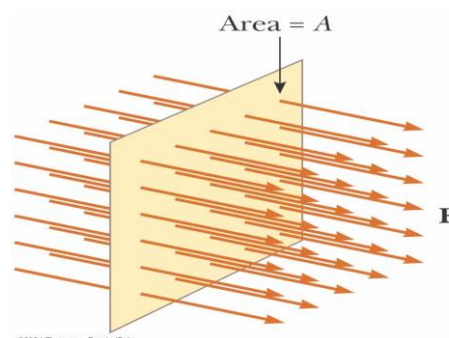# Chapter 24

# Gauss's Law

## 24.1 Electric Flux

Consider an electric field uniform in magnitude and direction; the field lines penetrate a rectangular surface of area $A$, which is perpendicular to the field.



Area = A

E

The number of lines per unit area (in other words, the *line density*) is proportional to the magnitude of the electric field. Therefore, the number of lines penetrating the surface is proportional to the product **EA**. This product of the magnitude of the electric field $E$ and surface area $A$ perpendicular to the field is called the **electric flux $\Phi_E$** (uppercase Greek phi):

$$\Phi_E = E\,A \qquad (\text{N.m}^2/\text{C})$$

Electric flux is proportional to the number of electric field lines penetrating some surface.

**Example**:

What is the electric flux through a sphere with a radius of 1.00 m and a charge of 1.00 µC at its center?

As we have studied in Chapter 23, the E is given as follows:

$$E = k_e \frac{q}{r^2} = (8.99 \times 10^9\,\text{N·m}^2/\text{C}^2)\,\frac{1.00 \times 10^{-6}\,\text{C}}{(1.00\,\text{m})^2}$$
$$= 8.99 \times 10^3\,\text{N/C}$$

Since the field points radially outward, they are everywhere perpendicular to the surface of the sphere. The flux through the sphere (whose surface area is $A = 4\pi r^2 = 12.6\,\text{m}^2$)thus

$$\Phi_E = EA = (8.99 \times 10^3\,\text{N/C})\,(12.6\,\text{m}^2)$$
$$= 1.13 \times 10^5\,\text{N·m}^2/\text{C}$$

## ➢ If the surface under consideration is not perpendicular to the field

Field lines represent a uniform electric field penetrating an area $A$ that is at an angle $\theta$ to the field. Because the number of lines that go through the area $A'$ is the same as the number that goes through $A$, the flux through $A'$ is equal to the flux through $A$ and is given by
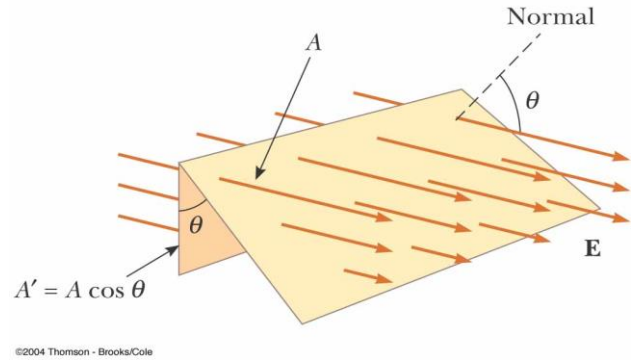
$$\Phi_E = EA \cos \theta$$

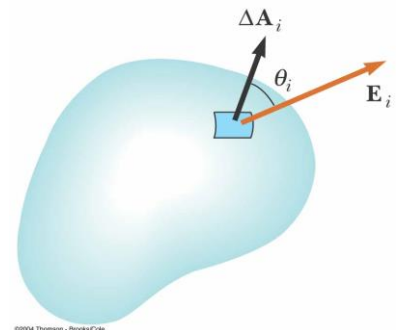$$\Phi_E = EA' = EA \cos \theta$$

## Note that:

# The flux through a surface of fixed area $A$ has a maximum value $EA$ when the surface is perpendicular to the field (in other words, when the normal to the surface is parallel to the field, that is $\theta = 0$.

# The flux is zero when the surface is parallel to the field (in other words, when the normal to the surface is perpendicular to the field, that is, $\theta = 90$.

## ➢ Arbitrary surface:

In the preceding discussion, we assumed a uniform electric field. In more general situations, the electric field may vary over a surface. Therefore, our definition of flux only means that it is over a small area.

A small element of surface area $\Delta A_i$. The electric field makes an angle $\theta$ with the vector $\Delta A_i$, defined as being expected to the surface element, and the flux through the element is equal to $E_i \Delta A_i \cos \theta$.

