

Chapter 6

Future Worth Analysis

Systematic Economic Analysis Technique

- 1. Identify the investment alternatives**
- 2. Define the planning horizon**
- 3. Specify the discount rate**
- 4. Estimate the cash flows**
- 5. Compare the alternatives**
- 6. Perform supplementary analyses**
- 7. Select the preferred investment**

Future Worth Analysis

Single Alternative

Future Worth Method

- converts all cash flows to a single sum equivalent at the end of the **planning horizon** using $i = MARR$
- used mostly for **financial planning**
- not a popular corporate DCF method

$$FW(i\%) = \sum_{t=0}^n A_t (1+i)^{n-t}$$

(take all cash flows to “time n ” and add them up!)

Example 6.1

A \$500,000 investment in a surface mount placement machine is being considered. Over a 10-year planning horizon, it is estimated the SMP machine will produce net annual savings of \$92,500. At the end of 10 years, it is estimated the SMP machine will have a \$50,000 salvage value. Based on a 10% MARR and future worth analysis, should the investment be made?

$$\begin{aligned}FW &= -\$500K(F|P \ 10\%,10) + \$92.5K(F|A \ 10\%,10) + \$50K \\ &= \$227,341.40 \\ &= FV(10\%,10,-92500,500000)+50000 \\ &= \$227,340.55\end{aligned}$$

Since $FW > \$0$, the investment is recommended

Example 6.2

How does future worth change over the life of the investment? How does future worth change when the salvage value decreases geometrically and as a gradient series?

Chapter 6 tables and figures (11-14-08) [Compatibility Mode] - Microsoft Excel

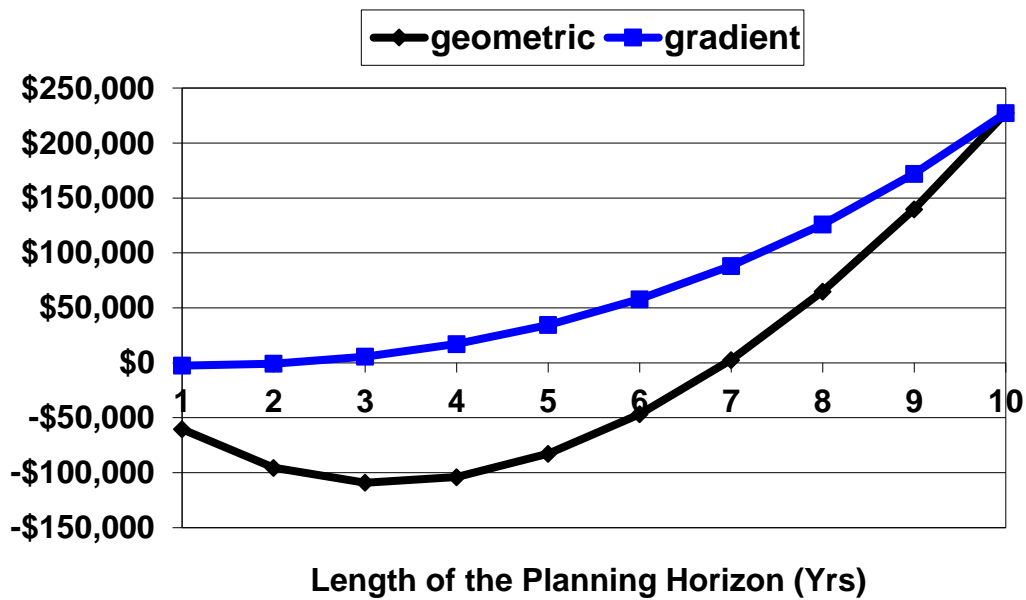
Home Insert Page Layout Formulas Data Review View Acrobat

D13 $=FV(\$B\$1,A13,-B13+C13,-\$B\$3)+\$C13$

	A	B	C	D	E	F	G	H	I	J
1	MARR =	10%								
2	Planning Horizon	CF	SV_{geometric}	FW_{geometric}	SV_{gradient}	FW_{gradient}				
3	0	-\$500,000	\$500,000	\$0.00	\$500,000	\$0.00	=B3+E3			
4	1	\$92,500	\$397,000	-\$60,500.00	\$455,000	-\$2,500.00	=FV(\$B\$1,A4,-B4,-\$B\$3)+\$E4			
5	2	\$92,500	\$315,218	-\$95,532.00	\$410,000	-\$750.00	=FV(\$B\$1,A5,-B5,-\$B\$3)+\$E5			
6	3	\$92,500	\$250,283	-\$109,041.91	\$365,000	\$5,675.00	=FV(\$B\$1,A6,-B6,-\$B\$3)+\$E6			
7	4	\$92,500	\$198,725	-\$104,032.72	\$320,000	\$17,242.50	=FV(\$B\$1,A7,-B7,-\$B\$3)+\$E7			
8	5	\$92,500	\$157,787	-\$82,745.78	\$275,000	\$34,466.75	=FV(\$B\$1,A8,-B8,-\$B\$3)+\$E8			
9	6	\$92,500	\$125,283	-\$46,803.32	\$230,000	\$57,913.43	=FV(\$B\$1,A9,-B9,-\$B\$3)+\$E9			
10	7	\$92,500	\$99,475	\$2,679.67	\$185,000	\$88,204.77	=FV(\$B\$1,A10,-B10,-\$B\$3)+\$E10			
11	8	\$92,500	\$78,983	\$65,008.32	\$140,000	\$126,025.24	=FV(\$B\$1,A11,-B11,-\$B\$3)+\$E11			
12	9	\$92,500	\$62,713	\$139,840.33	\$95,000	\$172,127.77	=FV(\$B\$1,A12,-B12,-\$B\$3)+\$E12			
13	10	\$142,500	\$50,000	\$227,340.55	\$50,000	\$227,340.55	=FV(\$B\$1,A13,-B13+C13,-\$B\$3)+\$E13			
14	FW =	\$227,340.55	=FV(B1,A13,,-NPV(B1,B4:B13)-B3)							

