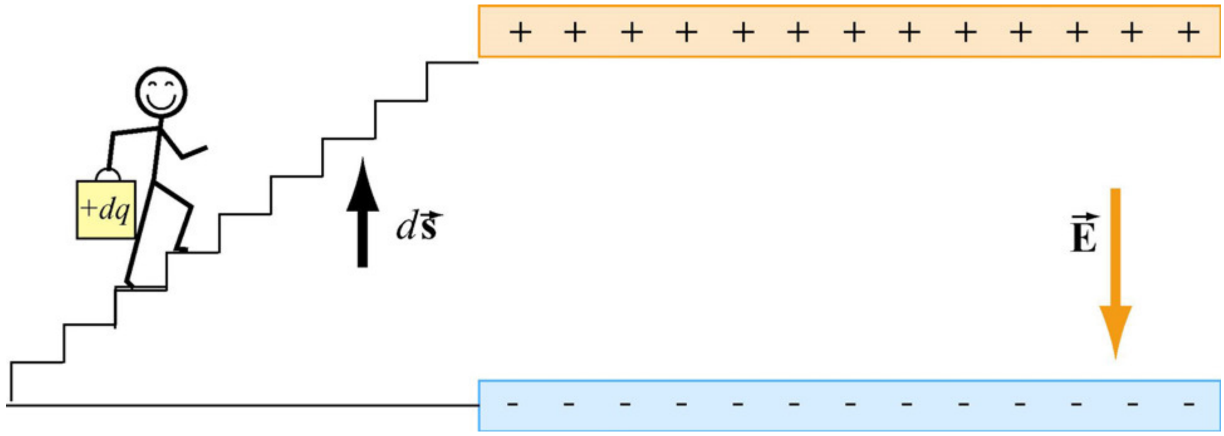
26.4 Energy stored in a charged capacitor

http://web.mit.edu/8.02t/www/802TEAL3D/visualizations/coursenotes/modules/guide05.pdf

As discussed in the introduction, capacitors can be used to stored electrical energy. The amount of energy stored is equal to the work done to charge it. During the charging process, the battery does work to remove charges from one plate and deposit them onto the other.



Let the capacitor be initially uncharged. In each plate of the capacitor, there are many negative and positive charges, but the number of negative charges balances the number of positive charges, so that there is no net charge, and therefore no electric field between the plates. We have a magic bucket and a set of stairs from the bottom plate to the top plate.

We start out at the bottom plate, fill our magic bucket with a charge $+dq$, carry the bucket up the stairs and dump the contents of the bucket on the top plate, charging it up positive to charge $+dq$. However, in doing so, the bottom plate is now charged to $-dq$. Having emptied the bucket of charge, we now descend the stairs, get another bucketful of charge $+dq$, go back up the stairs and dump that charge on the top plate. We then repeat this process over and over. In this way we build up charge on the capacitor, and create electric field where there was none initially.

Suppose the amount of charge on the top plate at some instant is $q+$, and the potential difference between the two plates is $|\Delta V|=q/C$. To dump another bucket of charge $+dq$ on the top plate, the amount of work done to overcome electrical repulsion is dW=ΔVdq. If at the end of the charging process, the charge on the top plate is $+Q$, then the total amount of work done in this process is

$$dW = Vdq$$

$$dW = (\frac{q}{C})dq$$

$$W = \int_0^q (\frac{q}{C})dq$$

$$W = \frac{1}{2}\frac{q^2}{C} \qquad \text{or} \qquad W = \frac{1}{2}CV^2$$

This is equal to the electrical potential energy of the system:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}qV$$

For parallel plat:

$$C = \varepsilon_0 \frac{A}{d} \qquad \text{and} \qquad V = Ed$$

$$U = \frac{1}{2}(\varepsilon_0 \frac{A}{d})(Ed)^2 = \frac{1}{2}\varepsilon_0(Ad)\, E^2$$

Since the quantity *Ad* represents the volume between the plates, we can define the electric energy density u as:

$$u = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

26.5 Capacitor with Dielectric

In many capacitors there is an insulating material such as paper or plastic between the plates. Such material, called a dielectric, can be used to maintain a physical separation of the plates. Since dielectrics break down less readily than air, charge leakage can be minimized, especially when high voltage is applied.
Experimentally it was found that capacitance *C* increases when the space between the conductors is filled with dielectrics. To see how this happens, suppose a capacitor has a capacitance when there is no material between the plates. When a

dielectric material is inserted to completely fill the space between the plates, the capacitance increases to

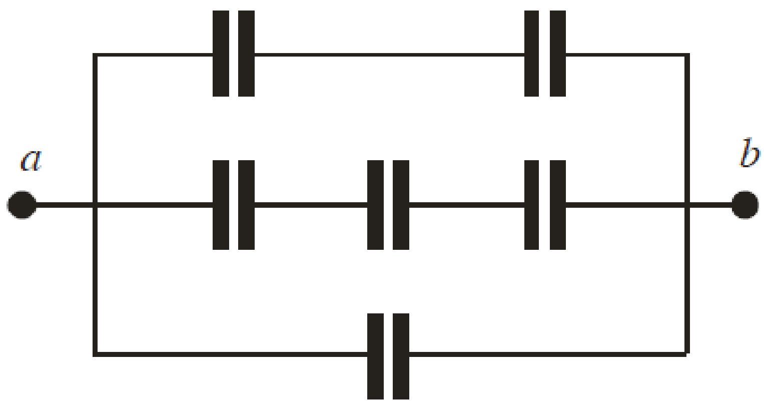
$$C=KC_0$$

Where K is called the dielectric constant. In the Table below, we show some dielectric materials with their dielectric constant. Experiments indicate that all dielectric materials have K>1. Note that every dielectric material has a characteristic dielectric strength which is the maximum value of electric field before breakdown occurs and charges begin to flow.

Material	K	Dielectric strength (10^6 V/m)
Air	1.00059	3
Paper	3.7	16
Glass	4–6	9
Water	80	–

Examples:

1- What is the equivalent capacitance of the capacitors combination in the drawing below, knowing that the capacitance of each is C?



2- When an insulating material, with a dielectric constant K=3, is inserted between the plates of a capacitor whose capacitance equals Co, what is the new capacitance, C?

3- If the stored energy of a capacitor, disconnected from the electric circuit, equals Uo, what is its stored energy, U, after inserting a dielectric material, whose K = 5, between its plates?

4- For a capacitor having C = 6 μF, d = 0.07 *mm*, and a dielectric material with $E_{max} = 14 \times 10^6$ *V/m*, what is the maximum charge that can accumulates on its plate?