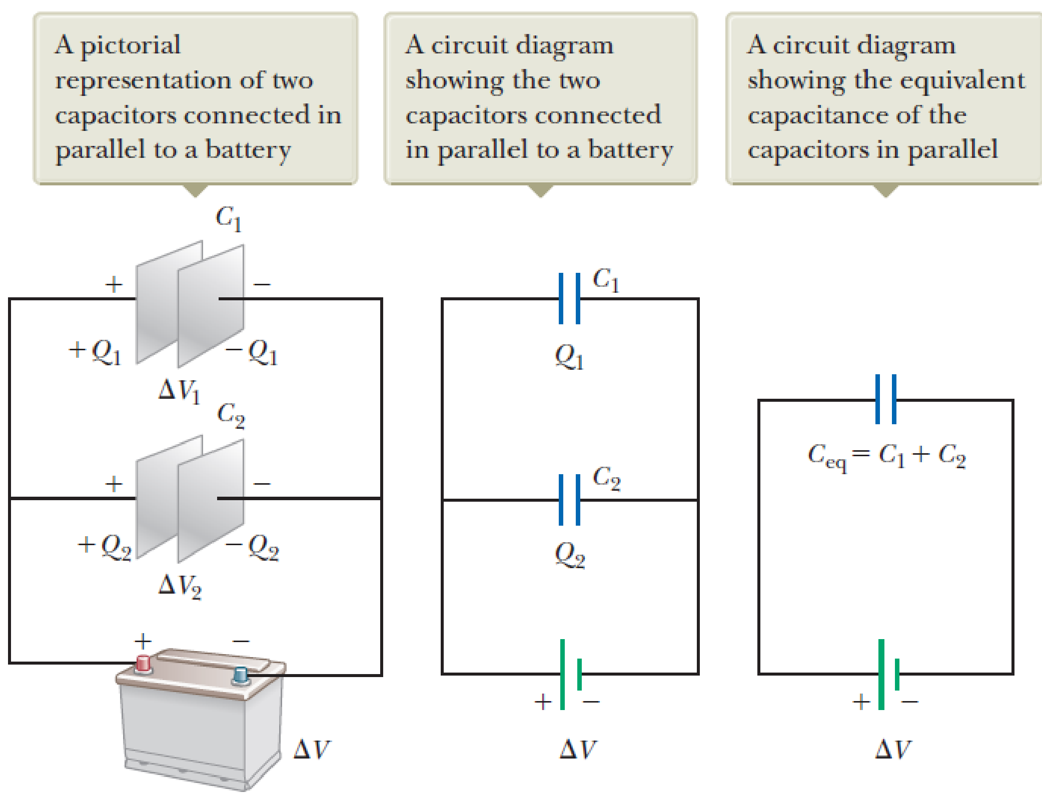
25.3 Combination of capacitors:

In electric circuits, capacitors can be combined in series, parallel, or a combination of both to achieve the desired capacitance. Understanding these combinations is crucial for designing circuits in various applications, including energy storage, filtering, and signal processing.

I- Parallel Combination:

As shown in the Figure, two connected capacitors form a parallel combination. The figure shows a circuit diagram of this capacitor combination. A conducting wire connects the left plates of the capacitors to the battery's positive terminal. They are, therefore, both at the same electric potential as the positive terminal. Likewise, the right plates are connected to the negative terminal and are, therefore, both at the same potential as the negative terminal. Thus, the individual potential differences across capacitors connected in parallel ***are all the same and are equal to the potential difference applied across the combination.***



When capacitors are connected **in parallel**, they all experience the **same voltage**, but the total charge is **divided** among them.