Figure 6.1(a) Figure 6.1(b) Figure 6.2 Figure 6.3 Figure 6.4 Figure 6.5 Figure

Future Worth of the SMP Machine Investment



	A	B	C	D	E	F	G	H	I	J
1	MARR =	10%								
	Planning Horizon	CF	SV_{geometric}	FW_{geometric}	SV_{gradient}	FW_{gradient}				
2										
3	0	-\$500,000	\$500,000	\$0.00	\$500,000	\$0.00	=B3+E3			
4	1	\$92,500	\$397,000	-\$60,500.00	\$455,000	-\$2,500.00	=FV(\$B\$1,A4,-B4,-\$B\$3)+\$E4			
5	2	\$92,500	\$315,218	-\$95,532.00	\$410,000	-\$750.00	=FV(\$B\$1,A5,-B5,-\$B\$3)+\$E5			
6	3	\$92,500	\$250,283	-\$109,041.91	\$365,000	\$5,675.00	=FV(\$B\$1,A6,-B6,-\$B\$3)+\$E6			
7	4	\$92,500	\$198,725	-\$104,032.72	\$320,000	\$17,242.50	=FV(\$B\$1,A7,-B7,-\$B\$3)+\$E7			
8	5	\$92,500	\$157,787	-\$82,745.78	\$275,000	\$34,466.75	=FV(\$B\$1,A8,-B8,-\$B\$3)+\$E8			
9	6	\$92,500	\$125,283	-\$46,803.32	\$230,000	\$57,913.43	=FV(\$B\$1,A9,-B9,-\$B\$3)+\$E9			
10	7	\$92,500	\$99,475	\$2,679.67	\$185,000	\$88,204.77	=FV(\$B\$1,A10,-B10,-\$B\$3)+\$E10			
11	8	\$92,500	\$78,983	\$65,008.32	\$140,000	\$126,025.24	=FV(\$B\$1,A11,-B11,-\$B\$3)+\$E11			
12	9	\$92,500	\$62,713	\$139,840.33	\$95,000	\$172,127.77	=FV(\$B\$1,A12,-B12,-\$B\$3)+\$E12			
13	10	\$142,500	\$50,000	\$227,340.55	\$50,000	\$227,340.55	=FV(\$B\$1,A13,-B13+C13,-\$B\$3)+\$E13			
14	FW =	\$227,340.55	=FV(B1,A13,,-NPV(B1,B4:B13)-B3)							
15										
16										
17										
18										
19										
20										
21										
22										
23										
24										

Notice, a negative FW occurs until the 3rd year with gradient decreases and until the 7th year with geometric decreases; also, FW achieves a minimum during the 1st year with gradient decreases and during the 3rd year with geometric decreases.

Example 6.3

A recent engineering graduate began investing at age 23, with a goal of achieving a net worth of \$5 million by age 58. If the engineer obtains an annual return of 6.5% and makes a first investment of \$5000, what gradient increase is required?

$$G(A|G 6.5\%,36) + \$5000 = \$5,000,000(A|F 6.5\%,36)$$

$$G = [\$5,000,000(A|F 6.5\%,36) - \$5000]/(A|G 6.5\%,36)$$

$$(A|F 6.5\%,36) = 0.065/[(1.065)^{36} - 1] = 0.0075133$$

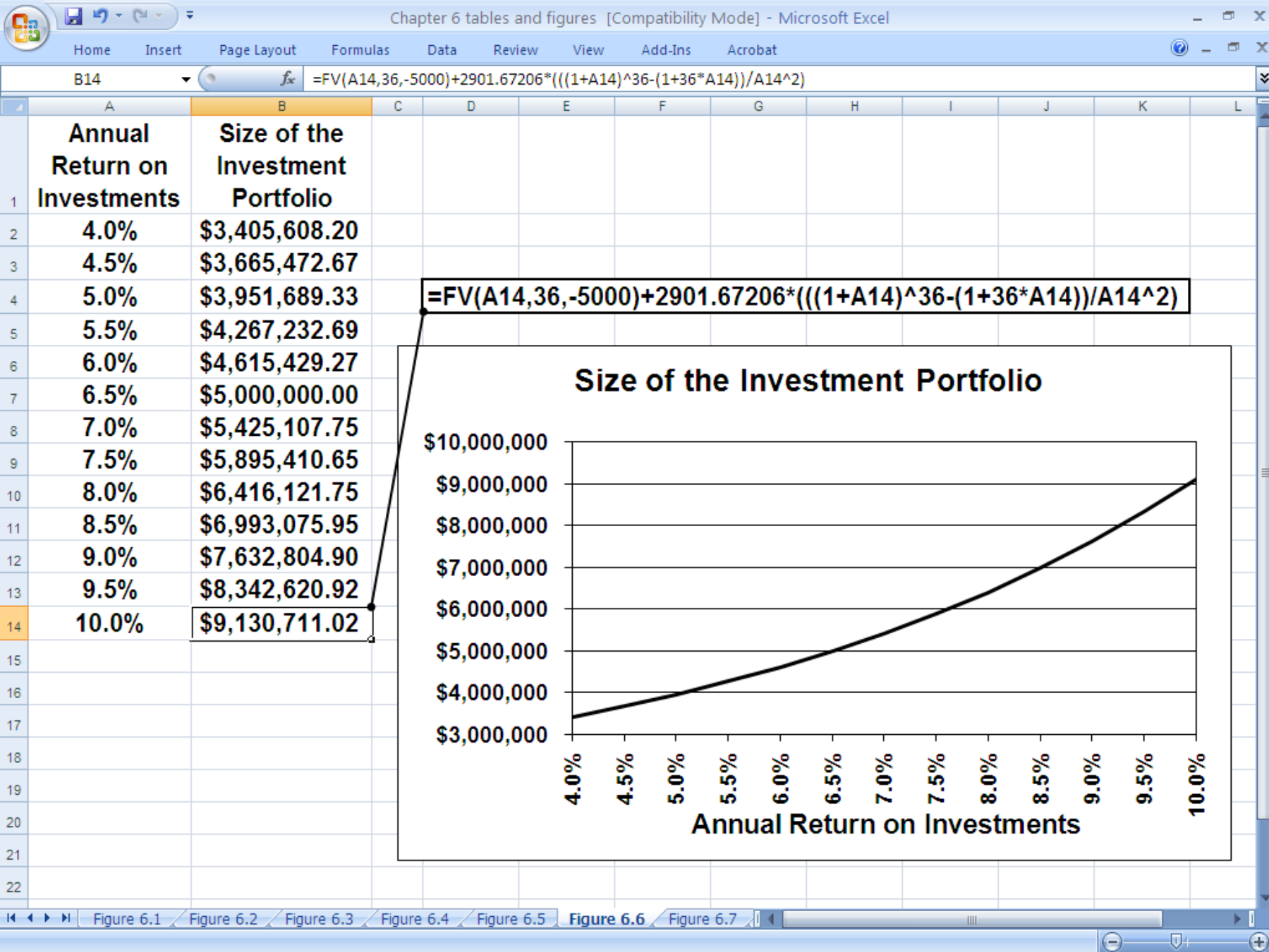
$$(A|G 6.5\%,36) = \{(1.065)^{36} - [1 + 36(0.065)]\}/\{0.065[(1.065)^{36} - 1]\}$$
$$= 11.22339$$

$$G = [\$5,000,000(0.0075133) - \$5000]/11.22339 = \$2901.66$$

Example 6.3 (Continued)

