

Chapter 25

Capacitance and Dielectric



25.1 Definition of capacitance

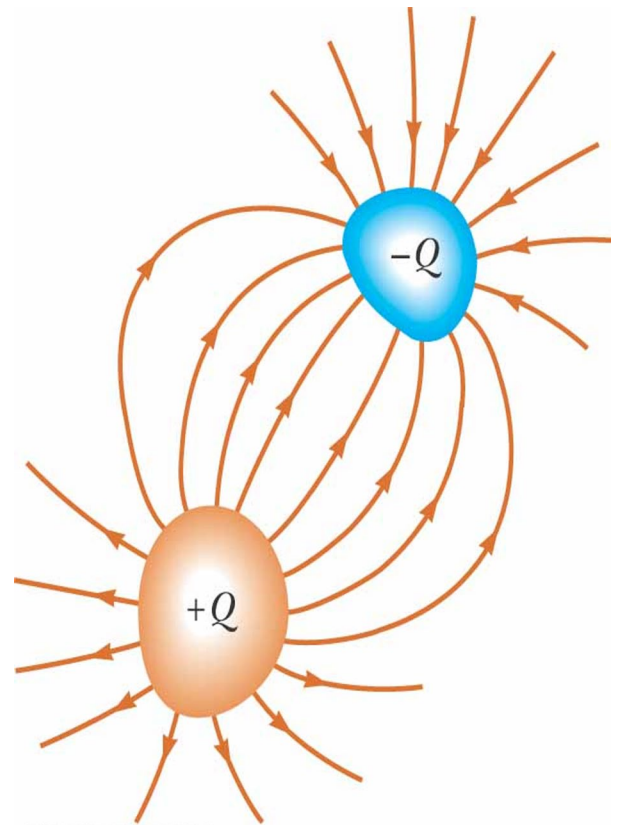
PhET Capacitor Lab: Visualize Charging, Voltage, Capacitance, and Electric Fields

https://phet.colorado.edu/sims/html/capacitor-lab-basics/latest/capacitor-lab-basics_en.html

In the front figure, you can see two conductors. Such a system is called a capacitor. Due to the different charges on the two plates, a difference in potential ΔV exists between the two plates. It has been found that,

$$q \propto \Delta V$$

So, $q = C\Delta V$ 25.1



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Where C is the capacitance. Capacitance is defined as the ratio of charge to potential difference.

$$C = \frac{q}{\Delta V}$$

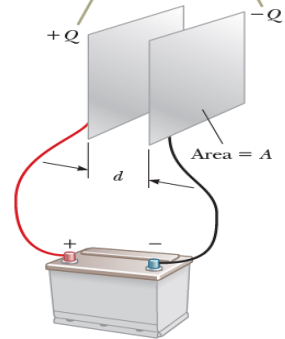
The unit of capacitance is Farad (F). In practice, the devices have capacitance ranging from microfarad ($\mu\text{F}=10^{-6}$ F) to picofarad ($\text{pF}=10^{-12}$ F).

How a capacitor charges when connected to a battery.

Step 1: Connecting the Capacitor to a Battery

- A capacitor consists of **two conductive plates** separated by an **insulating material** (dielectric).
- When the capacitor is **connected to a battery**, one plate is linked to the **positive terminal**, and the other plate is linked to the **negative terminal** through conducting wires.

When the capacitor is connected to the terminals of a battery, electrons transfer between the plates and the wires so that the plates become charged.



Step 2: Battery Creates an Electric Field

- The battery maintains a **potential difference (voltage, V)** between its terminals.
- This creates an **electric field** inside the wires, pushing **free electrons** to move.

Step 3: Electron Movement from the Negative Terminal

- Electrons from the **negative terminal** of the battery move through the wire **toward one plate** of the capacitor.
- As electrons accumulate, this plate **gains a negative charge**.

Step 4: Electron Removal from the Positive Plate

- At the same time, the **battery pulls electrons** away from the **other plate**, leaving it with a **positive charge**.
- The removal of electrons is driven by the attraction of positive ions inside the battery.

Step 5: Charge Accumulation and Electric Field Formation

- As the capacitor charges, the **potential difference** between its plates **increases**, opposing further electron flow.
- Inside the capacitor, an **electric field** forms **between the plates**, storing **electrostatic energy** in the dielectric material.

Step 6: Charging Stops When Equilibrium is Reached

- The capacitor continues charging until its voltage **matches the battery's voltage**.
- At this point, there are **no more current flows**, and the capacitor is **fully charged**.
- The charge stored is given by $Q=CV$, where:
 - Q is the charge,
 - C is the capacitance,
 - V is the battery voltage.

Comments:

- **A capacitor does not "store" electrons**, but rather **separates charge** across its plates.
- **Electrons flow through the wires but do not pass between the plates** due to the dielectric.
- **Energy is stored in the electric field** between the plates, which can later be discharged to power a circuit.

25.2 Calculating Capacitance

❖ Parallel-plate capacitors:

In the figure, two parallel plates carry equal and opposite charges ($\pm Q$) and are separated by a distance d . As we discussed in Chapter 23, the surface charge σ is $\frac{q}{A}$. Then, the electric field between the two plates is given by:

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A} \quad 25.2$$

Also, the electric potential can be written as,

$$V = \int_0^d E \, dx = E \, d \quad 25.3$$

Comparing Eq. (25.2) and (25.3), we can derive an expression for the capacitance of the parallel-plate as follows,

$$\begin{aligned} V &= \frac{q}{\epsilon_0 A} d = \frac{d}{\epsilon_0 A} q \\ q &= \frac{\epsilon_0 A}{d} V \end{aligned} \quad 25.4$$

$$\Rightarrow C = \frac{\epsilon_0 A}{d} \quad 25.5$$

Then, the capacitance of the parallel plate depends on:

- 1- **The distance between the two plates,**
- 2- **The area of each plate, and**
- 3- **The material between the two plates.**

Example 1: A capacitor stores charge Q at a potential difference ΔV . What happens if the voltage applied to the capacitor by a battery is doubled to $2\Delta V$?

