24.3 Applications of Gauss' Law

What is the charge density?

Charge density refers to the amount of electric charge present per unit of a given dimension (length, surface area, or volume) in a material or space.

- If a charge Q is uniformly distributed throughout a volume V, the volume charge density ρ is defined by:
  $\rho = \frac{Q}{V} \ (\frac{C}{m^3})$
- If a charge Q is uniformly distributed throughout a surface of area A, the surface charge density σ is defined by:
  $\sigma = \frac{Q}{A} \ (\frac{C}{m^3})$
- If a charge Q is uniformly distributed throughout a line of length *l*, the linear charge density λ is defined by:
  $\lambda = \frac{Q}{l} \ (\frac{C}{m^3})$

One of the approaches used to determine the electric filed is Gauss's law.

What conditions should the Gauss's surface satisfy when calculating electric filed due to charge distributions?

Each portion of the surface satisfies one or more of the following conditions:

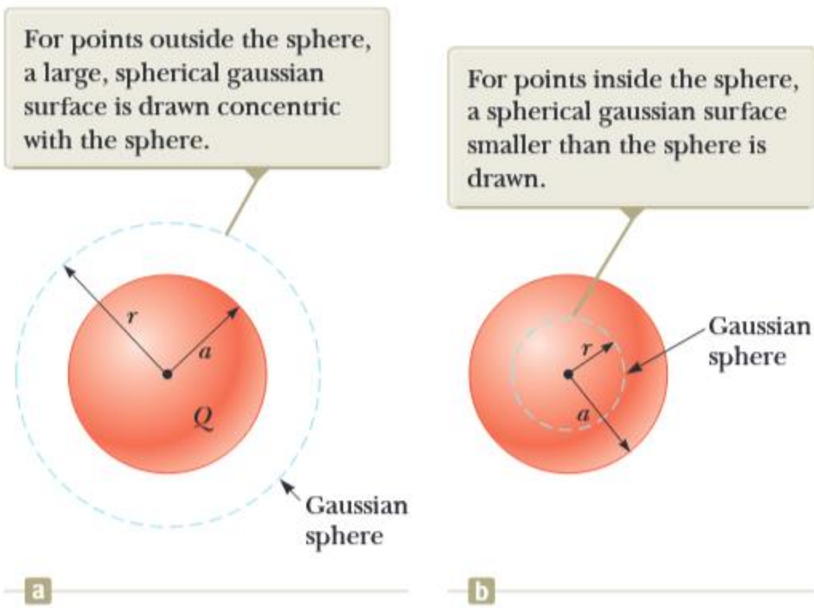
- The value of the electric field can be argued from symmetry to be constant over the surface.
- The dot product of $\vec{\boldsymbol{E}}.\overrightarrow{\Delta\boldsymbol{A}}$ can be expressed as a simple algebraic product EdA because $\vec{E}$and $\overrightarrow{\Delta A}$ are parallel.
- The dot product is 0 because $\vec{E}$and $\overrightarrow{\Delta A}$ are perpendicular.
- The electric field is constant over the portion of the surface.

I- Sphere of Uniform Charge

An insulating solid sphere of radius a has a uniform volume charge density r and carries a total positive charge Q.

(a) Calculate the magnitude of the electric field at a point outside the sphere.

(b) Find the magnitude of the electric field at a point inside the sphere.



$$\rho = \frac{Q}{V}$$

$$a)\ E = k_e \frac{Q}{r^2}, \qquad r \succ a$$

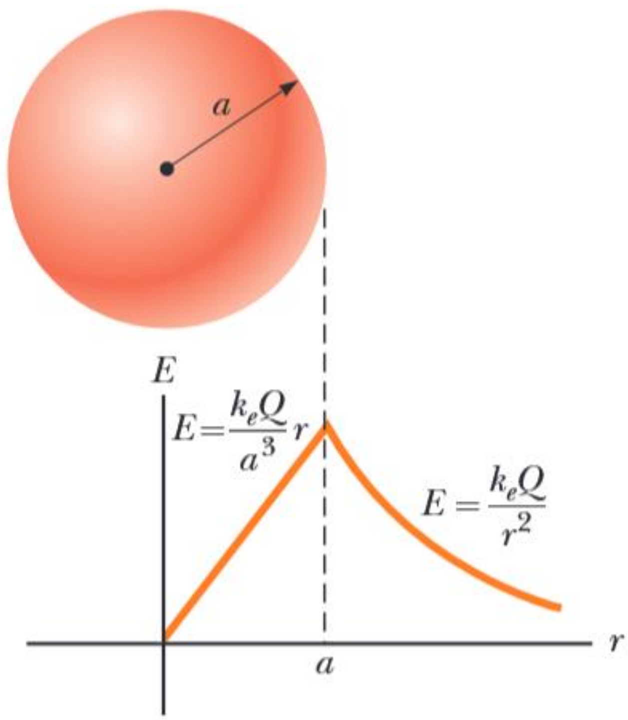
$$b)\ q_{in} = \rho V' = \rho\left(\frac{4}{3}\pi r^3\right), \qquad r \prec a$$

$$\phi = \frac{q_{in}}{\varepsilon_0}$$

$$E = \frac{q_{in}}{4\pi\varepsilon_0 r^2} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\varepsilon_0 r^2} = \frac{\rho}{3\varepsilon_0} r$$

$$\rho = \frac{Q}{\frac{4}{3}\pi a^3}$$

$$E = \frac{Q r}{4\pi\varepsilon_0 a^3} = k_e \frac{Q}{a^3} r \quad \Rightarrow E \to 0 \ as \ r \to o$$