$V=V_1=V_2=V_3$

$q= q_1 + q_2 +q_3$

$$CV = C_1V + C_2V + C_3V$$

$$CV = (C_1 + C_2 + C_3)V$$

$$C = (C_1 + C_2 + C_3)$$

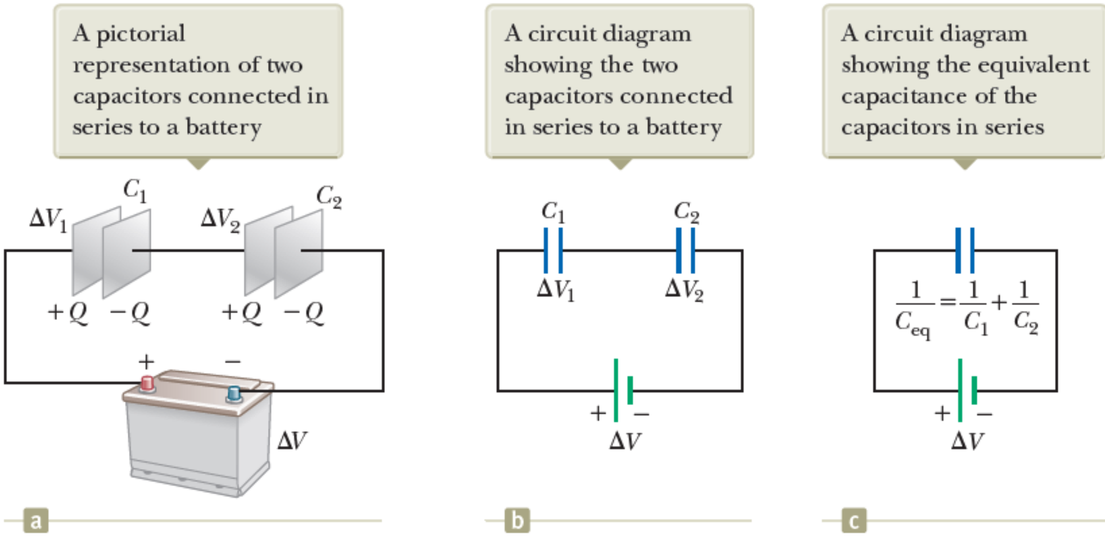
For an ensemble of capacitors:

$$C = \sum_{i=1}^{n} C_1 + C_2 + ... + C_n$$

Characteristics of Parallel Combination:

- **Same Voltage**: Each capacitor has the **same potential difference** V.
- **Charge Distribution**: The total charge is **split** among capacitors:
- **Increased Capacitance**: The equivalent capacitance is **larger than the largest capacitor** in the combination.

II- Series Combination:



Suppose three initially uncharged capacitors C_1 and C_2 are connected in series, as shown in the figure above. A potential difference ΔV is then applied across both capacitors. The left plate of capacitor 1 is connected to the positive terminal of the battery and becomes positively charged with a charge +*Q*, while the right plate of capacitor 2 is connected to the negative terminal and becomes negatively charged with charge –*Q* as electrons flow in. What about the inner plates? They were initially uncharged; now the outside plates each attract an equal and opposite charge. So the right plate of capacitor 1 will acquire a charge –*Q* and the left plate of capacitor +*Q*.

When capacitors are connected **in series**, the same charge Q is stored on each capacitor, but the total voltage is **divided** among them.

We see that the total potential difference is simply the sum of the two individual potential differences:

$$V = V_1 + V_2$$

$$q = q_1 = q_2$$

Since $V = \frac{q}{C}$

$$V = V_1 + V_2$$

$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

For an ensemble of capacitors:

$$\frac{1}{C} = \sum_{i=1}^{n} \frac{1}{C_1} + \frac{1}{C_1} ....+ \frac{1}{C_n}$$

Characteristics of Series Combination:

- **Same Charge**: Each capacitor stores the same amount of charge, Q.
- **Voltage Division**: The total voltage is **shared** among capacitors:
- **Decreased Capacitance**: The equivalent capacitance is **less than the smallest capacitor** in the series.

Examples:

Ex. 1::

Three capacitors (C_1=4μF, C_2=6μF, and C_3=12μF) are connected in **series**.
Find the equivalent capacitance. (2 μF)

Ex. 2:

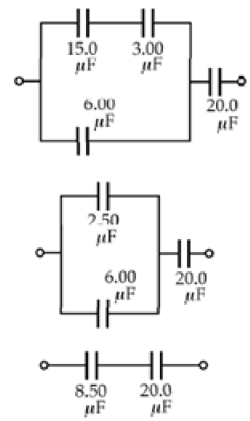
Three capacitors (C_1=4μF, C_2=6μF, and C_3=12μF) are connected in **parallel**.
Find the equivalent capacitance. (22 μF)

Ex. 3

Four capacitors are connected as shown in the figure.

(a) Find the equivalent capacitance between points *a* and *b*.

(b) Calculate the charge on each capacitor if ΔV_{ab} = 15.0 V.