Suppose the return on the investment is quite uncertain. Specifically, suppose it can be between 4% and 10%. What will be the impact on the value of the investment portfolio when the engineer is 58?

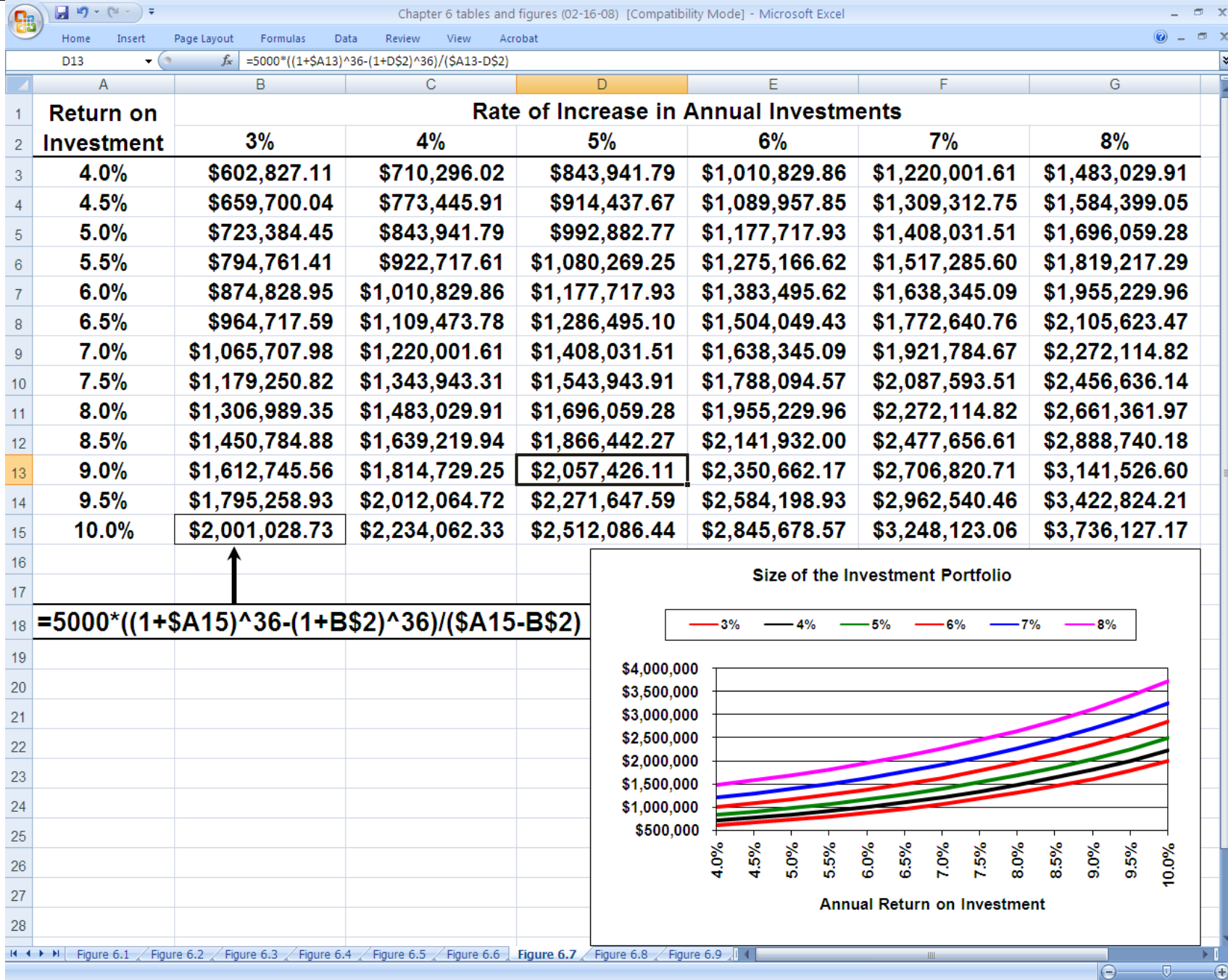
Answer: it will have a value between \$3.41 million and \$9.13 million. (Convert CF to FW at different MARR)



Example 6.3 (Continued)

Suppose the engineer makes geometric increases in annual investments. Specifically, suppose annual investments increase by 3%, 4%, 5%, 6%, 7%, or 8%. What will be the impact on the value of the investment portfolio when the engineer is 58?

Answer: it will have a value between \$0.6 million and \$3.7 million. (Same however considering different j for geometric series)



Example 6.3 (Continued)

Based on the results of the analysis, the engineer decides to increase by \$2500 the annual investment until age 40; the next 18 annual investments are 5% greater than the previous investment. What will be the impact on the value of the investment portfolio when the engineer is 58?