- (a) The capacitance falls to half its initial value, and the charge remains the same.
- (b) The capacitance and the charge both fall to half their initial values.
- (c) The capacitance and the charge both double.
- (d) The capacitance remains the same, and the charge doubles.

Example-2

A parallel-plate capacitor with air between the plates has an area $A = 2.00 \times 10^{-4} \text{ m}^2$ and a plate separation $d = 1.00 \text{ mm}$. Find its capacitance.

Solution:

$$C = \epsilon_0 A / d$$

$$\text{where } \epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

Calculation:

$$C = (8.85 \times 10^{-12})(2.00 \times 10^{-4}) / (1.00 \times 10^{-3})$$

$$C = 1.77 \times 10^{-12} \text{ F} = 1.77 \text{ pF}$$

Example-3

An air-filled capacitor consists of two parallel plates, each with an area of 7.60 cm^2 , separated by a distance of 1.80 mm . A 20.0 V potential difference is applied to these plates. Calculate (a) the electric field between the plates, (b) the surface charge density, (c) the capacitance, and (d) the charge on each plate.

Solution:

Given:

$$\text{Area } A = 7.60 \text{ cm}^2 = 7.60 \times 10^{-4} \text{ m}^2$$

$$\text{Separation } d = 1.80 \text{ mm} = 1.80 \times 10^{-3} \text{ m}$$

$$\text{Potential difference } V = 20.0 \text{ V}$$

(a) Electric field:

$$E = V / d = 20.0 / (1.80 \times 10^{-3})$$

$$E = 1.11 \times 10^4 \text{ V/m}$$

(b) Surface charge density:

$$\sigma = \epsilon_0 E = (8.85 \times 10^{-12})(1.11 \times 10^4)$$

$$\sigma = 9.82 \times 10^{-8} \text{ C/m}^2$$

(c) Capacitance:

$$C = \epsilon_0 A / d = (8.85 \times 10^{-12})(7.60 \times 10^{-4}) / (1.80 \times 10^{-3})$$

$$C = 3.74 \times 10^{-12} \text{ F} = 3.74 \text{ pF}$$

(d) Charge on each plate:

$$Q = C V = (3.74 \times 10^{-12})(20.0)$$

$$Q = 7.48 \times 10^{-11} \text{ C}$$

Example-4:

When a potential difference of 150 V is applied to the plates of a parallel-plate capacitor, the plates carry a surface charge density of 30.0 C/m². What is the spacing between the plates?

Solution:

Given:

Potential difference $V = 150 \text{ V}$

Surface charge density $\sigma = 30.0 \text{ C/m}^2$

Using:

$$E = \sigma / \epsilon_0$$

$$E = V / d$$

Therefore:

$$d = \epsilon_0 V / \sigma$$

Calculation:

$$d = (8.85 \times 10^{-12} \times 150) / 30.0$$

$$d = 4.43 \times 10^{-11} \text{ m}$$

Example-5:

What is the capacitance of the capacitor shown in the figure, knowing that the plates are square in shape?

Solution:

The capacitor consists of two square parallel plates.

Side length of each square plate = 10 cm = 0.10 m

Plate separation $d = 0.5 \text{ mm} = 5.0 \times 10^{-4} \text{ m}$

The space between the plates is air ($\epsilon \approx \epsilon_0$).

Step 1: Calculate the area of one plate

$$A = (\text{side length})^2 = (0.10)^2 = 0.010 \text{ m}^2$$

Step 2: Use the parallel-plate capacitance formula

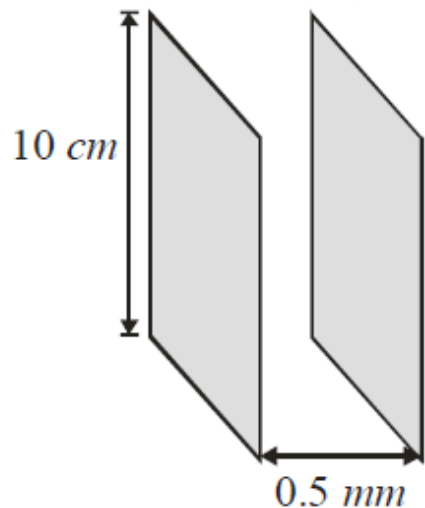
$$C = \epsilon_0 A / d$$

where $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$

Step 3: Substitute the values

$$C = (8.85 \times 10^{-12} \times 0.010) / (5.0 \times 10^{-4})$$

Step 4: Calculate



$$C = 1.77 \times 10^{-10} \text{ F}$$

Final Answer:

The capacitance of the capacitor is

$$C = 1.77 \times 10^{-10} \text{ F} = 177 \text{ pF}$$

Think about it: What is the difference between a battery and a capacitor?

CAPACITOR VS. BATTERY



CAPACITOR

- The potential energy is stored in the electric field



BATTERY

- The potential energy is stored in the form of chemical energy which is later converted to electric energy

Think about it !!

How Can Capacitors Significantly Reduce Costs and Improve Train Efficiency?



Think about it: Why are multiple capacitors used in electric circuits?

