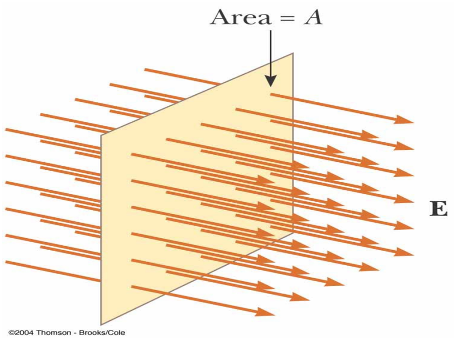
Chapter 23

Gauss's Law

23.2 Electric Flux



Consider an electric field uniform in magnitude and direction; the field lines penetrate a rectangular surface of area *A*, which is perpendicular to the field.

The number of lines per unit area (in other words, the *line density*) is proportional to the magnitude of the electric field. Therefore, the number of lines penetrating the surface is proportional to the product ***EA***. This product of the magnitude of the electric field ***E*** and surface area *A* perpendicular to the field is called the **electric flux Φ_E** (uppercase Greek phi):

$$\Phi_E = E\,A \qquad (N.m^2/C)$$

Electric flux is proportional to the number of electric field lines penetrating some surface.

Example:

What is the electric flux through a sphere with a radius of 1.00 m and a charge of 1.00 µC at its center?

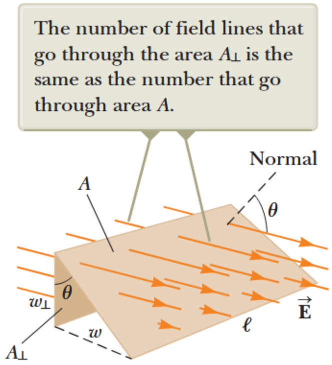
As we have studied in Chapter 23, the **E** is given as follows:

$$E = k_e \frac{q}{r^2} = 9x10^9 x \frac{1x10^{-6}}{(1)^2} = 9x10^3 \text{N/C}$$

Since the field points radially outward, they are everywhere perpendicular to the surface of the sphere. The flux through the sphere (whose surface area is $A = 4\pi r^2 = 12.6\ m^2$); thus:

$$\emptyset_E = EA = (9x10^3)x(12.6) = 1.13x10^5 N.m^2/C$$

➢ If the surface under consideration is not perpendicular to the field



Field lines represent a uniform electric field penetrating an area A that is at an angle θ to the field. Because the number of lines that go through the area A' is the same as the number that goes through A, the flux through A' is equal to the flux through A and is given by

$$\Phi_E = EA \cos\theta$$

$$\Phi_E = EA' = EA \cos\theta$$

Note that:

The flux through a surface of fixed area $\boldsymbol{A}$ has a **maximum value** $\boldsymbol{EA}$ when the surface is perpendicular to the field (in other words, when the normal to the surface is parallel to the field, that is $\boldsymbol{\theta = 0}$.

The flux is zero when the surface is parallel to the field (in other words, when the normal to the surface is perpendicular to the field, that is, $\boldsymbol{\theta = 90}$.

➢ Arbitrary surface:

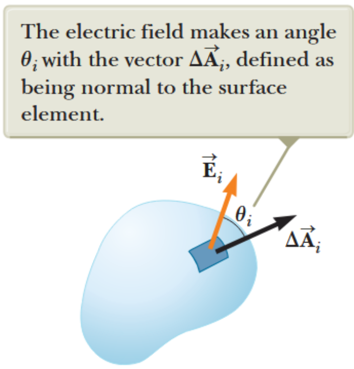
In the preceding discussion, we assumed a uniform electric field. In more general situations, the electric field may vary over a surface. Therefore, our definition of flux only means that it is over a small area.

A small element of surface area $\Delta \boldsymbol{A_i}$. The electric field makes an angle θ with the vector $\Delta \mathbf{A}_i$, defined as being expected to the surface element, and the flux through the element is equal to $\boldsymbol{E_i \, \Delta A_i \cos\theta}$.