Answer: The investment portfolio will equal **\$5,819,498.50.**

NEXT 18
YEARS

$$FW = 5000 (F/P, 8\%, 36) + G (F/G, 8\%, 18)(F/P, 8\%, 18) + 49875 (F/A1, 8\%, 5\%, 18)$$

5000+2500*17= 47500, CF at n=40
so n at 41 = (47500+(47500*.05))

	A	B	C	D	E	F	G	H	I	J	K
1	EOY	Deposit	Balance	EOY	Deposit	Balance					
2	23	\$5,000.00	\$5,000.00	41	\$49,875.00	\$908,552.04					
3	24	\$7,500.00	\$12,900.00	42	\$52,368.75	\$1,033,604.96	=E2*(1+\$C\$21)				
4	25	\$10,000.00	\$23,932.00	43	\$54,987.19	\$1,171,280.54					
5	26	\$12,500.00	\$38,346.56	44	\$57,736.55	\$1,322,719.53					
6	27	\$15,000.00	\$56,414.28	45	\$60,623.37	\$1,489,160.47					
7	28	\$17,500.00	\$78,427.43	46	\$63,654.54	\$1,671,947.85					
8	29	\$20,000.00	\$104,701.62	47	\$66,837.27	\$1,872,540.94					
9	30	\$22,500.00	\$135,577.75	48	\$70,179.13	\$2,092,523.35					
10	31	\$25,000.00	\$171,423.97	49	\$73,688.09	\$2,333,613.31					
11	32	\$27,500.00	\$212,637.89	50	\$77,372.49	\$2,597,674.87					
12	33	\$30,000.00	\$259,648.92	51	\$81,241.12	\$2,886,729.98					
13	34	\$32,500.00	\$312,920.83	52	\$85,303.18	\$3,202,971.56					
14	35	\$35,000.00	\$372,954.50	53	\$89,568.33	\$3,548,777.61					
15	36	\$37,500.00	\$440,290.86	54	\$94,046.75	\$3,926,726.57					
16	37	\$40,000.00	\$515,514.13	55	\$98,749.09	\$4,339,613.79	=E16+(1+\$C\$22)*F15				
17	38	\$42,500.00	\$599,255.26	56	\$103,686.54	\$4,790,469.43					
18	39	\$45,000.00	\$692,195.68	57	\$108,870.87	\$5,282,577.86					
19	40	\$47,500.00	\$795,071.34	58	\$114,314.41	\$5,819,498.50					
20		Gradient = \$2,500.00									
21		Geometric = 5.0%					=E18*(1+\$C\$21)				
22		ROI = 8.0%					=B19+(1+\$C\$22)*C18				

Future Worth Analysis

Multiple Alternatives

Example 6.4

Recall the example involving two design alternatives (A & B) for a new ride (The Scream Machine) in a theme park. A costs \$300,000, has revenue of \$55,000/yr, and has a negligible salvage value at the end of the 10-year planning horizon; B costs \$450,000, has revenue of \$80,000/yr, and has a negligible salvage value. Based on a FW analysis and a 10% MARR, which is preferred?

$$\begin{aligned}FW_A(10\%) &= -\$300,000(F/P\ 10\%,10) + \$55,000(F/A\ 10\%,10) \\ &= \$98,436.10 \\ &= FV(10\%,10,-55000,300000) = \$98,435.62\end{aligned}$$

$$\begin{aligned}FW_B(10\%) &= -\$450,000(F/P\ 10\%,10) + \$80,000(F/A\ 10\%,10) \\ &= \$107,810.60 \\ &= FV(10\%,10,-80000,450000) = \$107,809.86\end{aligned}$$



Example 6.4

Recall the example involving two design alternatives (A & B) for a new ride (The Scream Machine) in a theme park. A costs \$300,000, has revenue of \$55,000/yr, and has a negligible salvage value at the end of the 10-year planning horizon; B costs \$450,000, has revenue of \$80,000/yr, and has a negligible salvage value. Based on a FW analysis and a 10% MARR, which is preferred?

$$\begin{aligned}FW_A(10\%) &= -\$300,000(F/P\ 10\%,10) + \$55,000(F/A\ 10\%,10) \\ &= \$98,436.10\end{aligned}$$

$$=FV(10\%,10,-55000,300000) = \$98,435.62$$

$$\begin{aligned}FW_B(10\%) &= -\$450,000(F/P\ 10\%,10) + \$80,000(F/A\ 10\%,10) \\ &= \$107,810.60\end{aligned}$$

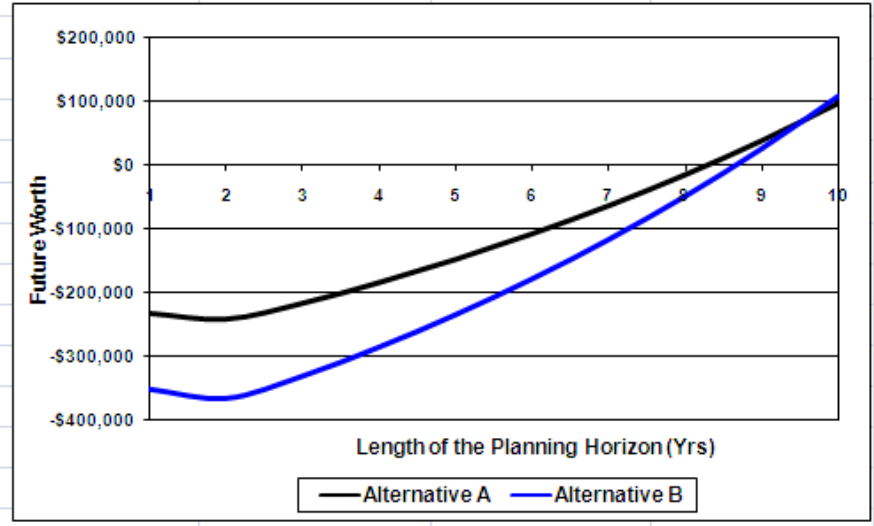
$$=FV(10\%,10,-80000,450000) = \$107,809.86$$

