24.2 Gauss's Law

In this section, we describe a general relationship between the net electric flux through a closed surface (often called a *Gaussian surface*) and the charge enclosed by the surface. This relationship, known as *Gauss's law,* is of fundamental importance in the study of electric fields as we will see in the next discussion.

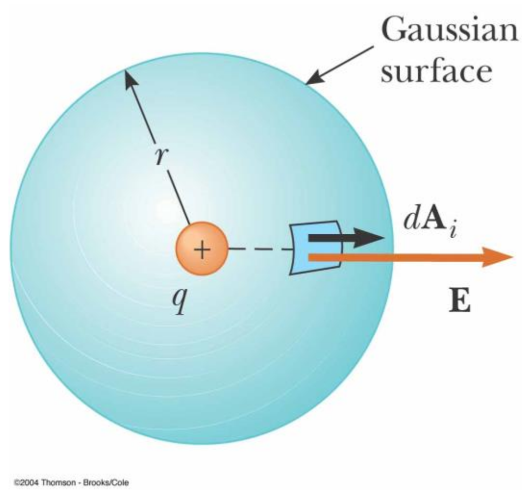
❖ Flux due to a positive point charge q

$$\phi_c = \oiint \vec{E}.d\vec{A} = \oiint E_n \; dA = \oiint E \, dA \cos\theta$$

$$\phi_c = E \oiint dA$$

$$\phi_c = \left(\frac{1}{4\pi\varepsilon_0}\frac{q_{in}}{r^2}\right)(4\pi r^2)$$

$$\phi_c = \frac{q_{in}}{\varepsilon_0}$$



$$\Longrightarrow \qquad \emptyset_E = \oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

A spherical Gaussian surface of radius *r* surrounding a point charge *q*. When the charge is at the center of the sphere, the electric field is everywhere normal to the surface and constant in magnitude.

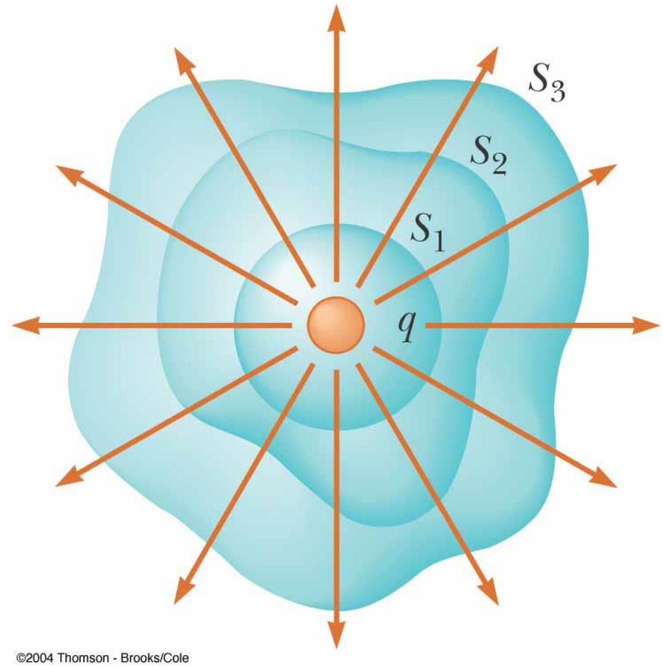
The permittivity: $\varepsilon_0 = 8.85 \; x \; 10^{-12} \; N.m^2/C^2$

$$K_e = \frac{1}{4\pi\varepsilon_0}$$

Gauss's Law: The total of the electric flux out of a closed surface is **equal to the charge enclosed divided by the permittivity.**

❖ Flux through various surfaces

As one can see in the figure in front, closed surfaces of various shapes surround a charge *q*. The net electric flux is the same through all surfaces.

