

Chapter 5

Present Worth Analysis

Systematic Economic Analysis Technique

- 1. Identify the investment alternatives**
- 2. Define the planning horizon**
- 3. Specify the discount rate**
- 4. Estimate the cash flows**
- 5. Compare the alternatives**
- 6. Perform supplementary analyses**
- 7. Select the preferred investment**

Measures of Economic Worth

- Present Worth ($\geq \$0$)
- Future Worth ($\geq \$0$)
- Annual Worth ($\geq \$0$)
- Capitalized Worth ($\geq \$0$)
- Discounted Payback Period (\leq Value, e.g. 2 yrs)
- Payback Period (\leq Value)
- Internal Rate of Return (\geq MARR)
- External Rate of Return (\geq MARR)
- Modified Internal Rate of Return (\geq MARR)
- Benefit/Cost Ratio (≥ 1.0)

Measures of Economic Worth

- **Ranking Methods**
 - Present Worth
 - Future Worth
 - Annual Worth
 - Capitalized Worth (represents PW for **Infinite planning Horizon**)
 - Discounted Payback Period (**Time?**)
 - Payback Period (**Time?**)
- **Incremental Methods**
 - Internal Rate of Return (%)
 - External Rate of Return (%)
 - Modified Internal Rate of Return (%)
 - Benefit/Cost Ratio

Measures of Economic Worth

- The following are consistent measures of economic worth, i.e., yield the same recommendation (*if performed correctly*)
 - Present Worth
 - Future Worth
 - Annual Worth
 - Internal Rate of Return
 - External Rate of Return
 - Benefit/Cost Ratio
- Capitalized worth yields the same recommendation if the planning horizon is *infinitely long* or equal to **a least common multiple of lives** of the investment alternatives

Present Worth Analysis

Single Alternative

Present Worth Method

- converts all cash flows to a single sum equivalent at time zero using $i = MARR$ over the planning horizon
- the most popular DCF method

$$PW(i\%) = \sum_{t=0}^n A_t (1+i)^{-t}$$

(bring all cash flows back to “time zero” and add them up!)

Example 5.2

A \$500,000 investment in a surface mount placement machine is being considered. Over a 10-year planning horizon, it is estimated the SMP machine will produce net annual savings of \$92,500. At the end of 10 years, it is estimated the SMP machine will have a \$50,000 salvage value. Based on a 10% MARR and a present worth analysis, should the investment be made?

$$\begin{aligned} \text{PW} &= -\$500\text{K} + \$92.5\text{K}(\text{P|A } 10\%, 10) + \$50\text{K}(\text{P|F } 10\%, 10) \\ &= \$87,650.50 \\ &= \text{PV}(10\%, 10, -92500, -50000) - 500000 \\ &= \$87,649.62 \end{aligned}$$

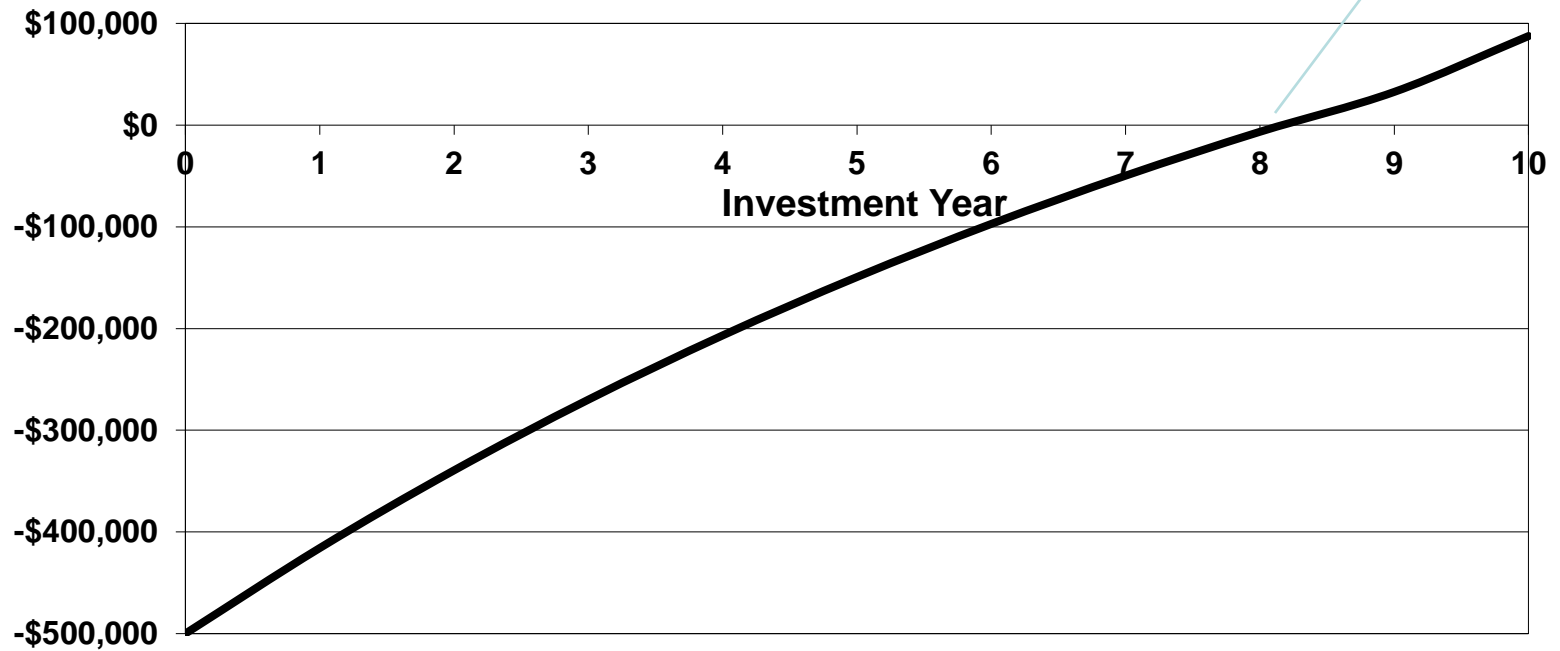
Solving with Excel®,

MARR =	10%	(ignores salvage value until EOY = 10)	
EOY	CF	Cum(PW)	
0	-\$500,000	-\$500,000.00	=B3
1	\$92,500	-\$415,909.09	=NPV(\$B\$1,\$B\$4:B4)+\$B\$3
2	\$92,500	-\$339,462.81	=NPV(\$B\$1,\$B\$4:B5)+\$B\$3
3	\$92,500	-\$269,966.19	=NPV(\$B\$1,\$B\$4:B6)+\$B\$3
4	\$92,500	-\$206,787.45	=NPV(\$B\$1,\$B\$4:B7)+\$B\$3
5	\$92,500	-\$149,352.22	=NPV(\$B\$1,\$B\$4:B8)+\$B\$3
6	\$92,500	-\$97,138.39	=NPV(\$B\$1,\$B\$4:B9)+\$B\$3
7	\$92,500	-\$49,671.26	=NPV(\$B\$1,\$B\$4:B10)+\$B\$3
8	\$92,500	-\$6,519.33	=NPV(\$B\$1,\$B\$4:B11)+\$B\$3
9	\$92,500	\$32,709.70	=NPV(\$B\$1,\$B\$4:B12)+\$B\$3
10	\$142,500	\$87,649.62	=NPV(\$B\$1,\$B\$4:B13)+\$B\$3
PW =	\$87,649.62	=NPV(B1,B4:B13)+B3	

Plotting Cumulative Present Worth,

**Present Worth of the Investment
(ignores salvage value until EOY = 10)**

Break even point



Present Worth Analysis

Multiple Alternatives

$$\text{Maximize } PW_j(i\%) = \sum_{t=0}^n A_{jt} (1+i)^{-t}$$

$\forall j$

Choose the alternative with the greatest present worth

Example 5.3

Two design alternatives (A & B) are being considered for a new ride (The Scream Machine) at a theme park in Florida. Alternative A requires a \$300,000 investment and will produce net annual revenue of \$55,000/yr. Alternative B requires a \$450,000 investment and will produce net annual revenue of \$80,000/yr. At the end of the 10-yr planning horizon, both designs will have negligible salvage values. Based on a 10% *MARR*, which should be chosen? (The “do nothing” alternative is feasible and assumed to have a PW of \$0.)

$$\begin{aligned}PW_A(10\%) &= -\$300,000 + \$55,000(P/A\ 10\%,10) = \$37,951.35 \\ &= PV(10\%,10,-55000)-300000 = \$37,951.19 > \$0\end{aligned}$$

(A is better than doing nothing)

$$\begin{aligned}PW_B(10\%) &= -\$450,000 + \$80,000(P/A\ 10\%,10) = \$41,565.60 \\ &= PV(10\%,10,-80000)-450000 = \$41,565.37 > PW_A\end{aligned}$$

(B is better than A)

Example 5.3

Two design alternatives (A & B) are being considered for a new ride (The Scream Machine) at a theme park in Florida. Alternative A requires a \$300,000 investment and will produce net annual revenue of \$55,000/yr. Alternative B requires a \$450,000 investment and will produce net annual revenue of \$80,000/yr. At the end of the 10-yr planning horizon, both designs will have negligible salvage values. Based on a 10% *MARR*, which should be chosen? (The “do nothing” alternative is feasible and assumed to have a PW of \$0.)