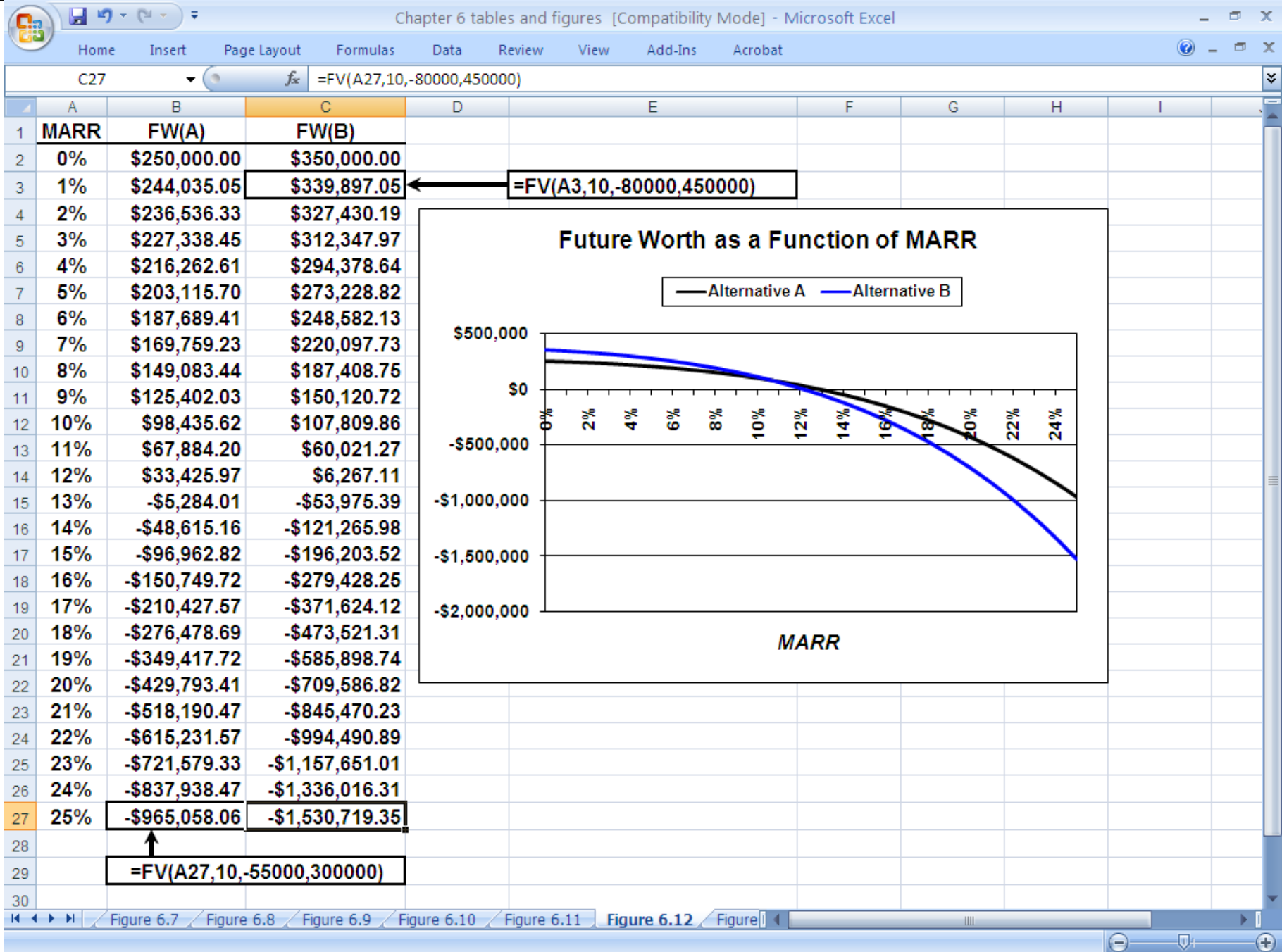
Analyze the impact on FW based on salvage values decreasing geometrically to 1¢ after 10 years; and analyze the impact of changes in the *MARR* on the recommendation.

B17 =FV(10%,A17,-B12,-\$B\$3)+C12

	A	B	C	D	E	F	G	H
1	Planning	Geometric Series Rate (A) = -85.800%			Geometric Series Rate (B) = -86.364%			
2	Horizon	CF(A)	SV(A)	FW(A)	CF(B)	SV(B)	FW(B)	FW(B-A)
3	0	-\$300,000.00	\$300,000.00	\$0.00	-\$450,000.00	\$450,000.00	\$0.00	\$0.00
4	1	\$55,000.00	\$42,599.95	-\$232,400.05	\$80,000.00	\$61,360.84	-\$353,639.16	-\$121,239.12
5	2	\$55,000.00	\$6,049.19	-\$241,450.81	\$80,000.00	\$8,367.00	-\$368,133.00	-\$126,682.18
6	3	\$55,000.00	\$858.98	-\$216,391.02	\$80,000.00	\$1,140.90	-\$333,009.10	-\$116,618.08
7	4	\$55,000.00	\$121.98	-\$183,853.02	\$80,000.00	\$155.57	-\$287,409.43	-\$103,556.40
8	5	\$55,000.00	\$17.32	-\$147,355.18	\$80,000.00	\$21.21	-\$236,300.29	-\$88,945.11
9	6	\$55,000.00	\$2.46	-\$107,107.29	\$80,000.00	\$2.89	-\$179,950.76	-\$72,843.47
10	7	\$55,000.00	\$0.35	-\$62,820.38	\$80,000.00	\$0.39	-\$117,948.62	-\$55,128.24
11	8	\$55,000.00	\$0.05	-\$14,102.75	\$80,000.00	\$0.05	-\$49,743.86	-\$35,641.11
12	9	\$55,000.00	\$0.01	\$39,486.93	\$80,000.00	\$0.01	\$25,281.70	-\$14,205.23
13	10	\$55,000.00	\$0.00	\$98,435.62	\$80,000.00	\$0.00	\$107,809.86	\$9,374.25

Planning Horizon	FW(A)	FW(B-A)
10	\$98,435.62	
10		\$9,374.25





Example 6.5

A recent 22-year old engineering graduate is choosing between 2 retirement plans: with plan 1, up to 4% of salary is matched by employer and, in the past, has earned 6% annual returns; with plan 2, a 1.5% fee is paid, matching up to 4% still occurs, and the investments being considered return between 2% and 12% annually. Her current salary is \$55,000; she assumes her salary will increase at an annual rate of 5%. Which should she choose?

$$FW_1(6\%) = 2(0.04)(\$55,000)(F|A_1 \ 6\%,5\%,40) = \$1,428,120.90$$

Fees paid 100-1.5



$$FW_2(2\%) = 2(0.04)(\$55,000)(0.985)(F|A_1 \ 2\%,5\%,40) = \$698,055.57$$

$$FW_2(12\%) = 2(0.04)(\$55,000)(0.985)(F|A_1 \ 12\%,5\%,40) = \$5,325,308.50$$

She chose the 2nd plan; which would you choose?