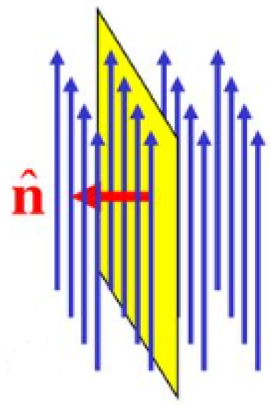
$$\Delta\phi = E_i \, \Delta A_i \cos\theta_i = \vec{E}_i \cdot \Delta\vec{A}_i$$

$$\phi = \lim_{\Delta A \to 0} \sum \vec{E}_i \cdot \Delta\vec{A}_i = \int \vec{E} \cdot d\vec{A}$$

$$= \int E \, dA \cos\theta$$



Please note that θ is the angle between the direction of the electric field and the normal to the surface ($\hat{n}$).

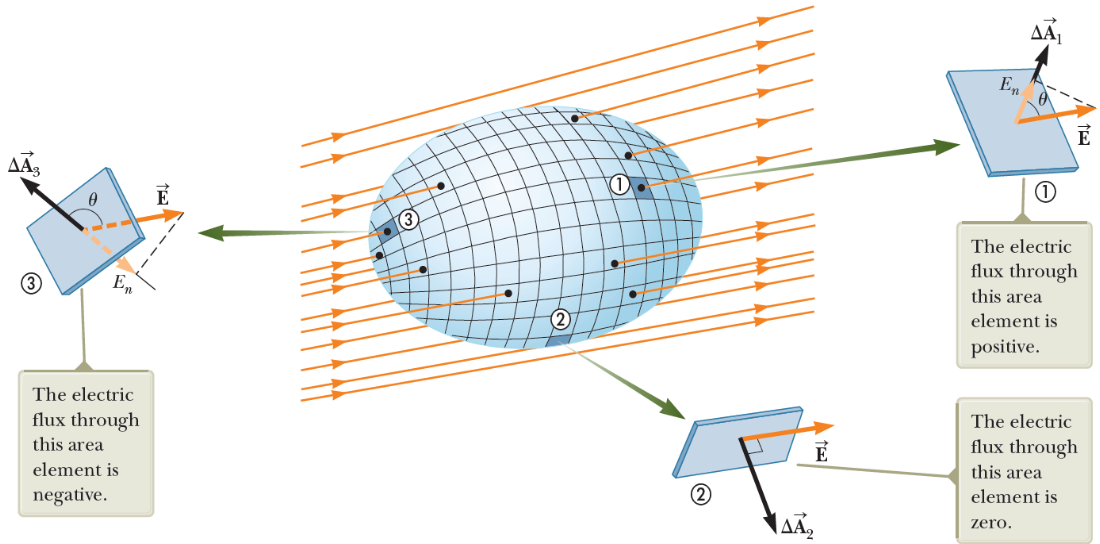


So, $$\emptyset = \int \vec{E}.\hat{n}\, dA = \int |\vec{E}| dA \cos\theta$$

$$\phi_c = \oint \vec{E} . d\vec{A} = \oint E_n\, dA = \oint E\, dA \cos\theta$$

$$\theta < 90^0 \quad \phi_c \Rightarrow +ve \qquad \& \quad \theta > 90^0 \quad \phi_c \Rightarrow -ve$$