How does PW change with changing *MARR*?

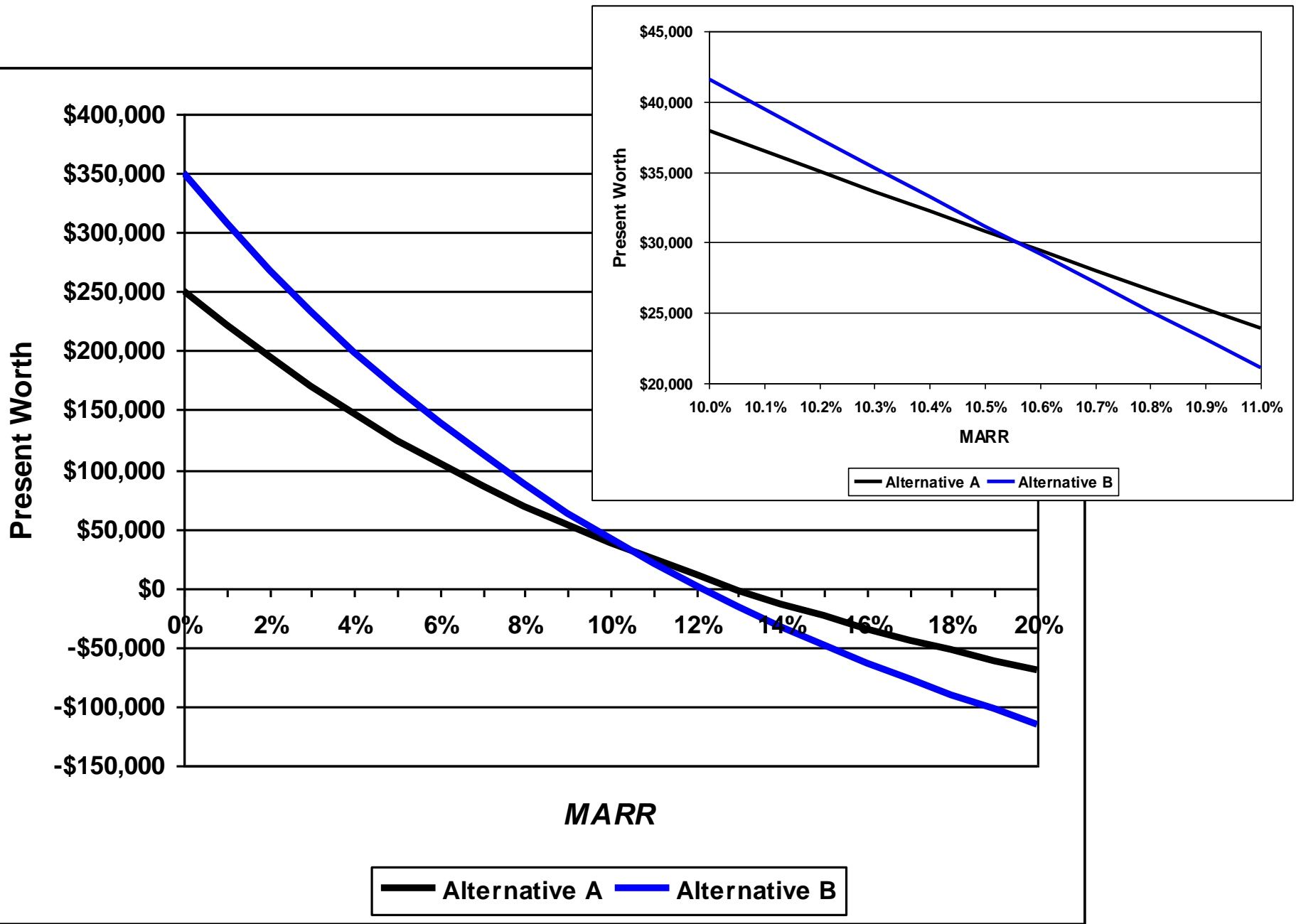
$$=PV(10\%,10,-55000)-300000 = \$37,951.19 > \$0$$

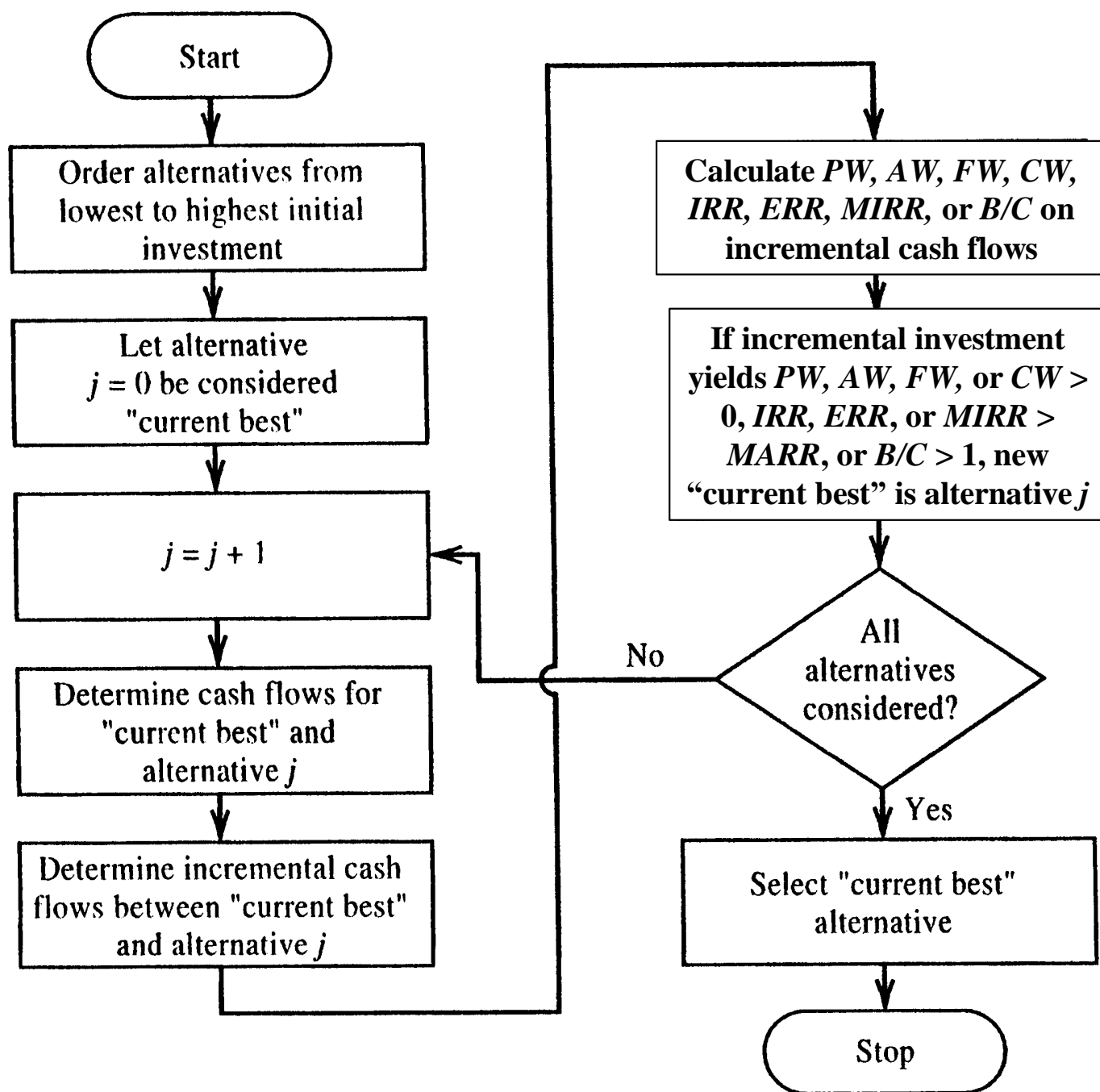
(A is better than doing nothing)

$$PW_B(10\%) = -\$450,000 + \$80,000(P/A\ 10\%,10) = \$41,565.60$$

$$=PV(10\%,10,-80000)-450000 = \$41,565.37 > PW_A$$

(B is better than A)





Flow Chart for the Incremental Comparison of Investment Alternatives

Example 5.4

Let's use an **incremental approach** to evaluate the two design alternatives for a new ride at a theme park. Recall, Alternative A required a \$300,000 investment and produced annual revenue of \$55,000; Alternative B required a \$450,000 investment and produced annual revenue of \$80,000. At the end of the 10-yr planning horizon, both had negligible salvage values. Based on a 10% *MARR*, which should be chosen?