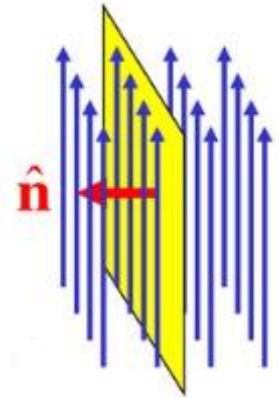$$\Delta \phi = E_i \, \Delta A_i \cos \theta_i = \vec{E}_i . \Delta \vec{A}_i$$

$$\phi = \lim_{\Delta A \to 0} \sum \vec{E}_i . \Delta \vec{A}_i = \int \vec{E}.d\vec{A}$$

$$= \int E \, dA \cos \theta$$

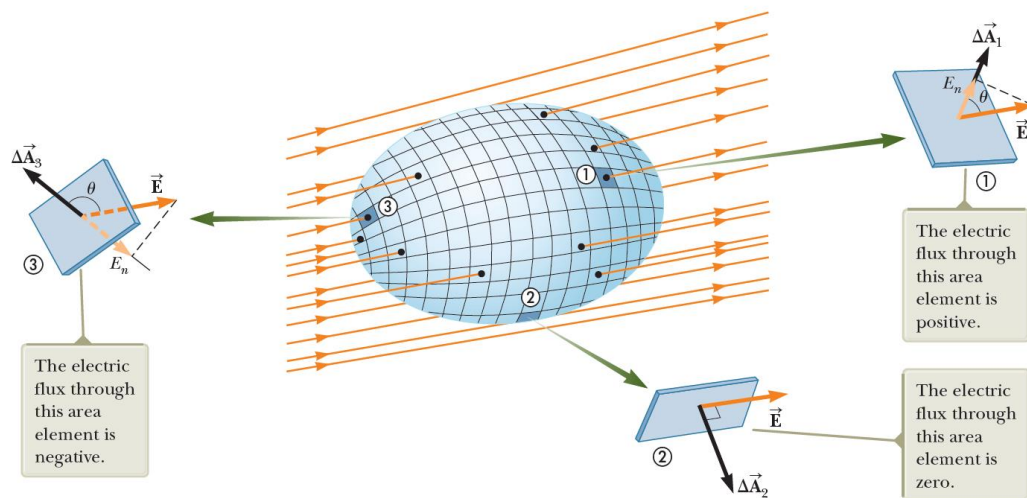Please note that θ is the angle between the direction of the electric field and the normal to the surface ($\hat{n}$).

So,    $\emptyset = \int \vec{E}.\hat{n}\, dA = \int |\vec{E}| dA \cos\theta$

$$\phi_c = \oint \vec{E}.d\vec{A} = \oint E_n\, dA = \oint E\, dA \cos\theta$$

$$\theta < 90^0 \quad \phi_c \Rightarrow +ve \quad \& \quad \theta > 90^0 \quad \phi_c \Rightarrow -ve$$

**A closed surface in an electric field**



The electric flux through this area element is positive. ①

The electric flux through this area element is zero. ②

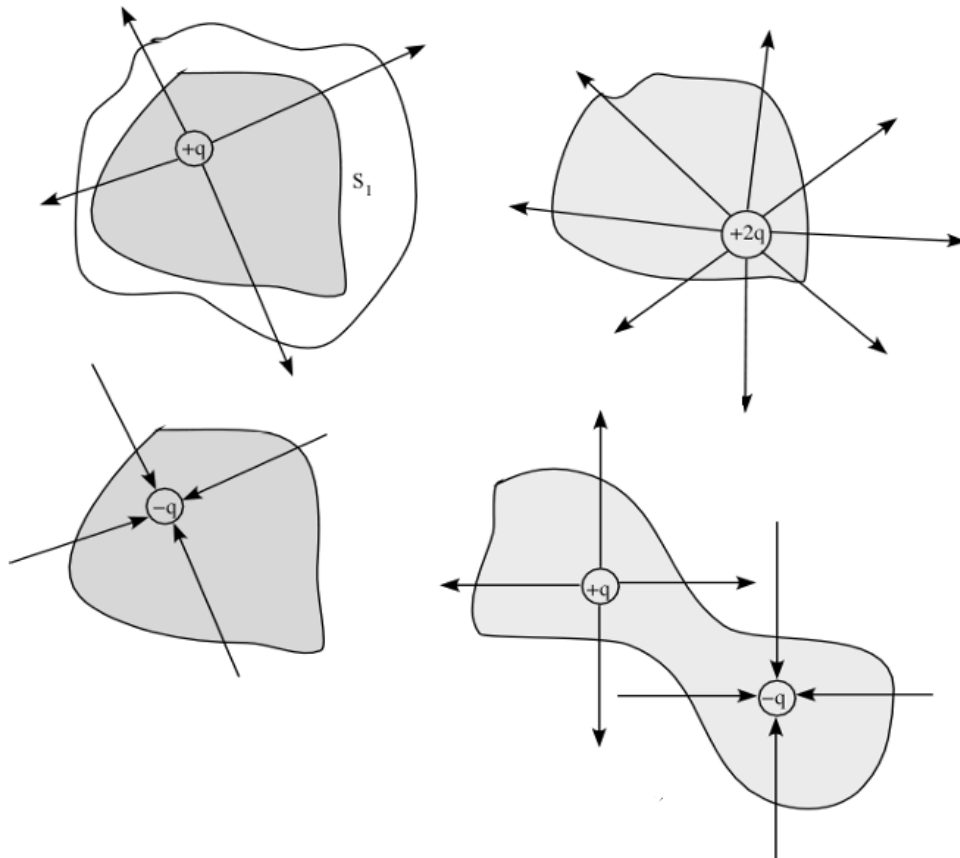The electric flux through this area element is negative. ③