A closed surface in an electric field

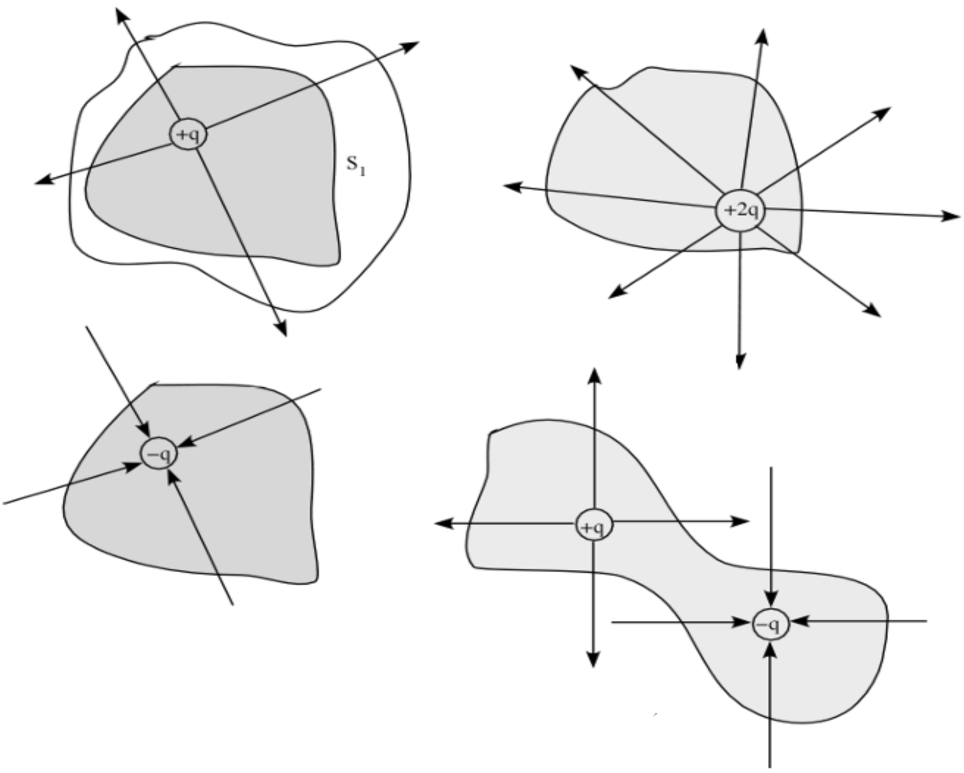


By convention, the area vectors Δ**A*i*** are normal to the surface and point outward. The flux through an area element can be positive (element 1), zero (element 2), or negative (element 3).

The role of the number of electric field lines:

The *net* flux through the surface is proportional to the net number of lines leaving the surface, where the net number means ***the number leaving the surface minus the number entering the surface.***

- If more lines are leaving than entering, the net flux is **positive**.
- If more lines are entering than leaving, the net flux is **negative**.



Example -2:

A flat surface of an area of 3.20 m^2 is rotated in a uniform electric field of magnitude 6.20x10^2 N/C.

1- **Determine the electric flux through this area when the electric field is perpendicular to the surface.**

$$\emptyset = EA\cos\theta$$
$$\theta = 0^0$$
$$\emptyset = 6.20x10^6\ 3.20\cos 0 = 1984\ N.m^2/C$$

2- **Determine the electric flux through this area when the electric field is parallel to the surface.**

$$\emptyset = EA\cos\theta$$
$$\theta = 90^0$$
$$\emptyset = 0$$

3- **Determine the electric flux through this area when the electric field is perpendicular to the area vector.**

$$\boldsymbol{\emptyset = EA\cos\theta}$$

$$\theta = 90^0$$

$$\emptyset = 0$$

Exampl-2

Consider a uniform electric field $\vec{E}$ oriented in the x direction in space. A cube of edge length l is placed in the field, oriented as shown in the Figure. Find the net electric flux through the surface of the cube.

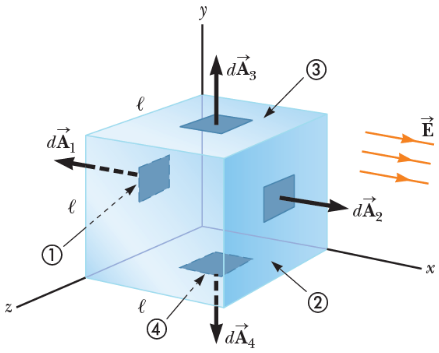


Figure 23.9 (Example 23.4) A closed surface in the shape of a cube in a uniform electric field oriented parallel to the x axis. Side ④ is the bottom of the cube, and side ① is opposite side ②.

Solution:

The electric flux through a surface is defined as

$\Phi_E = \oint E \cdot dA$,

where dA is the outward-pointing area vector of each surface element.