$$\begin{aligned}PW_A(10\%) &= -\$300,000 + \$55,000(P/A\ 10\%,10) = \$37,951.35 \\ &= PV(10\%,10,-55000)-300000 = \$37,951.19 > \$0 \\ &\text{(A is better than doing nothing)}\end{aligned}$$

$$\begin{aligned}PW_{B-A}(10\%) &= -\$150,000 + \$25,000(P/A\ 10\%,10) = \$3,614.25 \\ &= PV(10\%,10,-25000)-150000 = \$3,614.18 > \$0 \\ &\text{(B is better than A)}\end{aligned}$$

-450k- (-300k)

80 k-55k

Present Worth Analysis

“One Shot” Investments

Example 5.5

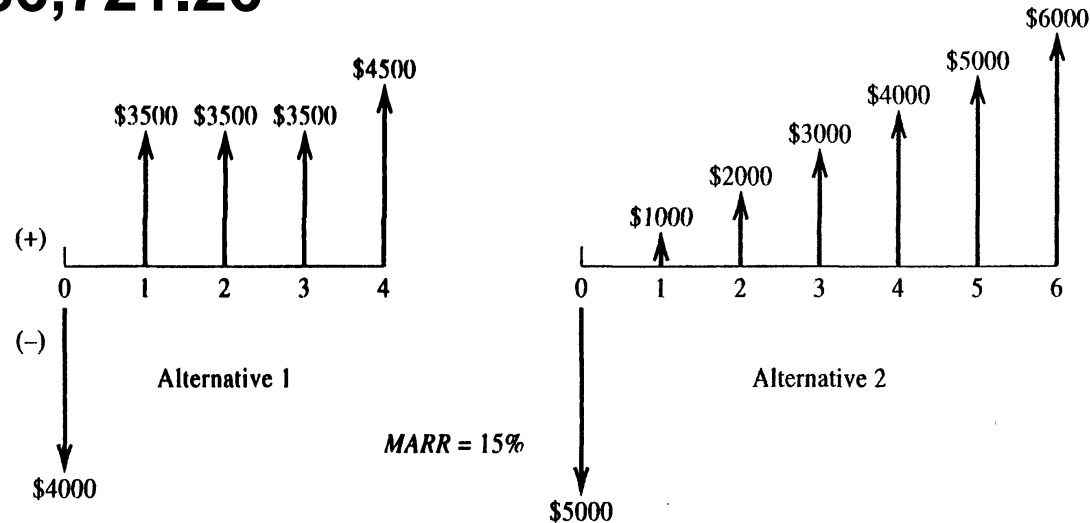
Two investment alternatives (1 & 2) are available, with the CFDs shown below. They are “one shot” investments. Using a 15% MARR, which should be chosen?

$$PW_1(15\%) = -\$4,000 + \$3,500(P|A \ 15\%,4) + \$1,000(P|F \ 15\%,4)$$

$$PW_1(15\%) = \$6,564.18$$

$$PW_2(15\%) = -\$5,000 + \$1,000(P|A \ 15\%,6) + \$1,000(P|G \ 15\%,6)$$

$$PW_2(15\%) = \$6,721.26$$



Example 5.5

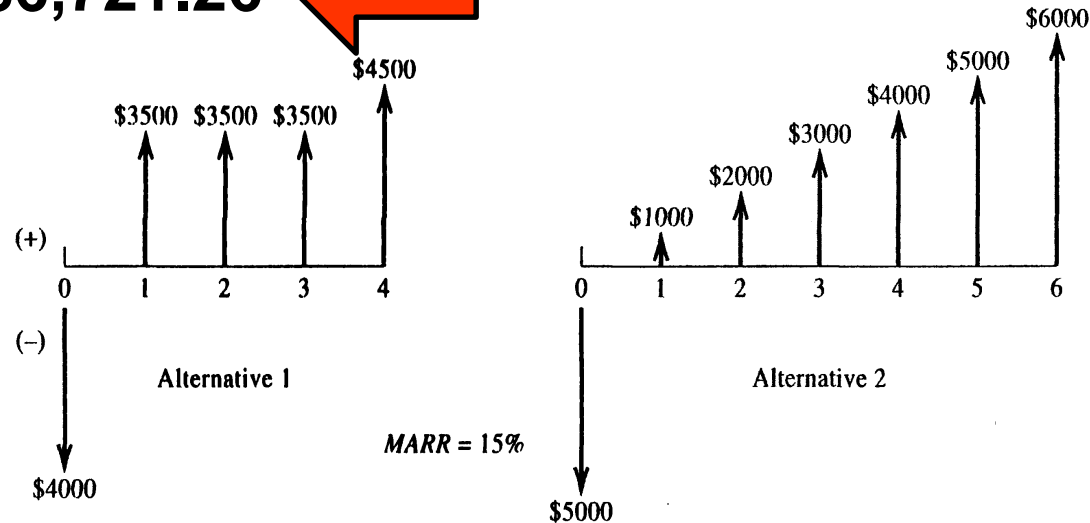
Two investment alternatives (1 & 2) are available, with the CFDs shown below. They are “one shot” investments. Using a 15% MARR, which should be chosen?

$$PW_1(15\%) = -\$4,000 + \$3,500(P|A \ 15\%,4) + \$1,000(P|F \ 15\%,4)$$

$$PW_1(15\%) = \$6,564.18$$

$$PW_2(15\%) = -\$5,000 + \$1,000(P|A \ 15\%,6) + \$1,000(P|G \ 15\%,6)$$

$$PW_2(15\%) = \$6,721.26$$



Discounted Payback Period Analysis

(does not consider values after break even point but PW does)

Single Alternative

Discounted Payback Period (DPP) Method

- determines **how long** it takes to fully recover an investment while considering the **time value of money (MARR)**
- increasing in popularity
- determine the **smallest value** of m such that

$$\sum_{t=0}^m A_t (1 + i)^{-t} \geq 0$$

(determine the point in time when cumulative discounted cash flow \geq \$0)

Discounted Payback Period

- EASTMAN calls this the *net present value payback year*
- Let's use Excel's® **SOLVER** and/or **GOAL SEEK** to determine the DPBP for the SMP investment with salvage value decreasing as geometric and gradient series.

Example 5.6

Based on a 10% MARR, how long does it take for the \$500,000 investment in a surface mount placement machine to be recovered, based on an annual savings of \$92,500 and a negligible salvage value, regardless of how long the machine is used? (either by trial and error or by using equation ch 2)

$$\begin{aligned}\# \text{ years} &= \text{NPER}(10\%, 92500, -500000) \\ &= 8.16 \text{ years}\end{aligned}$$

$$\text{PW}=0$$

$$-500 \text{ K} + 92,500 (P/A \ 10, n) = 0$$

Solving with Excel®,

MARR =	10%	(ignores salvage value until EOY = 10)	
EOY	CF	Cum(PW)	
0	-\$500,000	-\$500,000.00	=B3
1	\$92,500	-\$415,909.09	=NPV(\$B\$1,\$B\$4:B4)+\$B\$3
2	\$92,500	-\$339,462.81	=NPV(\$B\$1,\$B\$4:B5)+\$B\$3
3	\$92,500	-\$269,966.19	=NPV(\$B\$1,\$B\$4:B6)+\$B\$3
4	\$92,500	-\$206,787.45	=NPV(\$B\$1,\$B\$4:B7)+\$B\$3
5	\$92,500	-\$149,352.22	=NPV(\$B\$1,\$B\$4:B8)+\$B\$3
6	\$92,500	-\$97,138.39	=NPV(\$B\$1,\$B\$4:B9)+\$B\$3
7	\$92,500	-\$49,671.26	=NPV(\$B\$1,\$B\$4:B10)+\$B\$3
8	\$92,500	-\$6,519.33	=NPV(\$B\$1,\$B\$4:B11)+\$B\$3
9	\$92,500	\$32,709.70	=NPV(\$B\$1,\$B\$4:B12)+\$B\$3
10	\$142,500	\$87,649.62	=NPV(\$B\$1,\$B\$4:B13)+\$B\$3
PW =	\$87,649.62	=NPV(B1,B4:B13)+B3	

Example 5.6 (Continued)

How is the PW of the investment in the SMP machine affected when salvage value decreases from \$500,000 to \$50,000 over the 10-year planning horizon? Consider both geometric and gradient decreases.