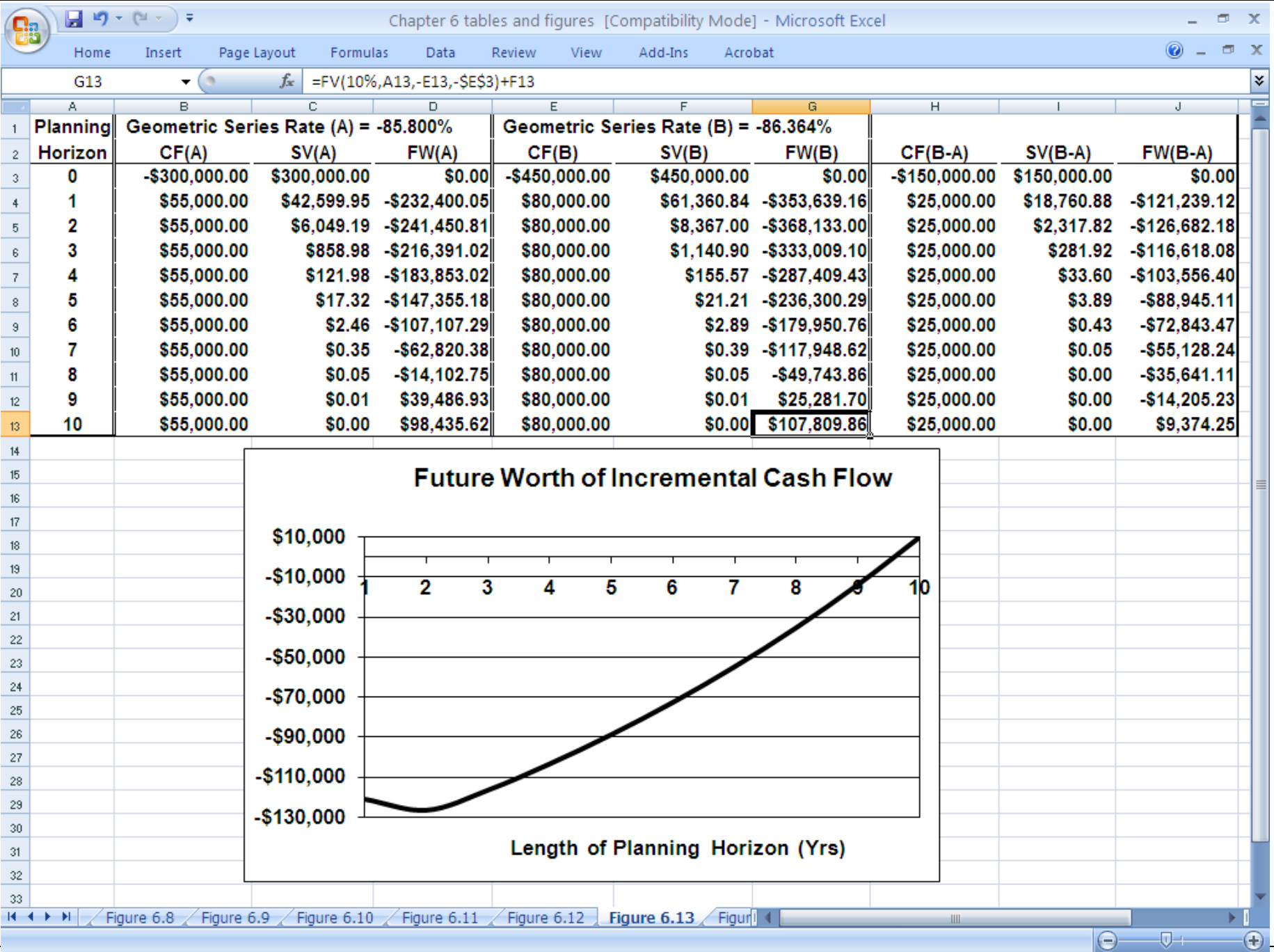
Example 6.6 (Incremental Approach)

Recall the example with two design alternatives for The Scream Machine: A costs \$300,000, has revenue of \$55,000/yr, and has a negligible salvage value at the end of the 10-year planning horizon; and B costs \$450,000, has revenue of \$80,000/yr, and has a negligible salvage value. Based on an incremental FW analysis and a 10% MARR, which is preferred?

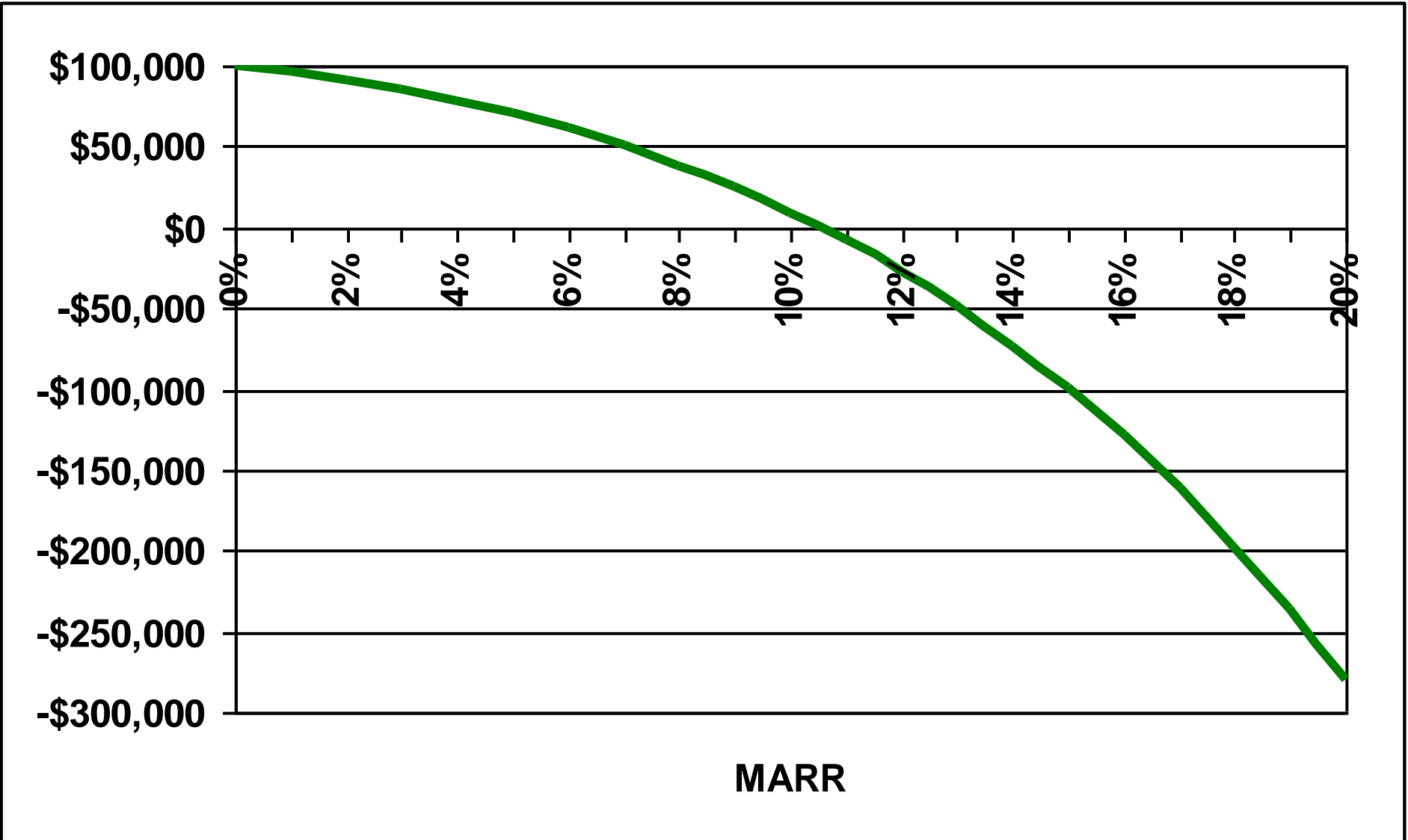
$$\begin{aligned}FW_A(10\%) &= -\$300,000(F/P\ 10\%,10) + \$55,000(F/A\ 10\%,10) \\ &= \$98,436.10 > \$0 \\ &= FV(10\%,10,-55000,300000) = \$98,435.62 > \$0 \\ &\text{(A is better than "do nothing")}\end{aligned}$$