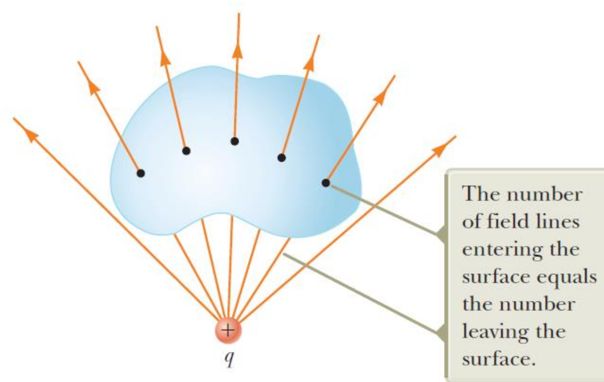


$\Phi_c \;\alpha\; q$

$$(\phi_c)_{s1} = (\phi_c)_{s2} = (\phi_c)_{s3} = \frac{q_{in}}{\varepsilon_0}$$

❖ Point charge outside closed surfaces

A point charge is located *outside* a closed surface. The number of lines entering the surface is equal to the number leaving the surface.



That means:

$$\phi_c = \frac{q_{in}}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$$

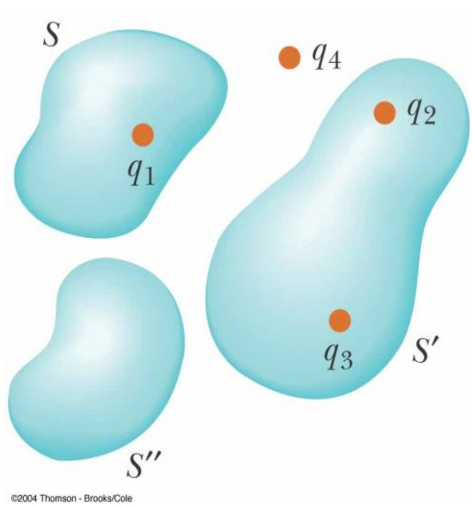
❖ Electric Field due to many charges:

The electric field due to many charges is the vector sum of the electric fields produced by the individual charges:

$$\oint \vec{E}.d\vec{A} = \oint (\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots \ldots.).d\vec{A}$$

The net electric flux through any closed surface depends only on the charge *inside* that surface.

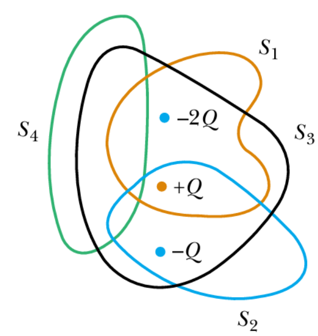
- The net flux through surface ***S* is q_1/ε_0 ,**
- The net flux through surface **S$^{/}$ is *(q* $_2$ + q$_3$)/ ε_0,**
- The net flux through surface ***S''* is zero**. Charge q_4 does not contribute to the flux through any surface because it is outside all surfaces.



Exercise:

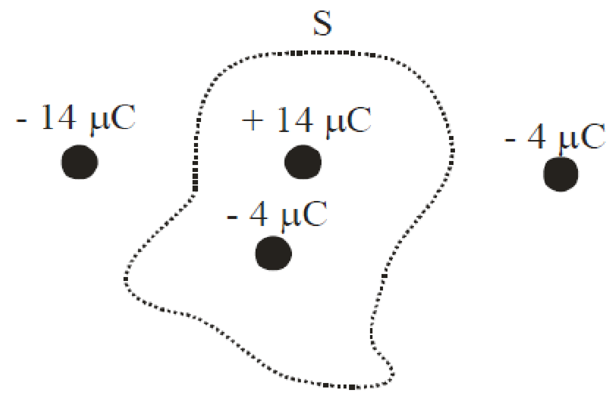
Four closed surfaces, S1 through S4, together with the charges 2Q , Q , and Q, are sketched in the Figure. (The colored lines are the intersections of the surfaces with the page.)

Find the electric flux through each surface.



Example-1:

The net electric flux (Φ) through the shown Gaussian surface (S) is:



Example-2:

A point charge of 177 μC is placed at the cube's center with an edge of 10 cm.

Calculate:

(a) Flux through each face.
(b) Flux through the whole cube.
(c) What changes if the charge is not at the center?

Solution:

$$\emptyset = \frac{q_{in}}{\varepsilon_0} = \frac{177x10^{-6}}{8.85x10^{-12}} = 1.92x10^{7} N.m^2/C$$

(a) $\emptyset(one\ face) = \frac{1}{6}\emptyset = \frac{1}{6}x1.92x10^{7} = 3.20x10^{6}\ N.m^2/C$

(b) $\emptyset(all\ faces) = \emptyset = 1.92x10^{7} N.\frac{m^2}{C}$

(c) The answer to (a) would change because the flux through each face of the cube would not be equal with an asymmetric charge distribution. The sides of the cube nearer the charge would have more flux and the ones further away would have less.