Example 5.6 (Continued)

How is the PW of the investment in the SMP machine affected when salvage value decreases from \$500,000 to \$50,000 over the 10-year planning horizon? Consider both geometric and gradient decreases.

$$G = (\$500,000 - \$50,000)/10 = \$45,000/\text{yr}$$

$$j = \text{RATE}(10, -, -500000, 50000) = -20.6\%/\text{yr}$$

1	MARR =	10%					
2	EOY	CF	SV _{geometric}	Cum(PW _{geom})	SV _{gradient}	Cum(PW _{grad})	
3	0	-\$500,000	\$500,000	n/a	\$500,000	n/a	
4	1	\$92,500	\$397,500	-\$54,545.45	\$455,000	-\$2,272.73	=PV(\$B\$1,A4,-B4,-E4)+\$B\$3
5	2	\$92,500	\$316,013	-\$78,295.45	\$410,000	-\$619.83	=PV(\$B\$1,A5,-B5,-E5)+\$B\$3
6	3	\$92,500	\$251,230	-\$81,213.42	\$365,000	\$4,263.71	=PV(\$B\$1,A6,-B6,-E6)+\$B\$3
7	4	\$92,500	\$199,728	-\$70,370.67	\$320,000	\$11,776.86	=PV(\$B\$1,A7,-B7,-E7)+\$B\$3
8	5	\$92,500	\$158,784	-\$50,760.10	\$275,000	\$21,401.14	=PV(\$B\$1,A8,-B8,-E8)+\$B\$3
9	6	\$92,500	\$126,233	-\$25,883.17	\$230,000	\$32,690.62	=PV(\$B\$1,A9,-B9,-E9)+\$B\$3
10	7	\$92,500	\$100,355	\$1,826.83	\$185,000	\$45,262.99	=PV(\$B\$1,A10,-B10,-E10)+\$B\$3
11	8	\$92,500	\$79,782	\$30,699.75	\$140,000	\$58,791.71	=PV(\$B\$1,A11,-B11,-E11)+\$B\$3
12	9	\$92,500	\$63,427	\$59,608.94	\$95,000	\$72,998.98	=PV(\$B\$1,A12,-B12,-E12)+\$B\$3
13	10	\$92,500	\$50,424	\$87,813.27	\$50,000	\$87,649.62	=PV(\$B\$1,A13,-B13,-E13)+\$B\$3

$$\begin{aligned}
 \text{Cum (PW}_{\text{geom}})_{t=1} &= -\$500,000 + \$92,500(P|F 10\%,1) + \$397,500(P|F 10\%,1) \\
 &= -\$54,545.45
 \end{aligned}$$

1	MARR =	10%					
2	EOY	CF	SV _{geometric}	Cum(PW _{geom})	SV _{gradient}	Cum(PW _{grad})	
3	0	-\$500,000	\$500,000	n/a	\$500,000	n/a	
4	1	\$92,500	\$397,500	-\$54,545.45	\$455,000	-\$2,272.73	=PV(\$B\$1,A4,-B4,-E4)+\$B\$3
5	2	\$92,500	\$316,013	-\$78,295.45	\$410,000	-\$619.83	=PV(\$B\$1,A5,-B5,-E5)+\$B\$3
6	3	\$92,500	\$251,230	-\$81,213.42	\$365,000	\$4,263.71	=PV(\$B\$1,A6,-B6,-E6)+\$B\$3
7	4	\$92,500	\$199,728	-\$70,370.67	\$320,000	\$11,776.86	=PV(\$B\$1,A7,-B7,-E7)+\$B\$3
8	5	\$92,500	\$158,784	-\$50,760.10	\$275,000	\$21,401.14	=PV(\$B\$1,A8,-B8,-E8)+\$B\$3
9	6	\$92,500	\$126,233	-\$25,883.17	\$230,000	\$32,690.62	=PV(\$B\$1,A9,-B9,-E9)+\$B\$3
10	7	\$92,500	\$100,355	\$1,826.83	\$185,000	\$45,262.99	=PV(\$B\$1,A10,-B10,-E10)+\$B\$3
11	8	\$92,500	\$79,782	\$30,699.75	\$140,000	\$58,791.71	=PV(\$B\$1,A11,-B11,-E11)+\$B\$3
12	9	\$92,500	\$63,427	\$59,608.94	\$95,000	\$72,998.98	=PV(\$B\$1,A12,-B12,-E12)+\$B\$3
13	10	\$92,500	\$50,424	\$87,813.27	\$50,000	\$87,649.62	=PV(\$B\$1,A13,-B13,-E13)+\$B\$3

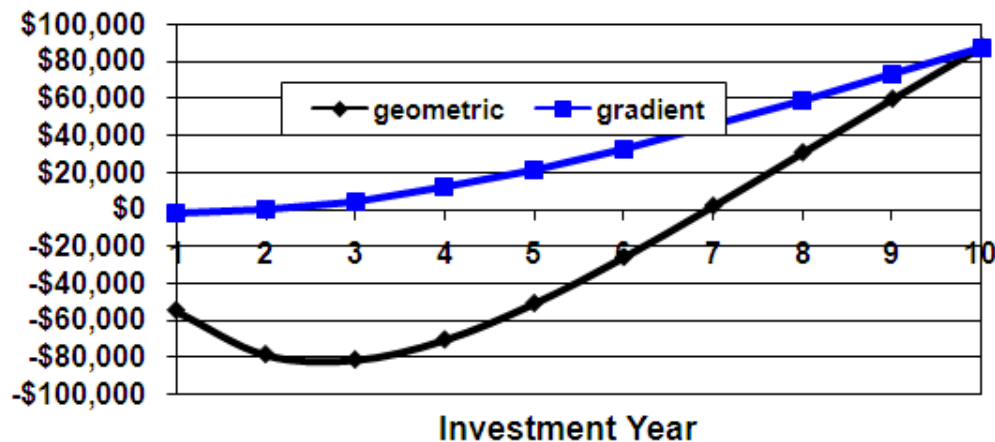
$$Cum (PW_{geom})_{t=1} = -\$500,000 + \$92,500(P|A 10\%,3) + \$251,230(P|F 10\%,3)$$

$$= -\$81213.55$$

D13 $=PV(\$B\$1,A13,-B13,-C13)+\$B\3

	A	B	C	D	E	F	G
1	MARR =	10%					
2	EOY	CF	SV _{geometric}	Cum(PW _{geom})	SV _{gradient}	Cum(PW _{grad})	
3	0	-\$500,000	\$500,000	n/a	\$500,000	n/a	
4	1	\$92,500	\$397,500	-\$54,545.45	\$455,000	-\$2,272.73	=PV(\$B\$1,A4,-B4,-E4)+\$B\$3
5	2	\$92,500	\$316,013	-\$78,295.45	\$410,000	-\$619.83	=PV(\$B\$1,A5,-B5,-E5)+\$B\$3
6	3	\$92,500	\$251,230	-\$81,213.42	\$365,000	\$4,263.71	=PV(\$B\$1,A6,-B6,-E6)+\$B\$3
7	4	\$92,500	\$199,728	-\$70,370.67	\$320,000	\$11,776.86	=PV(\$B\$1,A7,-B7,-E7)+\$B\$3
8	5	\$92,500	\$158,784	-\$50,760.10	\$275,000	\$21,401.14	=PV(\$B\$1,A8,-B8,-E8)+\$B\$3
9	6	\$92,500	\$126,233	-\$25,883.17	\$230,000	\$32,690.62	=PV(\$B\$1,A9,-B9,-E9)+\$B\$3
10	7	\$92,500	\$100,355	\$1,826.83	\$185,000	\$45,262.99	=PV(\$B\$1,A10,-B10,-E10)+\$B\$3
11	8	\$92,500	\$79,782	\$30,699.75	\$140,000	\$58,791.71	=PV(\$B\$1,A11,-B11,-E11)+\$B\$3
12	9	\$92,500	\$63,427	\$59,608.94	\$95,000	\$72,998.98	=PV(\$B\$1,A12,-B12,-E12)+\$B\$3
13	10	\$92,500	\$50,424	\$87,813.27	\$50,000	\$87,649.62	=PV(\$B\$1,A13,-B13,-E13)+\$B\$3

Present Worth of the Investment
(including salvage value)



Example 5.7

Based on a 10% MARR, how long does it take for the \$500,000 investment in a surface mount placement machine to be recovered, based on an annual savings of \$92,500 and a salvage value at the end of n years equal to

a) $\$500,000(1 - 0.206)^n$ and

b) $\$500,000 - \$45,000n$?

The Excel® **SOLVER** tool is used to solve the example.