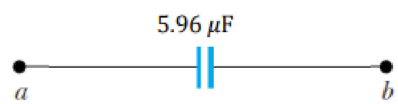


Solution:

(a) By looking at the circuits, we can see that the two capacitors (15 and 3) are connected in series and the equivalent capacitance can be calculated as follows:

$$\frac{1}{C_s} = \frac{1}{15} + \frac{1}{3} = \frac{1+5}{15} = \frac{6}{15}$$

Then, $C_s = \frac{15}{6} = 2.5\ \mu F$



The resultant capacitance (C_s) is connected in parallel with 6 µF, and the equivalent capacitance from this combination is calculated as follows:

$$C_p = 2.5 + 6 = 8.50\ \mu F$$

Great, we can see now this resultant capacitance is connected in series with 20 µF capacitor, and the final equivalent capacitance can be calculated as follows:

$$\frac{1}{C_{equivalent}} = \frac{1}{8.50} + \frac{1}{20}\ or\ \frac{2}{17} + \frac{1}{20} = \frac{40+17}{17x20} = \frac{57}{340}$$

Then, $C_{equivalent} = \frac{340}{57} = 5.96\mu F$

(b) To solve this part, we can do it by analyzing the circuits from bottom to up. The charge on the final equivalent capacitance is calculated as follows:

$$Q = CV = 5.96x10^{-6}x15 = 89.5x10^{-6}C = 89.5\ \mu C$$

Since 8.5 and 20 are connected in series, the charges on both capacitors are equal to 89.5 µC, but the potential difference on both capacitors is calculated as follows:

$V = \frac{Q}{C}$ $\quad V = \frac{89.5x10^{-6}}{8.5x10^{-6}} = 10.53\ V$ $\quad V = \frac{89.5x10^{-6}}{20x10^{-6}} = 4.47\ V$

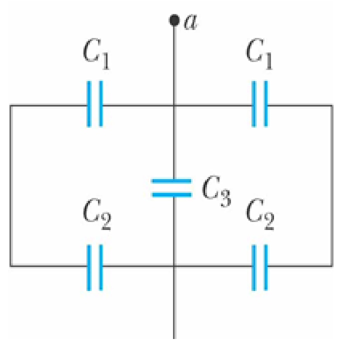
To check: 10.53+4.47 = 15

Coming back to the figure, we can say that the charges on 2.5 and 6 are different since they are connected in parallel and can be calculated as follows:
$Q = CV = 2.5x10^{-6}x10.53$ =26.325 µC
$Q = CV = 6x10^{-6}x10.53$ =63.18 µC
Note that: 26.325+63.18 = 89.5 µC
Finally, we can say that the charges on 15 and 3 are equal to the charge on 2.5 since they are connected in series.

Ex. 4

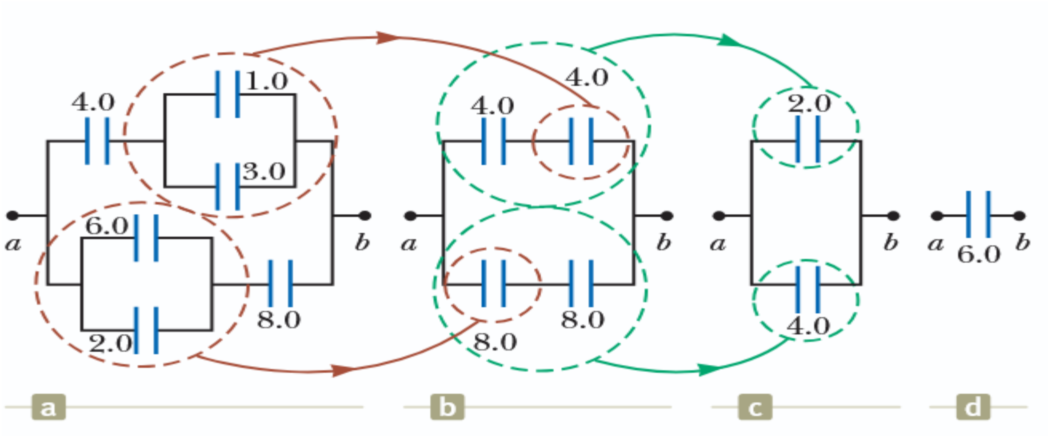
Find the equivalent capacitance between points a and b for the group of capacitors connected as shown in the figure. Take: C_1 = 5.00 µF, C_2 = 10.0 µF, and C_3 = 2.00 µF.

$$C_s = \left(\frac{1}{5.00} + \frac{1}{10.0}\right)^{-1} = 3.33\ \mu\text{F}$$
$$C_{p1} = 2(3.33) + 2.00 = 8.66\ \mu\text{F}$$
$$C_{p2} = 2(10.0) = 20.0\ \mu\text{F}$$
$$C_{eq} = \left(\frac{1}{8.66} + \frac{1}{20.0}\right)^{-1} = \boxed{6.04\ \mu\text{F}}$$



Example-5:

Find the equivalent capacitance between a and b for the combination of capacitors shown in the Figure. All capacitances are in microfarads.


4.0
1.0
3.0
6.0
2.0
8.0
a
b
4.0
4.0
8.0
8.0
a
b
2.0
4.0
a
b
a
6.0
b
a
b
c
d