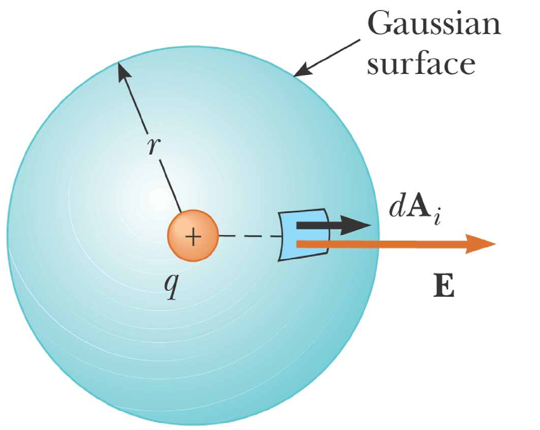
24.2 Gauss's Law

In this section we describe a general relationship between the net electric flux through a closed surface (often called a *Gaussian surface*) and the charge enclosed by the surface. This relationship, known as *Gauss's law,* is of fundamental importance in the study of electric fields as we will see in the next discussion.

❖ Flux due to a positive point charge q



$$\phi_c = \oint \vec{E}.d\vec{A} = \oint E_n \; dA = \oint E \, dA \cos\theta$$

$$\phi_c = E \oint dA$$

$$\phi_c = \left(\frac{1}{4\pi\varepsilon_0}\frac{q_{in}}{r^2}\right)(4\pi r^2)$$

$$\phi_c = \frac{q_{in}}{\varepsilon_0}$$

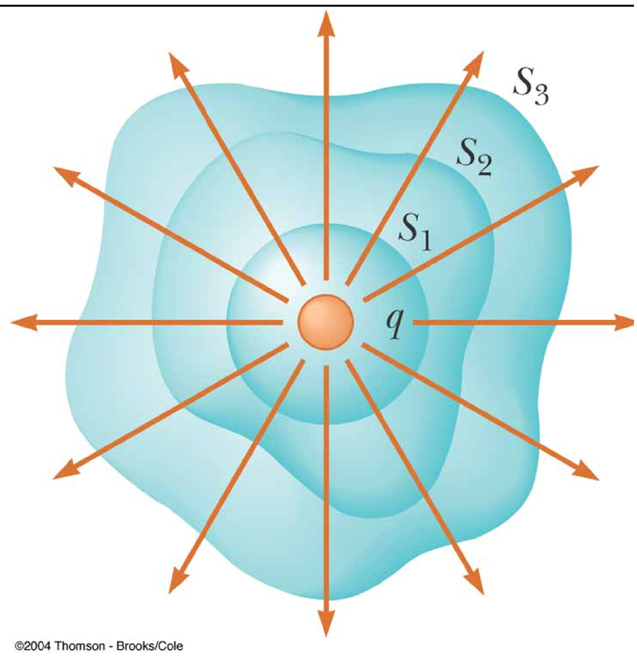
$$\Longrightarrow \quad \boxed{\Phi_E = \oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0}}$$

A spherical Gaussian surface of radius *r* surrounding a point charge *q*. When the charge is at the center of the sphere, the electric field is everywhere normal to the surface and constant in magnitude.

❖ Flux through various surfaces

As one can see in the figure in front closed surfaces of various shapes surrounding a charge *q*. The net electric flux is the same through all surfaces.

$\Phi_c \;\alpha\; q$



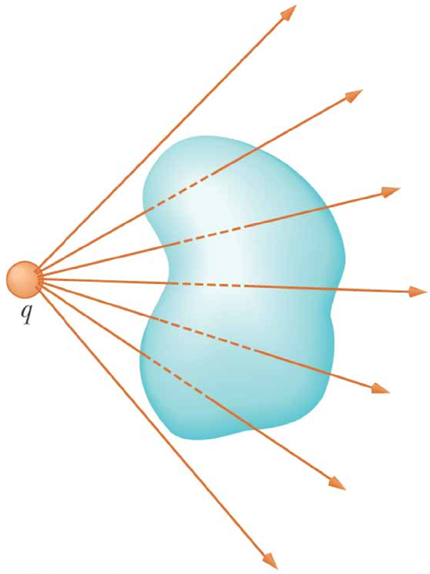
$$(\phi_c)_{s1} = (\phi_c)_{s2} = (\phi_c)_{s3} = \frac{q_{in}}{\varepsilon_0}$$

❖ Point charge outside closed surfaces

A point charge located *outside* a closed surface. The number of lines entering the surface equals the number leaving the surface.

That means:

$$\phi_c = \frac{q_{in}}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$$



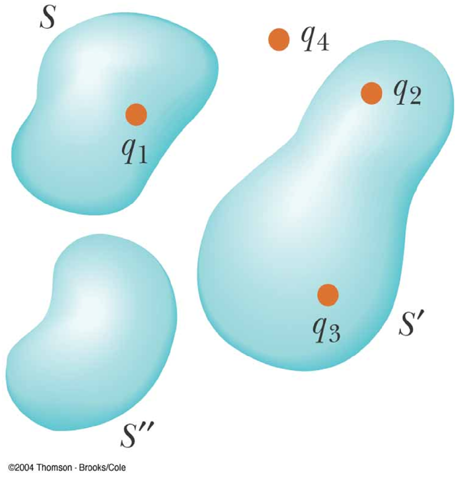
Guass's Law: The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity.

❖ Electric Filed due to many charges:

The electric field due to many charges is the vector sum of the electric fields produced by the individual charges:

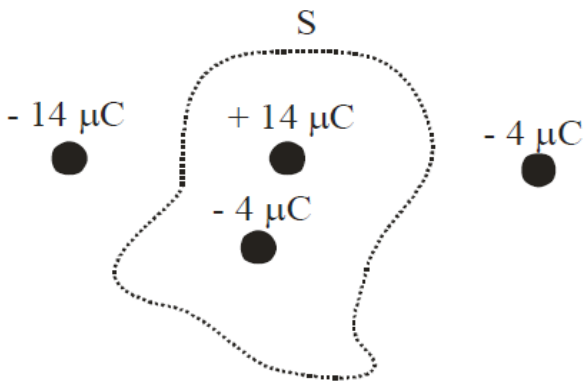
$$\oint \mathbf{E} \cdot d\mathbf{A} = \oint (\mathbf{E}_1 + \mathbf{E}_2 + \cdots) \cdot d\mathbf{A}$$

The net electric flux through any closed surface depends only on the charge *inside* that surface. The net flux through surface S is q_1/ε_0, the net flux through surface S' is $(q_2 + q_3)/\varepsilon_0$, and the net flux through surface S'' is zero. Charge q_4 does not contribute to the flux through any surface because it is outside all surfaces.



Example-1:

The net electric flux (Φ) through the shown Gaussian surface (S) is:



Example-2:

A point charge of 177 μC is places at the center of a cube of edge 10 cm. Calculate the electric flux through one face of the cube.