(note: same example 5.6 except Salvage value should be considered)

Chapter 5 tables and figures (09-30-08) [Compatibility Mode] - Microsoft Excel

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B12 $=PV(B1,B8,-B10,-B11)-B9$

	A	B	C
1	MARR:	10%	
2	# years used:	10	= DPBP
3	Annual Investment:	\$500,000	
4	Annual Savings:	\$92,500	
5	Salvage Value <small>geometric:</small>	\$49,794	=B3*(1-0.206)^B2
6	Present Worth:	\$87,570.11	=PV(B1,B2,-B4,-B5)-B3
7			
8	# years used:	10	= DPBP
9	Annual Investment:	\$500,000	
10	Annual Savings:	\$92,500	
11	Salvage Value <small>gradient:</small>	\$50,000	=B9-B8*45000
12	Present Worth:	\$87,649.62	=PV(B1,B8,-B10,-B11)-B9
13	The difference in present worths is due to round-off errors in the geometric series		
14	rate needed to obtain exactly a \$50,000 salvage value after 10 years of use.		
15			

Figure 5.4 Figure 5.5 Figure 5.6 Figure 5.7 Figure 5.8 Figure 5.9 Figure 5.10

	A	B	C
1	MARR:	10%	
2	# years used:	10 = DPBP	
3	Annual Investment:	\$500,000	
4	Annual Savings:	\$92,500	
5	Salvage Value:		
6	Present Worth:		
7			
8	# years used:		
9	Annual Investment:		
10	Annual Savings:		
11	Salvage Value:		
12	Present Worth:		
13	The difference in present worths is due to round-off errors in the geometric series		
14	rate needed to obtain exactly a \$50,000 salvage value after 10 years of use.		
15			

Solver Parameters

Set Target Cell:

Equal To: Max Min Value of:

By Changing Cells:

Subject to the Constraints:

Chapter 5 tables and figures (11-13-08) [Compatibility Mode] - Microsoft Excel

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B12 fx =PV(B1,B8,-B10,-B11)-B9

	A	B	C
1	MARR:	10%	
2	# years used:	6.95187521	= DPBP
3	Annual Investment:	\$500,000	
4	Annual Savings:	\$92,500	
5	Salvage Value <small>geometric:</small>	\$100,585	=B3*(1-0.206)^B2
6	Present Worth:	\$0.00	=PV(B1,B2,-B4,-B5)-B3
7			
8	# years used:	2.16997447	= DPBP
9	Annual Investment:	\$500,000	
10	Annual Savings:	\$92,500	
11	Salvage Value <small>gradient:</small>	\$402,351	=B9-B8*45000
12	Present Worth:	\$0.00	=PV(B1,B8,-B10,-B11)-B9
13	The difference in present worths is due to round-off errors in the geometric series		
14	rate needed to obtain exactly a \$50,000 salvage value after 10 years of use.		
15			

Figure 5.2 Figure 5.3 Figure 5.4 Figure 5.5 Figure 5.6 Figure 5.7 Figure 5.8

Payback Period Method (PBP)

- EASTMAN calls it the *cash payback year*
- determines the length of time required to recover the initial investment **without considering the time value of money (no interest been considered)**
- not equivalent to those already considered
- a popular method of valuing investments
- determine the smallest value of m such that

$$\sum_{t=0}^m A_t \geq 0$$

(ignores cash flows that occur after the payback period)

Why Use the Payback Period Method?

- **does not** require interest rate calculations
- **does not** require a decision concerning **the MARR**
- easily explained and understood
- reflects a manager's attitudes when capital is limited
- hedge against uncertainty of future cash flows
- provides a rough measure of the liquidity of an investment

Example 5.8

What is the payback period for the \$500,000 SMP investment, given an annual savings of \$92,500?

$$\begin{aligned} \text{PBP} &= \$500,000 / \$92,500 \\ &= 5.4054 \text{ years} \\ &= \text{NPER}(0\%, 92500, -500000) \\ &= 5.4054 \end{aligned}$$

Discounted Payback Period Analysis

Multiple Alternatives

Example 5.9

Now, suppose a third design (alternative C) is developed for The Scream Machine. As before, A requires a \$300,000 investment and produces revenue of \$55,000/yr; and B requires a \$450,000 investment and produces revenue of \$80,000/yr. The new design (C) requires a \$150,000 investment and produces 1st year revenue of \$45,000; thereafter, revenue decreases by \$5000/yr. Based on a 10% *MARR*, which design has the smallest DPBP?

$$\text{DPBP}_A(10\%) = \text{NPER}(10\%, -55000, 300000) = 8.273 \text{ years}$$

$$\text{DPBP}_B(10\%) = \text{NPER}(10\%, -80000, 450000) = 8.674 \text{ years}$$

$$\text{DPBP}_C(10\%) = 6.273 \text{ years (using the Excel® SOLVER tool)}$$

Alternative A		
EOY	CF	Cum (PW)
0	-\$300,000	-\$300,000
1	\$55,000	-\$250,000
2	\$55,000	-\$204,545
3	\$55,000	-\$163,223
4	\$55,000	-\$125,657
5	\$55,000	-\$91,507
6	\$55,000	-\$60,461
7	\$55,000	-\$32,237
8	\$55,000	-\$6,579
9	\$55,000	\$16,746
10	\$55,000	\$37,951

Alternative B		
EOY	CF	Cum (PW)
0	-\$450,000	-\$450,000
1	\$80,000	-\$377,273
2	\$80,000	-\$311,157
3	\$80,000	-\$251,052
4	\$80,000	-\$196,411
5	\$80,000	-\$146,737
6	\$80,000	-\$101,579
7	\$80,000	-\$60,526
8	\$80,000	-\$23,206
9	\$80,000	\$10,722
10	\$80,000	\$41,565

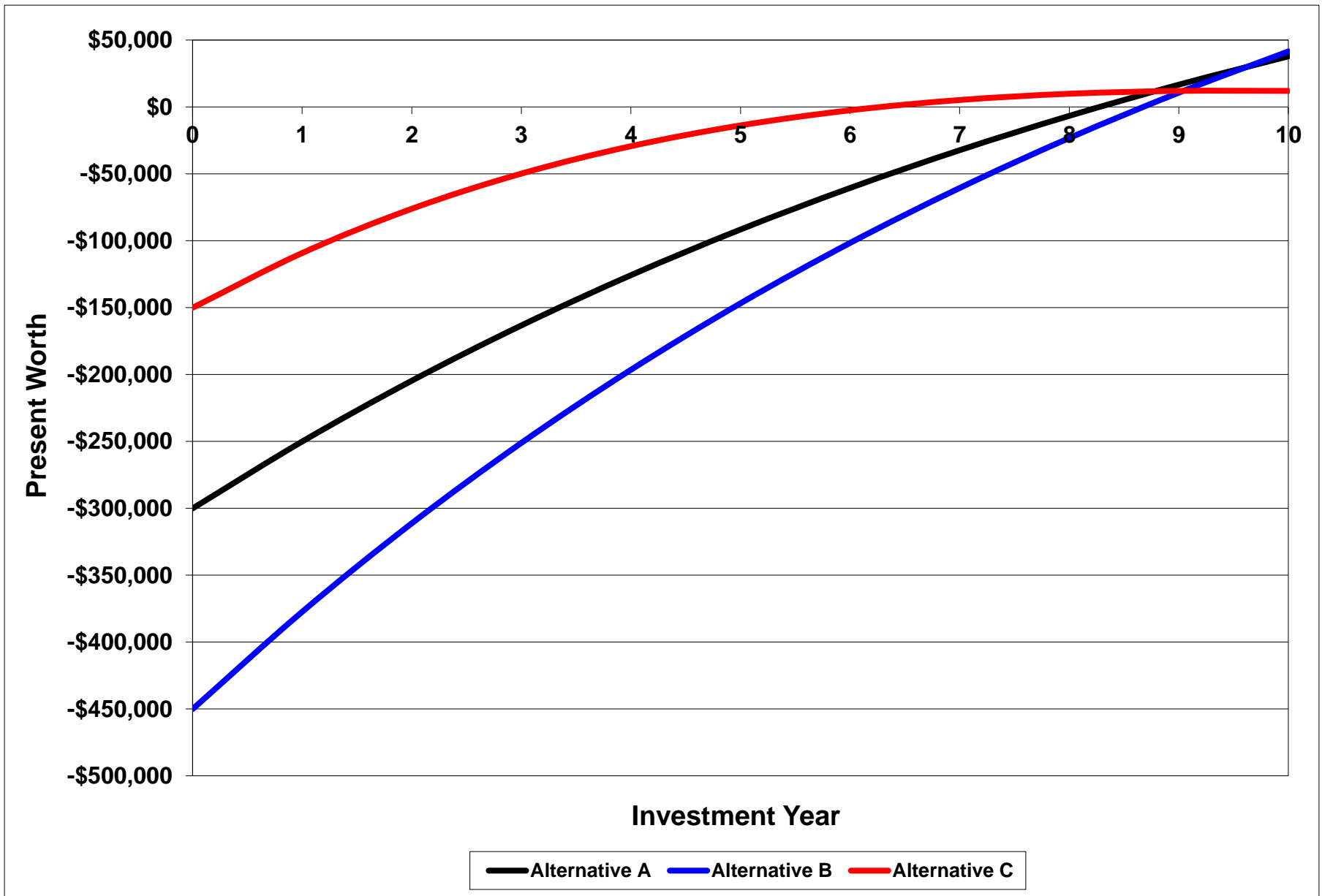
Alternative C		
EOY	CF	Cum (PW)
0	-\$150,000	-\$150,000
1	\$45,000	-\$109,091
2	\$40,000	-\$76,033
3	\$35,000	-\$49,737
4	\$30,000	-\$29,247
5	\$25,000	-\$13,724
6	\$20,000	-\$2,434
7	\$15,000	\$5,263
8	\$10,000	\$9,928
9	\$5,000	\$12,049
10	\$0	\$12,049