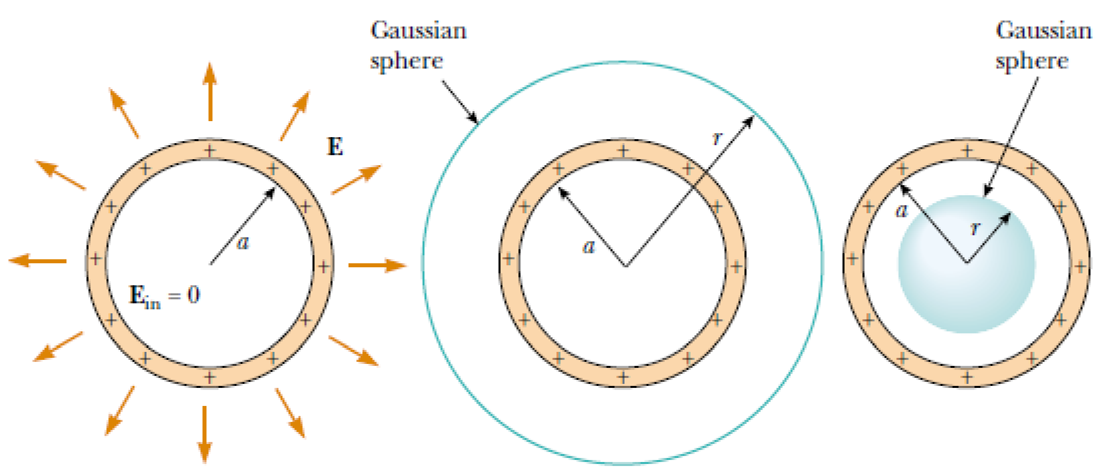


Comments:

Exercise:

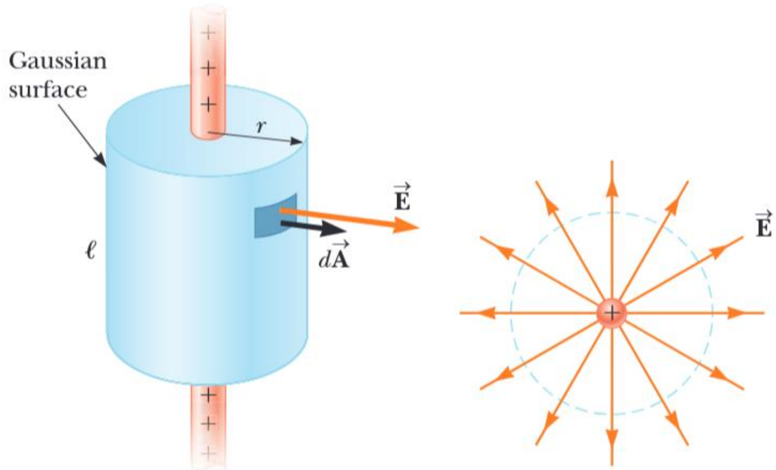
The Electric Field Due to a Thin Spherical Shell

A thin spherical shell of radius a has a total charge Q distributed uniformly over its surface (Fig. 24.13a). Find the electric field at points (A) outside and (B) inside the shell.



II- Electric Field of Line Charge

Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ.



$$\lambda = Const. \ , \quad Q = \lambda \ell, \quad dA \,||\, E$$

$$E \oint dA = \frac{q_{in}}{\varepsilon_0}$$

$$E(2\pi r \ell) = \frac{\lambda \ell}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi \varepsilon_0 r} = 2k_e \frac{\lambda}{r}$$

Comments:

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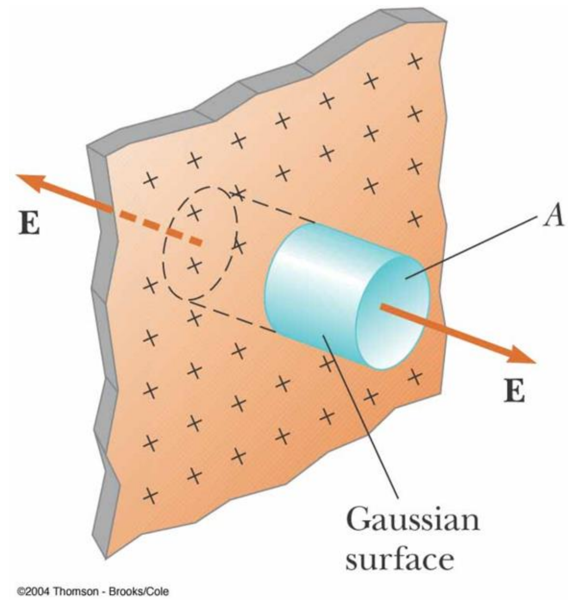
III A plan of charge

The electric field due to a non-conducting, infinite plane of positive charge with uniform surface charge density σ.



$$\sigma = \frac{q_{in}}{A}$$

$$\phi_c = E \oint dA = \frac{q_{in}}{\varepsilon_0}$$

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

In the figure below, can you calculate the electric field in the three regions?

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