By convention, the area vectors ΔAi are normal to the surface and point outward. The flux through an area element can be positive (element 1), zero (element 2), or negative (element 3).

**The role of the number of electric field lines:**

The *net* flux through the surface is proportional to the net number of lines leaving the surface, where the net number means *the number leaving the surface minus the number entering the surface.*

- If more lines are leaving than entering, the net flux is **positive**.

-  If more lines are entering than leaving, the net flux is **negative**.

**Example -1:**

**A flat surface of an area of 3.20 m² is rotated in a uniform electric field of magnitude 6.20x10² N/C.**

1- **Determine the electric flux through this area when the electric field is perpendicular to the surface.**

$$\emptyset = EA \cos \theta$$
$$\theta = 0^0$$
$$\emptyset = 6.20x10^6 \ 3.20 \cos 0 = 1984 \ N.m^2/C$$

2- **Determine the electric flux through this area when the electric field is parallel to the surface.**

$$\emptyset = EA \cos \theta$$
$$\theta = 90^0$$
$$\emptyset = 0$$

3- **Determine the electric flux through this area when the electric field is perpendicular to the area vector.**

$$\emptyset = EA \cos \theta$$

$$\theta = 0^0$$

$$\emptyset = 6.20x10^6 \ 3.20 \cos 0 = 1984 \ N.m^2/C$$

## Exampl-2

Consider a uniform electric field $\vec{E}$ oriented in the $x$ direction in
space. A cube of edge length $l$ is placed in the field, oriented as
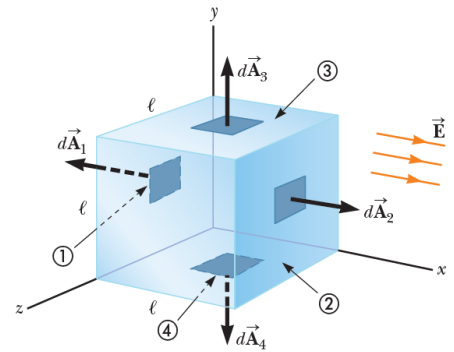shown in the Figure. Find the net electric flux through the surface of
the cube.

**Figure 23.9** (Example 23.4) A closed surface in the shape of a cube in a uniform electric field oriented parallel to the $x$ axis. Side ④ is the bottom of the cube, and side ① is opposite side ②.

## Example-3:

Consider a closed triangular box resting within a horizontal electric field
of magnitude E =7.80 $10^4$N/C as shown in Figure P24.4.

Calculate the electric flux through:
(a) The vertical rectangular surface,
(b) The slanted surface,
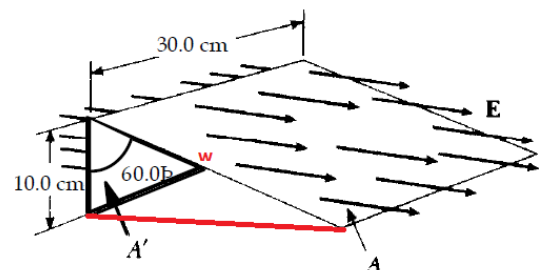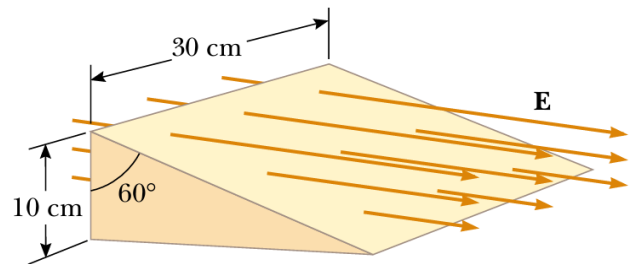and (c) The entire surface of the box.

FIG. P24.4