DPBP 8.282

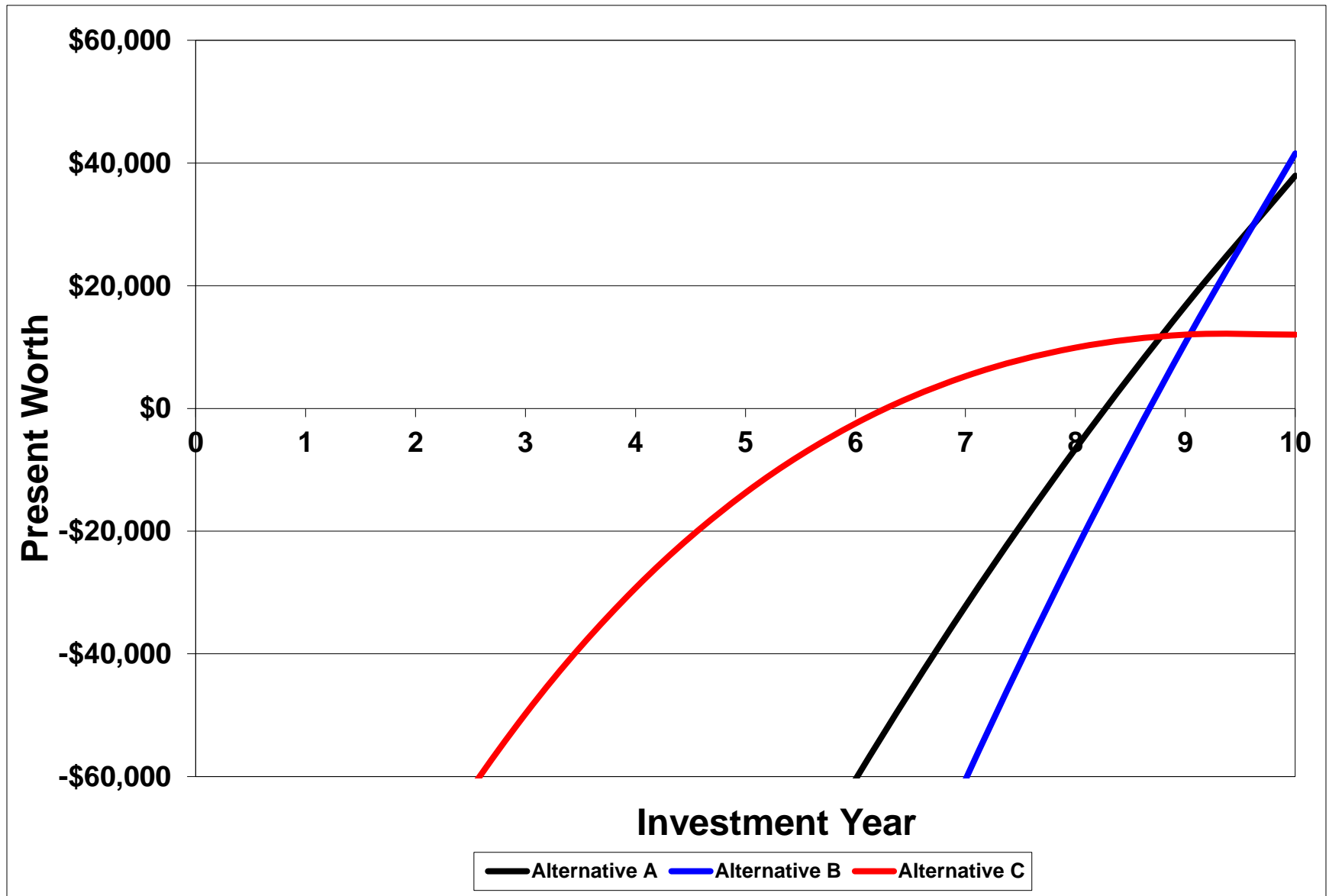
8.684

6.316

Present Worth as a Function of Investment Duration



Close-Up of Critical Region



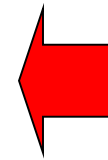
Example 5.9

Now, suppose a third design (alternative C) is developed for The Scream Machine. As before, A requires a \$300,000 investment and produces revenue of \$55,000/yr; and B requires a \$450,000 investment and produces revenue of \$80,000/yr. The new design (C) requires a \$150,000 investment and produces 1st year revenue of \$45,000; thereafter, revenue decreases by \$5000/yr. Based on a 10% *MARR*, which design has the smallest DPBP? (See the differences)

$$\text{DPBP}_A(10\%) = \text{NPER}(10\%, -55000, 300000) = 8.273 \text{ years}$$

$$\text{DPBP}_B(10\%) = \text{NPER}(10\%, -80000, 450000) = 8.674 \text{ years}$$

$$\text{DPBP}_C(10\%) = 6.273 \text{ years (using SOLVER)}$$



Note: $\text{PW}_C(10\%) = -\$150,000 + \$45,000(\text{P|A } 10\%, 10) - \$5,000(\text{P|G } 10\%, 10)$
 $\text{PW}_C(10\%) = \$12,048.81 < \text{PW}_A(10\%) < \text{PW}_B(10\%)$ B is best, not C!!

Example 5.10

Three investments are available, but only one can be pursued: invest \$10,000 and obtain \$5,000/yr for 2 yrs, plus \$1,000 after 5 yrs; invest \$10,000 and receive \$5,000, \$4,000, \$3,000, \$2,000, and \$1,000 over the next 5 yrs; invest \$10,000 and receive \$2,500/yr for 5 yrs, plus \$10,000 after 5 yrs. Which is best using PBP? using PW and a MARR of 10%?

EOY	CF(1)	CumCF(1)	CF(2)	CumCF(2)	CF(3)	CumCF(3)
0	-\$10,000	-\$10,000	-\$10,000	-\$10,000	-\$10,000	-\$10,000
1	\$5,000	-\$5,000	\$5,000	-\$5,000	\$2,500	-\$7,500
2	\$5,000	\$0	\$4,000	-\$1,000	\$2,500	-\$5,000
3	\$0	\$0	\$3,000	\$2,000	\$2,500	-\$2,500
4	\$0	\$0	\$2,000	\$4,000	\$2,500	\$0
5	\$1,000	\$1,000	\$1,000	\$5,000	\$12,500	\$12,500
<i>PBP</i> =		2 yrs		2.33 yrs		4 yrs
<i>PW</i> (10%) =	-\$701.39		\$2,092.13		\$5,686.18	

PBP ranking: 1, 2, 3

PW ranking: 3, 2, 1

Capitalized Worth Analysis

Single Alternative

Capitalized Worth Method

- a perpetuity is an investment that has **an infinite life**
- the capitalized worth is the present worth of a perpetuity
- the **capitalized worth** indicates the amount of money needed “**up front**” such that the interest earned will cover the cash flow requirements forever for the investment
- used mostly by government

$$CW(i) = AW(i) / i$$

Example 5.13

How much will it cost to endow a \$12,500 per year scholarship if the endowment earns 4.5% interest?

$$CW = \$12,500/0.045 = \$277,777.78$$

Example 5.11

Every 10 years the dome of the state capital building has to be cleaned, sand blasted, and re-touched. It costs \$750,000 to complete the work. Using a 5% MARR, what is the capitalized cost for the refurbishment of the capital dome?

$$CC = \$750,000 + \$750,000(P|F\ 5\%,10) + \$750,000(P|F\ 5\%,10) + \dots \text{ (PW for infinite planning horizon)}$$

or

$$CC = \$750,000(A|P\ 5\%,10)/0.05 = \$750,000(0.1295)/0.05 = \$1,942,500$$

$$CC = \text{PMT}(5\%,10,-750000)/0.05 = \$1,942,569 \text{ (convert the CF to annual divided by interest rate)}$$

or

$$CC = \$750,000 + \$750,000(A|F\ 5\%,10)/0.05 \text{ (AW/MARR)}$$
$$= \$750,000 + \$750,000(0.0795)/0.05 = \$1,942,500$$

$$CC = 750000 + \text{PMT}(5\%,10,-750000)/0.05 = \$1,942,569$$

$$\text{Recall, } (A|P\ i\%,n) = (A|F\ i\%,n) + i$$

Example 5.12

A new highway is to be constructed. Asphalt paving will be used. The asphalt will cost \$150/ft, including the material and paving operation. Due to heavy usage, the asphalt is expected to last 5 yrs before requiring resurfacing.

