividends of 0.32 per share, with the next dividend to be paid two months from now.

The continuously compounded annual risk free rate is 4%

Calculate the current one-year forward price for stock XYZ

(A) 34.37
(B) 35.77
(C) 36.43
(D) 37.23
(E) 37.92

Market maker in stock index forward contracts observed a 6-month forward price of 112 on the index. The index spot price is 110 and the continuously compounded annual dividend yield on the index is 2%. The continuously compounded risk-free interest rate is 5%. Describe actions the market maker could take to exploit an aibtrage opportunity and calculate the resulting profit (per index unit)

(A) Buy observed forward, sell synthetic forward, Profit=0.34

(B) Buy observed forward, sell synthetic forward, Profit=0.74

(C) Buy observed forward, sell synthetic forward, Profit=1.35

(D) Sell observed forward, buy synthetic forward, Profit=0.78

(E) Sell observed forward, buy synthetic forward, Profit=0.34

The following relates to one share of XYZ stock:

- The current price is 100

- The forward price for delivery in one year is 105

- P is the expected price in one year.

Determine which of the following statements about P is TRUE

(A) $P < 100$

(B) $P = 100$

(C) $100 < P < 105$

(D) $P = 105$

(E) $P > 105$

The current price of medical company's stock is 75. The expected value of a stock price in 3 years is 90 per share. The stock pays no divided.

You are also given:

- The risk-free interest rate is positive
- There are no transaction cost
- Investor requires compensation for risk.

The price a three-year forward contract on a share of this stock is X, and at this price investor willing to enter into the forward contract.

Determine what you can conclude about X

(A) $X < 75$

(B) $X = 75$

(C) $75 < X < 90$

(D) $X = 90$

(E) $X > 90$