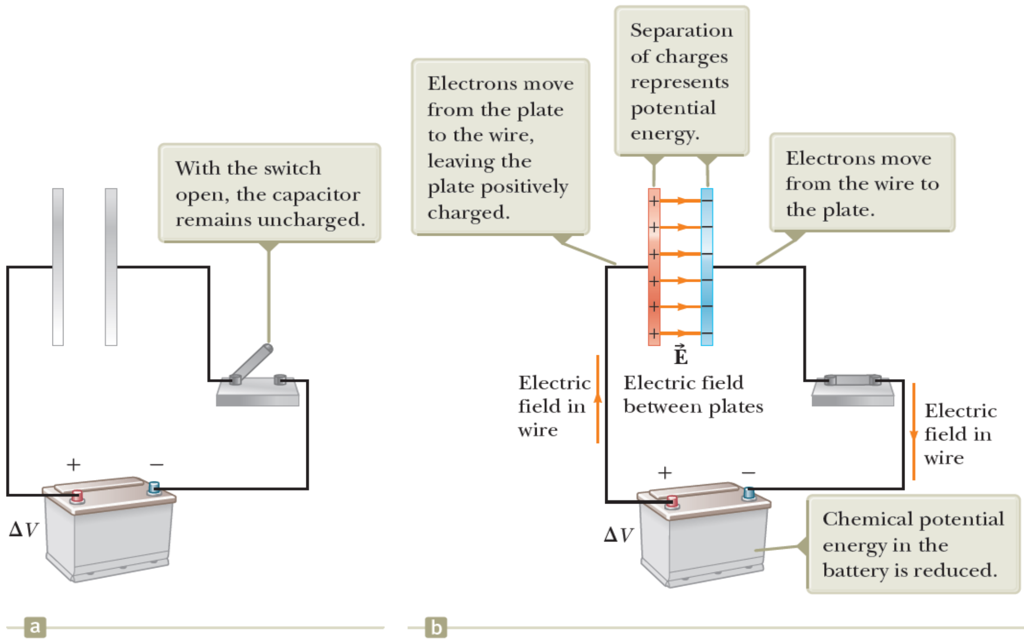
26.4 Energy stored in a charged capacitor

Capacitors are devices that store electrical energy by accumulating charge on their plates. The energy stored in a capacitor originates from the work required to move charges against the electric field.



- Calculation of Energy Stored

Consider a capacitor with charge Q and potential difference V. The small amount of work dW required to move an infinitesimal charge dq onto the capacitor is:

$$dW = Vdq$$

$$dW = \left(\frac{q}{C}\right)dq$$

To find the total work required to charge the capacitor from q=0 to q=Q, we integrate:

$$W = \int_0^q \left(\frac{q}{C}\right)dq$$

Solving the integral:

$$W = \frac{1}{2}\frac{q^2}{C} \qquad \text{or} \qquad W = \frac{1}{2}CV^2$$

This is equal to the electrical potential energy of the system:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\text{qV}$$

Where:

U is the **stored energy**
Q is the **charge** on the capacitor
V is the **potential difference**
C is the **capacitance**

- **Energy Density in a Capacitor**

The energy stored in a parallel-plate capacitor can be related to the electric field E.

Since: V=Ed and $C = \varepsilon_0 \frac{\text{A}}{\text{d}}$

The energy density u (energy per unit volume) can be calculated as follows:

$$U = \frac{1}{2}(\varepsilon_0 \frac{A}{\text{d}})(Ed)^2 = \frac{1}{2}\varepsilon_0(\text{Ad})\, E^2$$

Since the quantity *(Ad)* represents the volume between the plates, we can define the electric energy density u as:

$$u = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0\text{E}^2$$

Where:

ε_0 is the **permittivity of free space**
E is the **electric field** between the plates.

$$u \propto E^2$$

- **That is, the energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point.**
- **Think about it:**

  You have three capacitors and a battery. In which of the following combinations of the three capacitors is the maximum possible energy stored when the combination is attached to the battery?
  (a) series
  (b) parallel
  (c) no difference because both combinations store the same amount of energy.

🔍**The relationship between the energy stored in a capacitor and its capacitance depends on the conditions under which it changes**. Let's analyze different cases using the energy formula:

$$U = \frac{1}{2}CV^2$$

Where:

- U is the stored energy,
- C is the capacitance,
- V is the voltage applied across the capacitor.

Case 1: Constant Voltage (V is fixed)

If the voltage across the capacitor remains constant while increasing capacitance, the stored energy **increases** because:

$$U \propto C$$

Example:
If C doubles while V remains the same, then U also doubles.
Thus, a larger capacitance at the same voltage results in more stored energy.

Case 2: Constant Charge (Q is fixed)

Since capacitance is related to charge and voltage by: Q=CV

We can rewrite energy as: $U = \frac{Q^2}{2C}$

Here, **energy is inversely proportional to capacitance**. If capacitance increases while the charge remains constant, the stored energy **decreases**.

Example:
If C doubles while Q is fixed, then U is **halved**.

Conclusion

- **If voltage is held constant:** Increasing capacitance **increases** stored energy.
- **If a charge is held constant:** Increasing capacitance **decreases** stored energy.

So, whether energy increases or decreases depends on whether voltage or charge is controlled.

- **Applications**

- **Capacitors in Circuits**: Used in **power supplies**, **signal processing**, and **energy storage**.
- **Energy Storage in Electric Vehicles**: Supercapacitors store large amounts of energy for quick charge/discharge cycles.
- **Medical Devices**: Defibrillators use capacitors to deliver sudden bursts of electrical energy.

Exercise-1:

(a) A 3.00 μF capacitor is connected to a 12.0 V battery. How much energy is stored in the capacitor?

(b) How much energy would have been stored if the capacitor had been connected to a 6.00 V battery?

Exercise-2:

A uniform electric field E = 3000 V/m exists within a particular region. What volume of space contains an energy equal to 1.00×10^{-7} J? Express your answer in cubic meters and in liters.

Exercise-3:

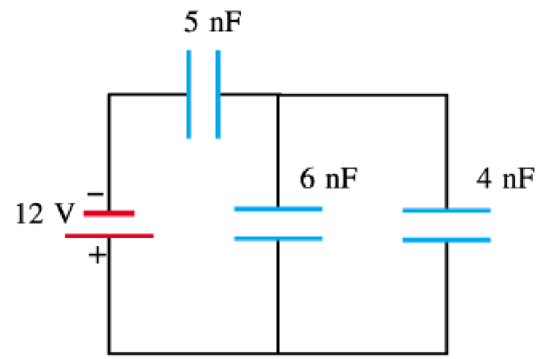
Two capacitors, C_1 = 4.0 μF and C_2 = 6.0 μF, are connected in series and charged to 100 V. Find the total energy stored.

Exercise-4:

A parallel-plate capacitor has a plate area of 0.02 m^2, a plate separation of 2mm, and is air-filled ($\varepsilon_0 = 8.85 \times 10^{-12}$ F/m). Find the energy density if the electric field between the plates is 5000 V/m.

Exercise-5:

For the system of capacitors shown in the figure.



(a) What is the magnitude of the electric potential difference across the 4 nF capacitor in (V) unit? (answer: 4)

(b) The total energy stored by the system of capacitors in (nJ) unit? (answer: 240)