The cost of resurfacing will be the same/ft. Paved ditches must be installed on each side of the highway and will cost \$7.75/ft to install; ditches will have to be re-paved in 15 yrs at a cost equal to the initial cost. Four pipe culverts are required/mile; each costs \$8,000 and will last 10 yrs; replacements will cost \$10,000, each, forever. Annual maintenance of the highway will cost \$9,000/mi. Cleaning each culvert will cost \$1,250/yr.

Cleaning and maintaining each ditch will cost \$3.75/ft every year. Using a 5% MARR, what is the capitalized cost (CC) per mile for the highway?

Example 5.12 (Solution)

Paving Highway and Ditches/mile

$$\begin{aligned} \text{CC} &= 5,280 \text{ ft/mi} [\$150/\text{ft}(\text{A|P } 5\%, 5) + \$7.75/\text{ft}(\text{A|P } 5\%, 15)] / 0.05 \\ &= \$3,737,409 \\ &= 5280 * (\text{PMT}(5\%, 5, -150) + \text{PMT}(5\%, 15, -7.75)) / 0.05 = \$3,737,487 \end{aligned}$$

Highway Maintenance/mile

$$\text{CC} = \$9,000 / 0.05 = \$180,000$$

Ditch Maintenance/mile

$$\text{CC} = 2(5,280 \text{ ft/mi})(\$3.75/\text{ft}) / 0.05 = \$792,000$$

Culverts/mile

$$\begin{aligned} \text{CC} &= 4 [(\$8,000 + \$1,250 / 0.05 + \$10,000(\text{A|F } 5\%, 10) / 0.05)] \\ &= \$195,600 \\ &= 4 * (8000 + 1250 / 0.05 + \text{PMT}(5\%, 10, -10000) / 0.05) \\ &= \$195,604 \end{aligned}$$

Highway/mile

$$\begin{aligned} \text{CC} &= \$3,737,487 + \$180,000 + \$792,000 + \$195,604 \\ &= \$4,905,091 \end{aligned}$$

Example 5.14

Suppose, instead of endowing a scholarship, you wish to establish a fund that will pay for the cost of a scholarship for 100 years. How much must you contribute to a fund that earns interest at an annual rate of 9%, if the size of the scholarship grows at an annual rate of 4.5%?

	A	B	C	D	E	F	G	H	I	J	K	L
1	Beginning Scholarship Amount =			\$12,500.00	Annual Endowment Growth Rate =			9.0%				
2	Size of the Gift =			\$300,000.00	Annual Increase in Tuition =			4.5%				
3												
4	EOY	Scholarship Cost	Size of Endowment	EOY	Scholarship Cost	Size of Endowment	EOY	Scholarship Cost	Size of Endowment	EOY	Scholarship Cost	Size of Endowment
5	1	\$12,500.00	\$314,500.00	26	\$37,567.93	\$1,081,281.01	51	\$112,907.95	\$4,423,077.96	76	\$339,337.45	\$23,411,238.06
6	2	\$13,062.50	\$329,742.50	27	\$39,258.49	\$1,139,337.81	52	\$117,988.81	\$4,703,166.16	77	\$354,607.64	\$25,163,641.85
7	3	\$13,650.31	\$345,769.01	28	\$41,025.12	\$1,200,853.10	53	\$123,298.31	\$5,003,152.81	78	\$370,564.98	\$27,057,804.63
8	4	\$14,264.58	\$362,623.65	29	\$42,871.25	\$1,266,058.63	54	\$128,846.73	\$5,324,589.83	79	\$387,240.41	\$29,105,766.64
9	5	\$14,906.48	\$380,353.29	30	\$44,800.46	\$1,335,203.45	55	\$134,644.83	\$5,669,158.08	80	\$404,666.23	\$31,320,619.41
10	6	\$15,577.27	\$399,007.81	31	\$46,816.48	\$1,408,555.28	56	\$140,703.85	\$6,038,678.46	81	\$422,876.21	\$33,716,598.95
11	7	\$16,278.25	\$418,640.27	32	\$48,923.22	\$1,486,402.04	57	\$147,035.53	\$6,435,123.99	82	\$441,905.63	\$36,309,187.23
12	8	\$17,010.77	\$439,307.12	33	\$51,124.76	\$1,569,053.46	58	\$153,652.12	\$6,860,633.03	83	\$461,791.39	\$39,115,222.69
13	9	\$17,776.26	\$461,068.50	34	\$53,425.38	\$1,656,842.89	59	\$160,566.47	\$7,317,523.53	84	\$482,572.00	\$42,153,020.73
14	10	\$18,576.19	\$483,988.48	35	\$55,829.52	\$1,750,129.23	60	\$167,791.96	\$7,808,308.69	85	\$504,287.74	\$45,442,504.85
15	11	\$19,412.12	\$508,135.32	36	\$58,341.85	\$1,849,299.02	61	\$175,342.60	\$8,335,713.87	86	\$526,980.69	\$49,005,349.60
16	12	\$20,285.66	\$533,581.84	37	\$60,967.23	\$1,954,768.70	62	\$183,233.02	\$8,902,695.11	87	\$550,694.82	\$52,865,136.24
17	13	\$21,198.52	\$560,405.69	38	\$63,710.76	\$2,066,987.13	63	\$191,478.50	\$9,512,459.16	88	\$575,476.09	\$57,047,522.42
18	14	\$22,152.45	\$588,689.75	39	\$66,577.74	\$2,186,438.23	64	\$200,095.03	\$10,168,485.45	89	\$601,372.51	\$61,580,426.93
19	15	\$23,149.31	\$618,522.51	40	\$69,573.74	\$2,313,643.93	65	\$209,099.31	\$10,874,549.83	90	\$628,434.27	\$66,494,231.08
20	16	\$24,191.03	\$649,998.51	41	\$72,704.56	\$2,449,167.33	66	\$218,508.78	\$11,634,750.54	91	\$656,713.82	\$71,821,998.06
21	17	\$25,279.63	\$683,218.75	42	\$75,976.26	\$2,593,616.12	67	\$228,341.68	\$12,453,536.41	92	\$686,265.94	\$77,599,711.94
22	18	\$26,417.21	\$718,291.22	43	\$79,395.19	\$2,747,646.38	68	\$238,617.05	\$13,335,737.64	93	\$717,147.91	\$83,866,538.11
23	19	\$27,605.98	\$755,331.45	44	\$82,967.98	\$2,911,966.58	69	\$249,354.82	\$14,286,599.21	94	\$749,419.56	\$90,665,106.98
24	20	\$28,848.25	\$794,463.02	45	\$86,701.54	\$3,087,342.03	70	\$260,575.78	\$15,311,817.35	95	\$783,143.44	\$98,041,823.17
25	21	\$30,146.43	\$835,818.27	46	\$90,603.11	\$3,274,599.71	71	\$272,301.69	\$16,417,579.22	96	\$818,384.90	\$106,047,202.36
26	22	\$31,503.01	\$879,538.90	47	\$94,680.25	\$3,474,633.44	72	\$284,555.27	\$17,610,606.08	97	\$855,212.22	\$114,736,238.36
27	23	\$32,920.65	\$925,776.75	48	\$98,940.86	\$3,688,409.59	73	\$297,360.26	\$18,898,200.37	98	\$893,696.77	\$124,168,803.04
28	24	\$34,402.08	\$974,694.58	49	\$103,393.19	\$3,916,973.26	74	\$310,741.47	\$20,288,296.93	99	\$933,913.12	\$134,410,082.20
29	25	\$35,950.17	\$1,026,466.92	50	\$108,045.89	\$4,161,454.97	75	\$324,724.84	\$21,789,518.82	100	\$975,939.21	\$145,531,050.38
30												
31		=B28*(1+\$L\$1)						=L28*1.09-H29				
32												
33												
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35												
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Home Insert Page Layout Formulas Data Review View Acrobat