$$\begin{aligned}FW_{B-A}(10\%) &= -\$150,000(F/P\ 10\%,10) + \$25,000(F/A\ 10\%,10) \\ &= \$9374.50 > \$0 \\ &= FV(10\%,10,-25000,150000) \\ &= \$9374.25 > \$0 \\ &\text{(B is better than A)}\end{aligned}$$

Prefer B



Incremental Future Worth as a Function of the *MARR*



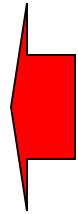
Example 6.8

Perform an investment portfolio analysis for the investment involving two design alternatives for The Scream Machine.

$$\begin{aligned}FW_{\text{do nothing}}(10\%) &= \$450,000(F|P\ 10\%,10) = \$1,167,183.00 \\ &= FV(10\%,10,,-450000) = \$1,167,184.11\end{aligned}$$

$$\begin{aligned}FW_B(10\%) &= \$80,000(F/A\ 10\%,10) = \$1,274,993.60 \\ &= FV(10\%,10,-80000) = \$1,274,993.97\end{aligned}$$

$$\begin{aligned}FW_A(10\%) &= \$55,000(F/A\ 10\%,10) + \$150,000(F/P \\ &10\%,10) \\ &= \$1,265,619.10 \\ &= FV(10\%,10,-55000)+FV(10\%,10,-150000) \\ &= \$1,265,619.72\end{aligned}$$



More on Unequal Lives

Principle #8

Compare investment alternatives over a common period of time

Example 6.9

If an investor's MARR is 12%, which mutually exclusive investment alternative maximizes the investor's future worth, given the parameters shown below?

EOY	CF(1)	CF(2)	CF(3)
0	-\$10,000	-\$14,500	-\$20,000
1	\$5,000	\$5,000	\$0
2	\$5,000	\$5,000	\$3,000
3	\$10,000	\$5,000	\$6,000
4		\$5,000	\$9,000
5		\$5,000	\$12,000
6		\$7,500	\$15,000

What planning horizon should be used? What assumptions are made regarding Alt. 1 for years 4, 5, and 6?

Example 6.9 (Continued)

If we use a 6-year planning horizon and assume no cash flows will occur in years 4, 5, and 6 for Alt. 1, the future worths will be as follows:

$$\begin{aligned}FW_1(12\%) &= -\$10,000(F|P\ 12\%,6) \\ &\quad + [\$5000(F|A\ 12\%,3) + \$5000](F|P\ 12\%,3) = \$10,990.43 \\ &= FV(12\%,6,-5000,10000) + FV(12\%,3,5000,-5000) = \\ &\mathbf{\$10,990.36}\end{aligned}$$

$$\begin{aligned}FW_2(12\%) &= -\$14,500(F|P\ 12\%,6) + \$5000(F|A\ 12\%,6) = \$11,955.56 \\ &= FV(12\%,6,-5000,14500) = \mathbf{\$11,955.52}\end{aligned}$$

$$\begin{aligned}FW_3(12\%) &= -\$20,000(F|P\ 12\%,6) + \$3000(A|G\ 12\%,6)(F|A\ 12\%,6) \\ &= \$13,403.40 \\ &= FV(12\%,6,-1000 * NPV(12\%,0,3,6,9,12,15) + 20000) = \\ &\mathbf{\$13,403.27}\end{aligned}$$



Example 6.9 (Continued)

If we use a 6-year planning horizon and assume Alt. 1 repeats with identical cash flows for years 4, 5, and 6 for Alt. 1, the cash flow profiles will be as follows:

EOY	CF(1')	CF(2)	CF(3)
0	-\$10,000	-\$14,500	-\$20,000
1	\$5,000	\$5,000	\$0
2	\$5,000	\$5,000	\$3,000
3	\$0	\$5,000	\$6,000
4	\$5,000	\$5,000	\$9,000
5	\$5,000	\$5,000	\$12,000
6	\$10,000	\$7,500	\$15,000

Example 6.9 (Continued)

Under the assumption that Alt. 1 is repeated with identical cash flows for years 4, 5, and 6, the future worths will be as follows:

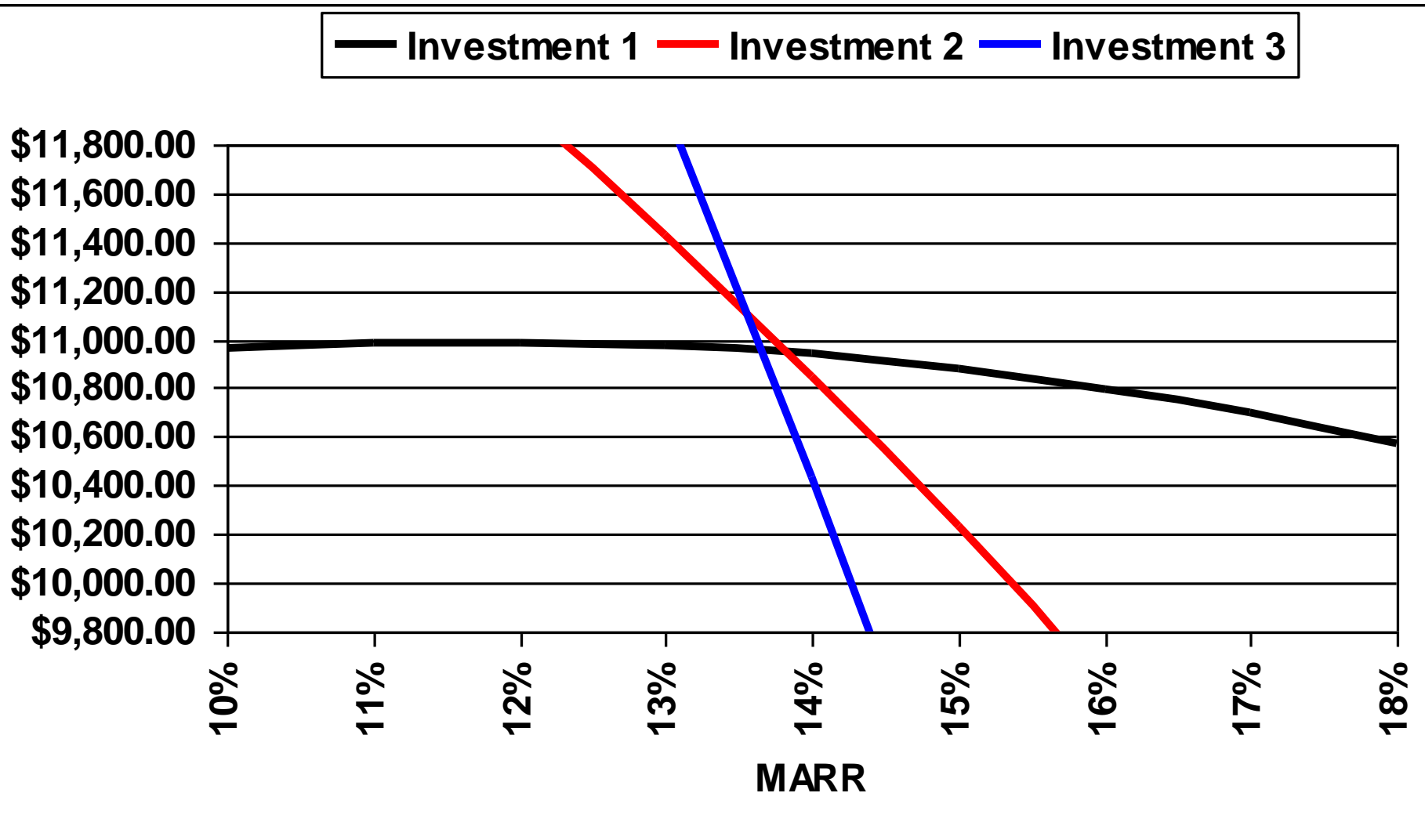
$$\begin{aligned}FW_1(12\%) &= -\$10,000(F|P\ 12\%,6) + \$5000(F|A\ 12\%,6) - \$5000(F|P\ 12\%,3) \\ &\quad + \$5000 \\ &= FV(12\%,6,-5000,10000) + FV(12\%,3,,5000) + 5000 \\ &= \$18,813.08\end{aligned}$$

$$\begin{aligned}FW_2(12\%) &= -\$14,500(F|P\ 12\%,6) + \$5000(F|A\ 12\%,6) \\ &= FV(12\%,6,-5000,14500) \\ &= \$11,955.52\end{aligned}$$

$$\begin{aligned}FW_3(12\%) &= -\$20,000(F|P\ 12\%,6) + \$3000(A|G\ 12\%,6)(F|A\ 12\%,6) \\ &= FV(12\%,6,-1000*NPV(12\%,0,3,6,9,12,15) + 20000) \\ &= \$13,403.27\end{aligned}$$

Is it reasonable to assume an investment alternative equivalent to Alt. 1 will be available in 3 years? If so, why was the MARR set equal to 12%?

Future Worths Assuming Investment 1 Is Not Repeated



Pit Stop #6—It's Time to Put the Peddle to the Metal!

1. True or False: Future worth analysis is the most popular *DCF* measure of economic worth.
2. True or False: Unless non-monetary considerations dictate otherwise, choose the mutually exclusive investment alternative that has the greatest future worth, regardless of the lives of the alternatives.
3. True or False: If $FW > 0$ when the $MARR = 20\%$, then $DPBP < 5$ years.
4. True or False: If $FW < 0$, then $PW < 0$.
5. True or False: If $FW(A) > FW(B)$, then $DPBP(A) < DPBP(B)$, and $PBP(A) < PBP(B)$.
6. True or False: When using future worth analysis with mutually exclusive alternatives having unequal lives, use a planning horizon equal to the least common multiple of lives.

Pit Stop #6—It's Time to Put the Peddle to the Metal!

1. True or False: Future worth analysis is the most popular *DCF* measure of economic worth. **FALSE**
2. True or False: Unless non-monetary considerations dictate otherwise, choose the mutually exclusive investment alternative that has the greatest future worth, regardless of the lives of the alternatives. **FALSE**
3. True or False: If $FW > 0$ when the $MARR = 20\%$, then $DPBP < 5$ years. **FALSE**
4. True or False: If $FW < 0$, then $PW < 0$. **TRUE**
5. True or False: If $FW(A) > FW(B)$, then $DPBP(A) < DPBP(B)$, and $PBP(A) < PBP(B)$. **FALSE**
6. True or False: When using future worth analysis with mutually exclusive alternatives having unequal lives, use a planning horizon equal to the least common multiple of lives. **FALSE (it is situation and circumstance dependent)**