The cube has six faces. Because the electric field is uniform and directed along the x-axis, we analyze the contribution of each face separately.

Faces parallel to the electric field:

Four faces of the cube are parallel to the electric field. For these faces, the area vector dA is perpendicular to E, so the angle between E and dA is 90°. Therefore,

$E \cdot dA = E\, dA \cos(90°) = 0$.

Hence, the electric flux through each of these four faces is zero.

Face 1 (field entering the cube):

For the face where the electric field enters the cube, the electric field is opposite to the outward area vector. Thus, the angle between E and dA is 180°. The flux through this face is

$\Phi_1 = \int E \cos(180°)\, dA = -E\, A = -E\, \ell^2$.

Face 2 (field leaving the cube):

For the opposite face, the electric field is in the same direction as the outward area vector. The angle between E and dA is 0°. The flux through this face is

$\Phi_2 = \int E \cos(0°)\, dA = +E\, A = +E\, \ell^2$.

Net electric flux:

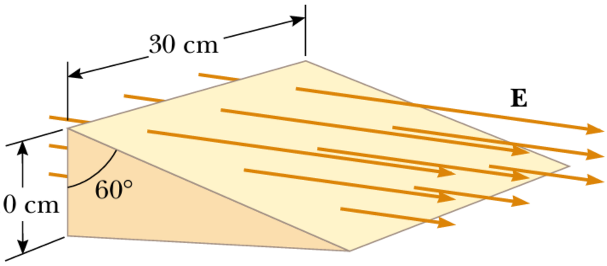
The net electric flux through the cube is the sum of the fluxes through all six faces:

$\Phi_E = (-E\, \ell^2) + (E\, \ell^2) + 0 + 0 + 0 + 0 = 0$.

Therefore, the net electric flux through the closed surface of the cube is zero.

Example-3:

Consider a closed triangular box resting within a horizontal electric field of magnitude E =7.80 10^4N/C as shown in the figure.



Calculate the electric flux through:
(a) The vertical rectangular surface,
(b) The slanted surface,
and (c) The entire surface of the box.

Solution:

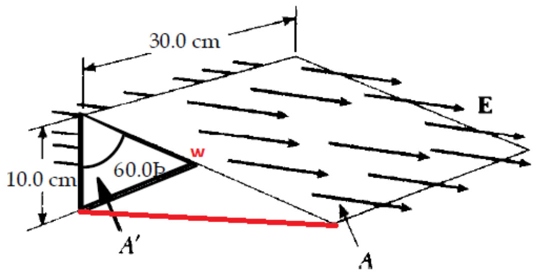


FIG. P24.4

The electric flux through a surface is given by
$\Phi_E = E\,A\cos\theta$,
where θ is the angle between the electric field vector and the outward normal to the surface.

(a) Flux through the vertical rectangular surface:

The area of the vertical surface is
$A' = (0.10\text{ m})(0.30\text{ m}) = 0.03\text{ m}^2$.
The electric field is directed opposite to the outward normal of this surface, so θ = 180°.

$\Phi_{E,A'} = E\,A'\cos(180°)$
$\Phi_{E,A'} = (7.80 \times 10^4)(0.03)(-1)$
$\Phi_{E,A'} = -2.34 \times 10^3\text{ N·m}^2\text{/C}$.

(b) Flux through the slanted surface:

The slanted surface has an effective area given by
$A = (0.30\text{ m})(0.10\text{ m}) / \cos(60°) = 0.06\text{ m}^2$.
The angle between the electric field and the outward normal is θ = 60°.

$\Phi_{E,A} = E\,A\cos(60°)$
$\Phi_{E,A} = (7.80 \times 10^4)(0.06)(0.5)$
$\Phi_{E,A} = 2.34 \times 10^3\text{ N·m}^2\text{/C}$.

(c) Flux through the entire surface:

The remaining surfaces of the box are either parallel to the electric field or have normals perpendicular to it, so the electric flux through them is zero.

Therefore, the total electric flux through the closed surface is
$\Phi_{E,total} = (-2.34 \times 10^3) + (2.34 \times 10^3) + 0 = 0$.