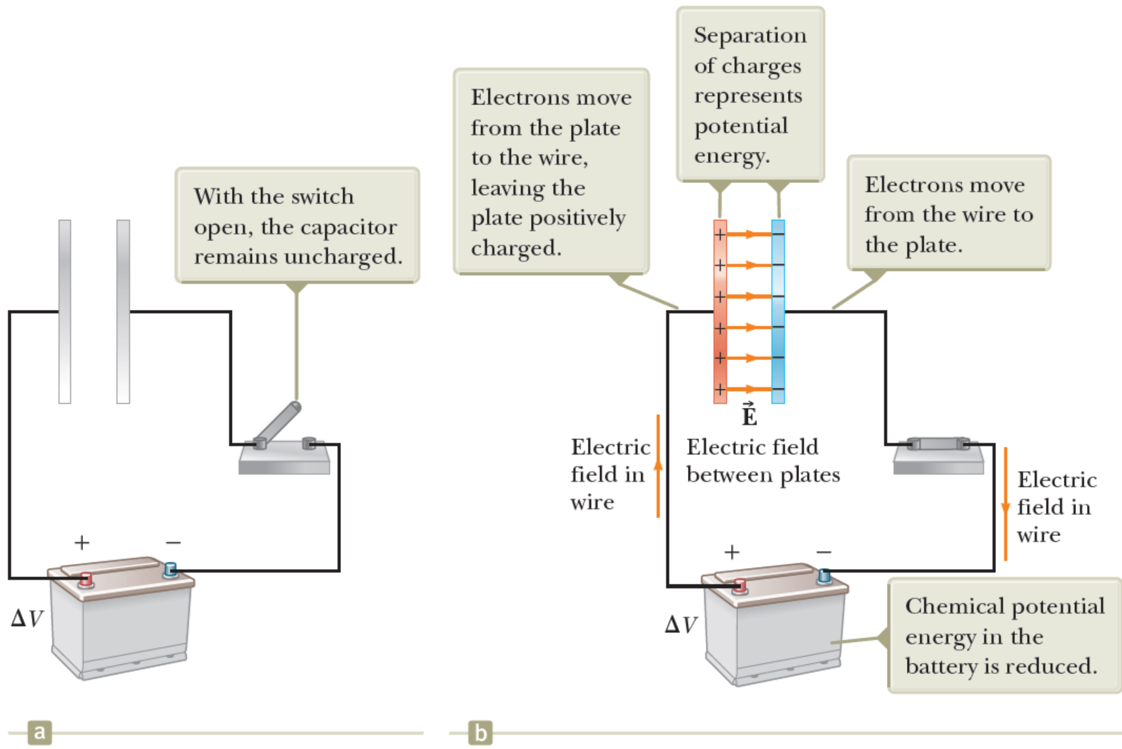
26.4 Energy stored in a charged capacitor

Capacitors are devices that store electrical energy by accumulating charge on their plates. The energy stored in a capacitor originates from the work required to move charges against the electric field.



- Calculation of Energy Stored

Consider a capacitor with charge Q and potential difference V. The small amount of work dW required to move an infinitesimal charge dq onto the capacitor is:

$$dW = V dq$$

$$dW = (\frac{q}{C}) dq$$

To find the total work required to charge the capacitor from q=0 to q=Q, we integrate:

$$W = \int_0^q (\frac{q}{C}) dq$$

Solving the integral:

$$W = \frac{1}{2}\frac{q^2}{C} \qquad \text{or} \qquad W = \frac{1}{2}\mathrm{C}V^2$$

This is equal to the electrical potential energy of the system:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\mathrm{qV}$$

Where:

- U is the **stored energy**
- Q is the **charge** on the capacitor
- V is the **potential difference**
- C is the **capacitance**

- Energy Density in a Capacitor

The energy stored in a parallel-plate capacitor can be related to the electric field E.

Since: V=Ed and $C = \varepsilon_0 \frac{\mathrm{A}}{\mathrm{d}}$

The energy density u (energy per unit volume) can be calculated as follows:

$$U = \frac{1}{2}(\varepsilon_0 \frac{A}{\mathrm{d}})(Ed)^2 = \frac{1}{2}\varepsilon_0(\mathrm{Ad})\, E^2$$

Since the quantity (*Ad)* represents the volume between the plates, we can define the electric energy density u as:

$$u = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 \mathrm{E}^2$$

Where:

- ε_0 is the **permittivity of free space**
- E is the **electric field** between the plates

This formula is crucial in understanding how energy is distributed in electric fields.

The relationship between energy stored in a capacitor and its capacitance depends on the conditions under which it changes. Let's analyze different cases using the energy formula:

$$U = \frac{1}{2}CV^2$$

Where:

- U is the stored energy,
- C is the capacitance,
- V is the voltage applied across the capacitor.

Case 1: Constant Voltage (V is fixed)

If the voltage across the capacitor remains constant while increasing capacitance, the stored energy **increases** because:

$$U \propto C$$

Example:
If C doubles while V remains the same, then U also doubles.
Thus, a larger capacitance at the same voltage results in more stored energy.

Case 2: Constant Charge (Q is fixed)

Since capacitance is related to charge and voltage by: Q=CV

We can rewrite energy as: $U = \frac{Q^2}{2C}$

Here, **energy is inversely proportional to capacitance**. If capacitance increases while the charge remains constant, the stored energy **decreases**.

Example:
If C doubles while Q is fixed, then U is **halved**.

Conclusion

- **If voltage is held constant:** Increasing capacitance **increases** stored energy.
- **If a charge is held constant,** Increasing capacitance **decreases** stored energy.

So, whether energy increases or decreases depends on whether voltage or charge is controlled.

- Applications

- **Capacitors in Circuits**: Used in **power supplies**, **signal processing**, and **energy storage**.
- **Energy Storage in Electric Vehicles**: Supercapacitors store large amounts of energy for quick charge/discharge cycles.
- **Medical Devices**: Defibrillators use capacitors to deliver sudden bursts of electrical energy.

Example-1:

(a) A 3.00 μF capacitor is connected to a 12.0 V battery. How much energy is stored in the capacitor?

(b) How much energy would have been stored if the capacitor had been connected to a 6.00 V battery?

Example-2:

A uniform electric field E = 3000 V/m exists within a particular region. What volume of space contains an energy equal to 1.00×10^{-7} J? Express your answer.

Example-3:

Two capacitors, C_1 = 4.0 μF and C_2 = 6.0 μF, are connected in series and charged to 100 V. Find the total energy stored.

Example-4:

A parallel-plate capacitor has a plate area of 0.02 m^2, a plate separation of 2 mm, and is air-filled ($\varepsilon_0 = 8.85 \times 10^{-12}$ F/m). Find the energy density if the electric field between the plates is 5000 V/m.