

## Chapter 25

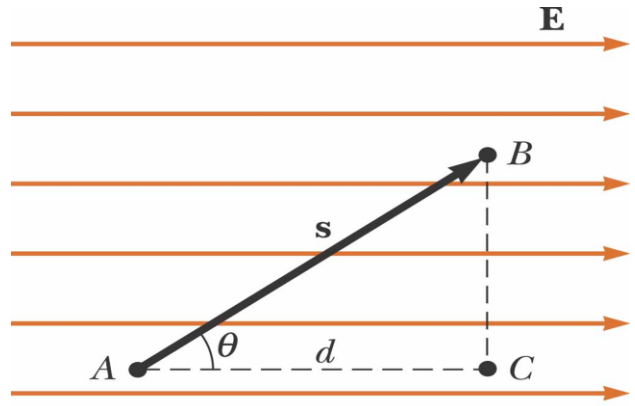
### Electrical Potential

#### 25.1 Potential Difference and Electric Potential

- What is the electric potential?
- What is the electrical potential energy?
- What is the potential difference?

Let's place a positive test charge in a uniform electric field.

What's the work done moving the charge from A to B?



$$dW = F dl \cos\theta$$

However, the total work done is given by:

$$W = \int_A^B F dl \cos\theta$$

Since  $F = Eq$  one can rewrite  $W$  as:

$$W = \int_A^B qE dl \cos\theta$$

But work is the negative of the change in potential energy,

$$\Delta U = U_f - U_i = -\int_A^B qE dl \cos\theta = -\int_A^B q\vec{E} \cdot d\vec{l}$$

This means that the potential energy is changed by an amount of  $-q \vec{E} \cdot d\vec{l}$

**Electric potential** is defined as potential energy per unit charge:

$$V = \frac{\Delta U}{q} = -\int_A^B \vec{E} \cdot d\vec{l}$$

Electric potential is a scalar characteristic of an electric field, independent of any charges that may be placed in the field.

Because electric potential is a measure of potential energy per unit charge, the SI unit of both electric potential and potential difference is joules per coulomb, which is defined as a volt (V):

$$1 \text{ V} \equiv 1 \text{ J/C}$$

## 25.2 Potential difference in Uniform electric field

Often what one is interested in is the *difference* in potential between two particular points. In words, a potential difference is the ratio of the work you have to do to move a charge from A to B, divided by the magnitude of that charge

$$\Delta V_{AB} = -\int_A^B \mathbf{E} d\mathbf{l} \cos \theta = V_B - V_A$$
$$\Delta V_{AB} = -E \int_A^B dl \cos \theta = -Ed$$

Often the potential difference is taken with respect to ground potential (recall  $mgh$  for gravitational potential energy)

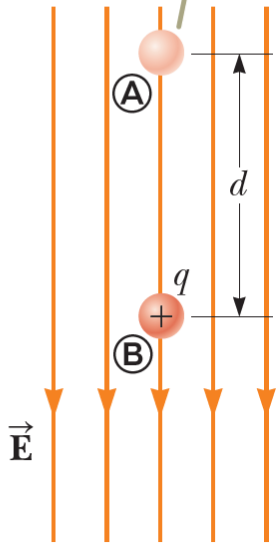
This expression rearranged gives the work required to move a charge:

$$W = V_{ab}q.$$

**Note:**

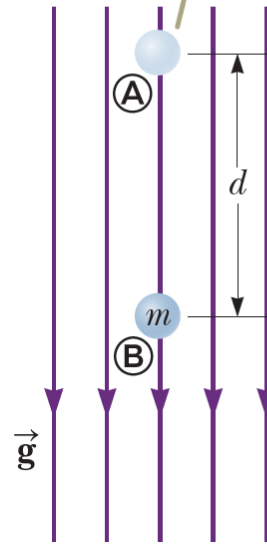
**The Sign of  $\Delta V$ :** The negative sign is because we started at point **(A)** and moved to a new point in the *same* direction as the electric field lines. If we started from **(B)** and moved to **(A)**, the potential difference would be  $1Ed$ . In a uniform electric field, the magnitude of the potential difference is  $Ed$ , and the direction of travel can determine the sign.

When a positive charge moves from point (A) to point (B), the electric potential energy of the charge–field system decreases.



a

When an object with mass moves from point (A) to point (B), the gravitational potential energy of the object–field system decreases.



b

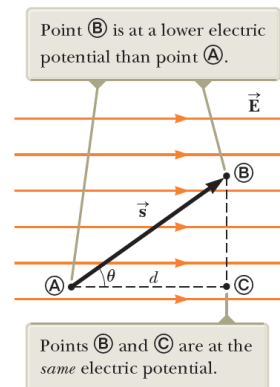
**Electric potential is often compared to gravitational potential:**

- A mass in a gravitational field has potential energy based on its height.
- Similarly, a charge in an electric field has electric potential based on its position.

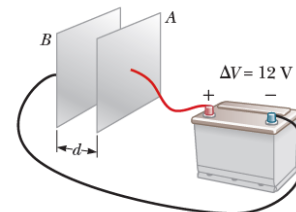
If we take the point A at infinity, one can write the electric potential at any point B as:

$$V_B - 0 = -\int_{\infty}^B E dl \cos \theta = V_B$$

Prove that  $V_B = V_C$ .



**Example-1:** A battery has a specified potential difference  $\Delta V$  between its terminals and establishes that potential difference between conductors attached to the terminals. A 12-V battery is connected between two parallel plates as shown in the Figure. The separation between the plates is  $d = 0.30$  cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and we do not consider locations near the plate edges.) Find the magnitude of the electric field between the plates.



**Example-2:** A proton is released from rest at point  $\textcircled{\text{A}}$  in a uniform electric field that has a magnitude of  $8.0 \times 10^4$  V/m (see the figure). The proton undergoes a displacement of magnitude  $d = 0.50$  m to point  $\textcircled{\text{B}}$  in the direction of  $\vec{E}$ . Find the speed of the proton after completing the displacement.