The answer to (b) would remain the same, as the overall flux would remain unchanged.

Example- 3 (conceptual)

A spherical Gaussian surface surrounds a point charge **q**. Describe what happens to the total flux through the surface in the following conditions:

Before solving such a problem, remember that Gauss's law states that:

$$\Phi_c = \frac{q_{in.}}{\varepsilon_o}$$

(A) The charge is tripled,

The flux through the surface is tripled because the flux is proportional to the amount of charge inside the surface.

(B) The radius of the sphere is doubled,

The flux does not change because all electric field lines from the charge pass through the sphere, regardless of its radius.

(C) The surface is changed to a cube,

The flux does not change when the shape of the Gaussian surface changes because all electric field lines from the charge pass through the surface, regardless of its shape.

(D) The charge is moved to another location inside the surface.

The flux does not change when the charge is moved to another location inside that surface because Gauss's law refers to the total charge enclosed, regardless of where the charge is located inside the surface.

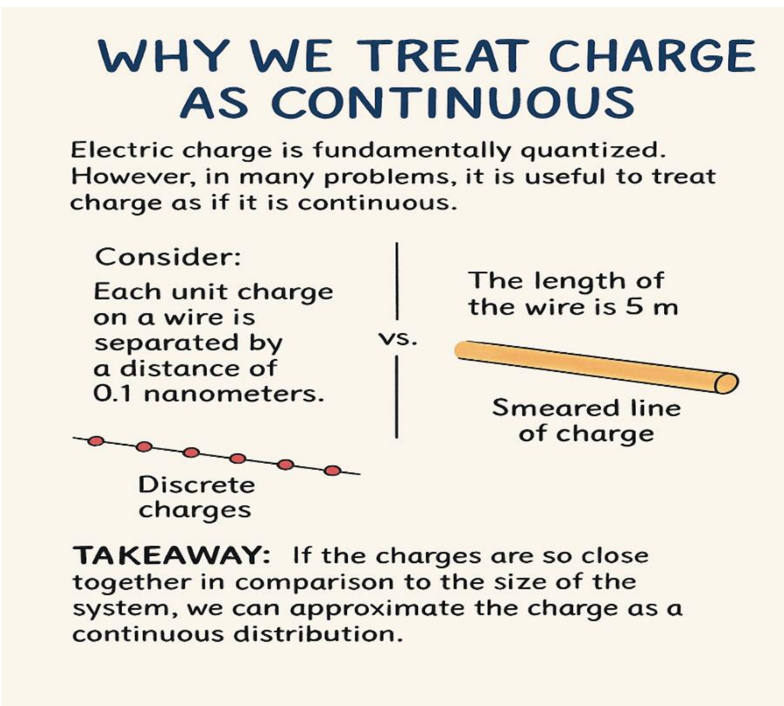
Exercise-4:

The following charges are located inside a submarine: 5.00 μC, -9.00 μC, 27.0 μC, and - 84.0 μC.

(a) Calculate the net electric flux through the hull of the submarine.

(b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?

Continuous Charge Distribution



- **Microscopic reality:** Charge is quantized.
- **Macroscopic reality:** Charges are so numerous and so close together that it is an excellent approximation to treat them as **continuous distributions**.

What is the charge density?

We use charge density because it enables us to describe how charge is distributed over a line, surface, or volume, making it easier to calculate electric fields without tracking each individual charge.

Charge density refers to the amount of electric charge present per unit of a given dimension (length, surface area, or volume) in a material or space.

- If a charge Q is uniformly distributed throughout a volume V, the volume charge density ρ is defined by:
  $\rho = \frac{Q}{V} \ \ (\frac{C}{m^3})$
- If a charge Q is uniformly distributed throughout a surface of area A, the surface charge density σ is defined by:
  $\sigma = \frac{Q}{A} \ \ (\frac{C}{m^2})$
- If a charge Q is uniformly distributed throughout a line of length *l*, the linear charge density λ is defined by:
  $\lambda = \frac{Q}{l} \ \ (\frac{C}{m})$