

Chapter 24

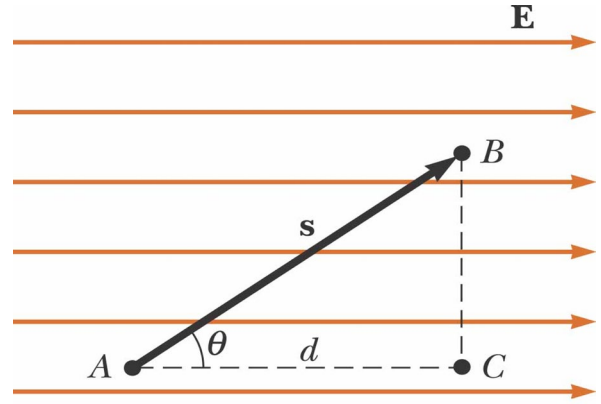
Electrical Potential

24.1 Potential Difference and Electric Potential

- What is the electric potential?
- What is the electrical potential energy?
- What is the potential difference?

Let's place a positive test charge in a uniform electric field.

What's the work done moving the charge from A to B?



$$dW = F ds \cos \theta$$

However, the total work done is given by:

$$W = \int_A^B F ds \cos \theta$$

Since $F = Eq$, one can rewrite W as:

$$W = \int_A^B qE ds \cos \theta$$

But work is the negative of the change in potential energy,

$$\Delta U = U_f - U_i = - \int_A^B qE ds \cos \theta = -q \int_A^B \vec{E} \cdot \vec{ds}$$

This means that the potential energy is changed by an amount of $-q \vec{E} \cdot \vec{ds}$

Electric potential is defined as potential energy per unit charge:

$$V = \frac{\Delta U}{q} = - \int_A^B \vec{E} \cdot \vec{ds}$$

Electric potential is a scalar characteristic of an electric field, independent of any charges that may be placed in the field.

- **Unit of electric potential and potential difference:**

Because electric potential is a measure of potential energy per unit charge, the SI unit of both electric potential and potential difference is **joules per coulomb**, which is defined as a volt (V):

$$1 \text{ V} \equiv 1 \text{ J/C}$$

A unit of energy commonly used in atomic and nuclear physics is the electron volt (eV), which is defined as the energy a charge–field system gains or loses when a charge of

$$1 \text{ eV} = (1.6 \times 10^{-19} \text{ C})(1 \text{ V}) = 1.6 \times 10^{-19} \text{ J}$$

24.2 Potential difference in a Uniform electric field

Often, what someone is interested in is the difference in potential between two specific points. In simple terms, a potential difference is the ratio of the work required to move a charge from point A to point B, divided by the magnitude of that charge.

$$\Delta V_{AB} = - \int_A^B E ds \cos \theta = V_B - V_A$$

$$\Delta V_{AB} = -E \int_A^B ds \cos 0 = -Ed$$

Often the potential difference is taken with respect to ground potential (recall mgh for gravitational potential energy)

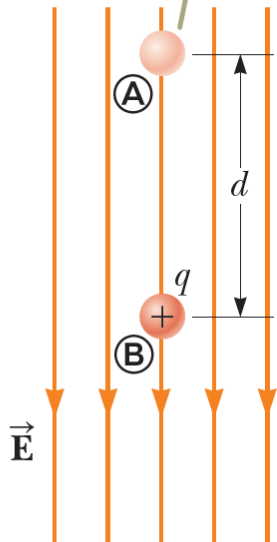
This expression gives the work required to move a charge:

$$W = V_{ab}q.$$

Note:

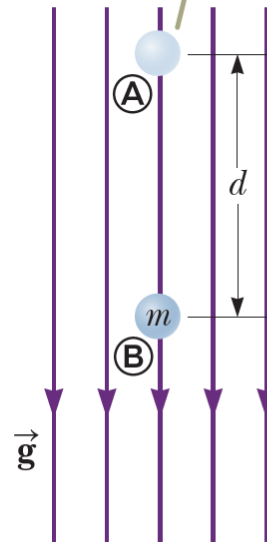
The Sign of ΔV : The negative sign is because we started at point **(A)** and moved to a new point in the *same* direction as the electric field lines. If we started from **(B)** and moved to **(A)**, the potential difference would be Ed . In a uniform electric field, the magnitude of the potential difference is Ed , and the direction of travel can determine the sign.

When a positive charge moves from point (A) to point (B), the electric potential energy of the charge–field system decreases.



a

When an object with mass moves from point (A) to point (B), the gravitational potential energy of the object–field system decreases.



b

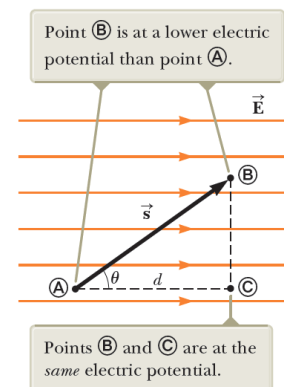
Electric potential is often compared to gravitational potential:

- A mass in a gravitational field has potential energy based on its height.
- Similarly, a charge in an electric field has an electric potential based on **its position**.

If we take point A at infinity, one can write the electric potential at any point B as:

$$V_B - V_A = - \int_{\infty}^B E ds \cos \theta = V_B$$

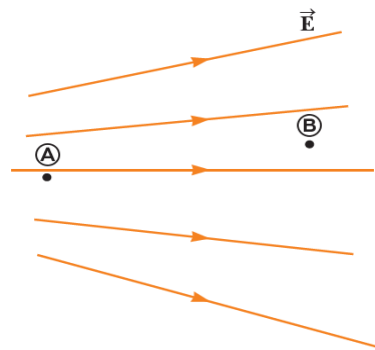
Prove that $V_B = V_C$.



Example-1: In the Figure, two points (A) and (B) are located within a region in which there is an electric field.

(i) How would you describe the potential difference $\Delta V = V_B - V_A$?

(a) It is positive. (b) It is negative. (c) It is zero.



Solution:

We know that:

$$V = - \int_A^B \vec{E} \cdot d\vec{s}$$

- The electric field $\vec{E}$ points to the **right**.
- The path from **A** $\rightarrow$ **B** is also to the **right** (A is left of B on the same horizontal line).
- Therefore, $\vec{E} \cdot d\vec{s} > 0$ along the path, so

ΔV_{AB} is negative value

(ii) A negative charge is placed at (A) and then moved to (B). How would you describe the change in potential energy of the charge–field system for this process? Choose from the same possibilities.

Solution:

Now the negative charge moved from A to B

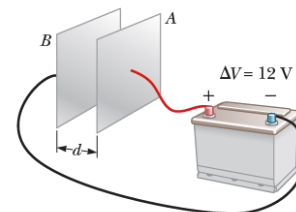
Potential-energy change:

$$\Delta U = q \Delta V.$$

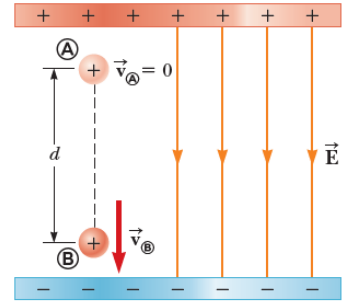
Here $q < 0$ and $\Delta V < 0$, so $\Delta U = (\text{negative})(\text{negative}) = \text{positive}$

So the potential energy *increases* when the negative charge is moved from A to B.

Example 2: A battery has a specified potential difference ΔV between its terminals and establishes that potential difference between conductors attached to the terminals. A 12-V battery is connected between two parallel plates as shown in the Figure. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and we do not consider locations near the plate edges.) Find the magnitude of the electric field between the plates.



Example-3: A proton is released from rest at a point **(A)** in a uniform electric field that has a magnitude of 8.0×10^4 V/m (see the figure). The proton undergoes a displacement of magnitude $d = 0.50$ m to a point **(B)** in the direction of $\vec{E}$. Find the speed of the proton after completing the displacement.



Solution:

Given:

$$E = 8.0 \times 10^4 \text{ V/m}$$

$$d = 0.50 \text{ m}$$

The proton is released from rest:

$$v_A = 0$$

For a proton:

$$q = +e = 1.60 \times 10^{-19} \text{ C}$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

The proton moves **in the direction of the electric field**, so the electric field does positive work on the proton.

The work done by the electric field is:

$$W = qEd$$

Since the proton starts from rest, its initial kinetic energy is zero:

$$K_i = 0$$

The work done becomes its final kinetic energy:

$$W = K_f - K_i$$

Therefore,

$$qEd = \frac{1}{2} m_p v_B^2$$

Solving for v_B :

$$v_B = \sqrt{\frac{2qEd}{m_p}}$$

Substitute the values:

$$v_B = \sqrt{\frac{2(1.60 \times 10^{-19})(8.0 \times 10^4)(0.50)}{1.67 \times 10^{-27}}}$$

First, calculate the numerator:

$$2(1.60 \times 10^{-19})(8.0 \times 10^4)(0.50) = 1.28 \times 10^{-14} \text{ J}$$

Thus,

$$\begin{aligned} v_B &= \sqrt{\frac{1.28 \times 10^{-14}}{1.67 \times 10^{-27}}} \\ v_B &= \sqrt{7.66 \times 10^{12}} \\ v_B &\approx 2.77 \times 10^6 \text{ m/s} \end{aligned}$$

Final Answer

$$v_B \approx 2.8 \times 10^6 \text{ m/s}$$

Because the proton is **positively charged**, the electric force is in the **same direction as the electric field**:

$$\vec{F} = q\vec{E}$$

Since the proton moves in the direction of the force, the electric field does **positive work**, increasing the proton's kinetic energy.

So:

$$\text{Electric potential energy decreases} \rightarrow \text{kinetic energy increases}$$

Exercise-4: An ion with charge $+5|e|$ and $4.2 \times 10^{-26} \text{ Kg}$ is accelerated from rest through a potential difference ΔV . Its speed reaches $4 \times 10^4 \text{ m/s}$. The potential difference ΔV in (V) unit is:

- A. 12 B. 24 C. 42 D. 86

Exercise-5: An electron is released in a region with varying electric potential. The electron moves towards the region with:

- A. higher electric potential and it loses potential energy.
- B. higher electric potential and it gains potential energy.
- C. lower electric potential and it loses potential energy.
- D. lower electric potential and it gains potential energy.