L29 $=L28*(1+\$H\$1)-K29$

	A	B	C	D	E	F	G	H	I	J	K	L
1	Beginning Scholarship Amount = \$12,500.00			Annual Endowment Growth Rate = 9.0%								
2	Size of the Gift = \$300,000.00			Annual Increase in Tuition = 4.5%								
	EOY	Scholarship Cost	Size of Endowment	EOY	Scholarship Cost	Size of Endowment	EOY	Scholarship Cost	Size of Endowment	EOY	Scholarship Cost	Size of Endowment
4	1	\$12,500.00	\$314,500.00	26	\$37,567.93	\$1,081,281.01	51	\$112,907.95	\$4,423,077.96	76	\$339,337.45	\$23,411,238.06
5	2	\$13,062.50	\$329,742.50	27	\$39,258.49	\$1,139,337.81	52	\$117,988.81	\$4,703,166.16	77	\$354,607.64	\$25,163,641.85
6	3	\$13,650.31	\$345,769.01	28	\$41,025.12	\$1,200,853.10	53	\$123,298.31	\$5,003,152.81	78	\$370,564.98	\$27,057,804.63
7	4	\$14,264.58	\$362,623.65	29	\$42,871.25	\$1,266,058.63	54	\$128,846.73	\$5,324,589.83	79	\$387,240.41	\$29,105,766.64
8	5	\$14,906.48	\$380,353.29	30	\$44,800.46	\$1,335,203.45	55	\$134,644.83	\$5,669,158.08	80	\$404,666.23	\$31,320,619.41
9	6	\$15,577.27										\$33,716,598.95
10	7	\$16,278.25										\$36,309,187.23
11	8	\$17,010.77										\$39,115,222.69
12	9	\$17,776.26										\$42,153,020.73
13	10	\$18,576.19										\$45,442,504.85
14	11	\$19,412.12										\$49,005,349.60
15	12	\$20,285.66										\$52,865,136.24
16	13	\$21,198.52										\$57,047,522.42
17	14	\$22,152.45										\$61,580,426.93
18	15	\$23,149.31										\$66,494,231.08
19	16	\$24,191.03										\$71,821,998.06
20	17	\$25,279.63										\$77,599,711.94
21	18	\$26,417.21										\$83,866,538.11
22	19	\$27,605.98										\$90,665,106.98
23	20	\$28,848.25										\$98,041,823.17
24	21	\$30,146.43										\$106,047,202.36
25	22	\$31,503.01										\$114,736,238.36
26	23	\$32,920.65										\$124,168,803.04
27	24	\$34,402.08										\$134,410,082.20
28	25	\$35,950.17										\$145,531,050.38
29												
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Solver Parameters

Set Target Cell:

Equal To: Max Min Value of:

By Changing Cells:

Subject to the Constraints:

$=B28*(1+\$L\$1)$

Figure 5.7 Figure 5.8 Figure 5.9 Figure 5.10 Figure 5.10 data Figure 5.11

	A	B	C	D	E	F	G	H	I	J	K	L
1	Beginning Scholarship Amount = \$12,500.00			Annual Endowment Growth Rate = 9.0%								
2	Size of the Gift = \$273,678.79			Annual Increase in Tuition = 4.5%								
3												
4	EOY	Scholarship Cost	Size of Endowment	EOY	Scholarship Cost	Size of Endowment	EOY	Scholarship Cost	Size of Endowment	EOY	Scholarship Cost	Size of Endowment
5	1	\$12,500.00	\$285,809.88	26	\$37,567.93	\$833,883.79	51	\$112,907.95	\$2,289,751.77	76	\$339,337.45	\$5,015,394.26
6	2	\$13,062.50	\$298,470.27	27	\$39,258.49	\$869,674.84	52	\$117,988.81	\$2,377,840.62	77	\$354,607.64	\$5,112,172.10
7	3	\$13,650.31	\$311,682.28	28	\$41,025.12	\$906,920.46	53	\$123,298.31	\$2,468,547.97	78	\$370,564.98	\$5,201,702.61
8	4	\$14,264.58	\$325,469.11	29	\$42,871.25	\$945,672.05	54	\$128,846.73	\$2,561,870.55	79	\$387,240.41	\$5,282,615.44
9	5	\$14,906.48	\$339,854.85	30	\$44,800.46	\$985,982.08	55	\$134,644.83	\$2,657,794.07	80	\$404,666.23	\$5,353,384.60
10	6	\$15,577.27	\$354,864.51	31	\$46,816.48	\$1,027,903.99	56	\$140,703.85	\$2,756,291.68	81	\$422,876.21	\$5,412,313.01
11	7	\$16,278.25	\$370,524.06	32	\$48,923.22	\$1,071,492.13	57	\$147,035.53	\$2,857,322.41	82	\$441,905.63	\$5,457,515.55
12	8	\$17,010.77	\$386,860.46	33	\$51,124.76	\$1,116,801.66	58	\$153,652.12	\$2,960,829.30	83	\$461,791.39	\$5,486,900.56
13	9	\$17,776.26	\$403,901.64	34	\$53,425.38	\$1,163,888.43	59	\$160,566.47	\$3,066,737.47	84	\$482,572.00	\$5,498,149.61
14	10	\$18,576.19	\$421,676.60	35	\$55,829.52	\$1,212,808.87	60	\$167,791.96	\$3,174,951.88	85	\$504,287.74	\$5,488,695.33
15	11	\$19,412.12	\$440,215.37	36	\$58,341.85	\$1,263,619.82	61	\$175,342.60	\$3,285,354.95	86	\$526,980.69	\$5,455,697.22
16	12	\$20,285.66	\$459,549.09	37	\$60,967.23	\$1,316,378.38	62	\$183,233.02	\$3,397,803.88	87	\$550,694.82	\$5,396,015.15
17	13	\$21,198.52	\$479,709.99	38	\$63,710.76	\$1,371,141.68	63	\$191,478.50	\$3,512,127.72	88	\$575,476.09	\$5,306,180.43
18	14	\$22,152.45	\$500,731.44	39	\$66,577.74	\$1,427,966.69	64	\$200,095.03	\$3,628,124.19	89	\$601,372.51	\$5,182,364.16
19	15	\$23,149.31	\$522,647.96	40	\$69,573.74	\$1,486,909.95	65	\$209,099.31	\$3,745,556.05	90	\$628,434.27	\$5,020,342.66
20	16	\$24,191.03	\$545,495.25	41	\$72,704.56	\$1,548,027.29	66	\$218,508.78	\$3,864,147.32	91	\$656,713.82	\$4,815,459.68
21	17	\$25,279.63	\$569,310.19	42	\$75,976.26	\$1,611,373.48	67	\$228,341.68	\$3,983,578.90	92	\$686,265.94	\$4,562,585.11
22	18	\$26,417.21	\$594,130.90	43	\$79,395.19	\$1,677,001.90	68	\$238,617.05	\$4,103,483.95	93	\$717,147.91	\$4,256,069.87
23	19	\$27,605.98	\$619,996.70	44	\$82,967.98	\$1,744,964.10	69	\$249,354.82	\$4,223,442.69	94	\$749,419.56	\$3,889,696.59
24	20	\$28,848.25	\$646,948.14	45	\$86,701.54	\$1,815,309.33	70	\$260,575.78	\$4,342,976.74	95	\$783,143.44	\$3,456,625.85
25	21	\$30,146.43	\$675,027.05	46	\$90,603.11	\$1,888,084.06	71	\$272,301.69	\$4,461,542.96	96	\$818,384.90	\$2,949,337.28
26	22	\$31,503.01	\$704,276.47	47	\$94,680.25	\$1,963,331.38	72	\$284,555.27	\$4,578,526.55	97	\$855,212.22	\$2,359,565.42
27	23	\$32,920.65	\$734,740.70	48	\$98,940.86	\$2,041,090.35	73	\$297,360.26	\$4,693,233.68	98	\$893,696.77	\$1,678,229.54
28	24	\$34,402.08	\$766,465.29	49	\$103,393.19	\$2,121,395.29	74	\$310,741.47	\$4,804,883.25	99	\$933,913.12	\$895,357.07
29	25	\$35,950.17	\$799,496.99	50	\$108,045.89	\$2,204,274.98	75	\$324,724.84	\$4,912,597.90	100	\$975,939.21	\$0.00
30												
31		=B28*(1+\$L\$1)						=L28*1.09-H29				
32												
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Capitalized Worth Analysis

Multiple Alternatives

Example 5.15

In a developing country, two alternatives are being considered for delivering water from a mountainous area to an arid area. A pipeline can be installed at a cost of \$125 million; major replacements every 15 years will cost \$10 million. Annual O&M costs are estimated to be \$5 million. Alternately, a canal can be constructed at a cost of \$200 million; annual O&M costs are estimated to be \$1 million; upgrades of the canal will be required every 10 years at a cost of \$5 million. Using a 5% MARR and a capitalized cost analysis, which alternative should be chosen?

Example 5.15 (Solution)

Pipeline

$$\begin{aligned} CC &= \$125,000,000 + [\$10,000,000(A|F 5\%,15) \\ &+ \$5,000,000]/0.05 = \$234,268,000.00 \\ &= 125000000 + (\text{PMT}(5\%,15,,-10000000) + 5000000)/0.05 \\ &= \$234,268,457.52 \end{aligned}$$

Canal

$$\begin{aligned} CC &= \$200,000,000 + [\$5,000,000(A|F 5\%,10) \\ &+ \$1,000,000]/0.05 = \$227,950,000.00 \\ &= 200000000 + (\text{PMT}(5\%,10,,-5000000) + 1000000)/.05 \\ CC &= \$227,950,457.50 \end{aligned}$$

Pit Stop #5— Open Road Ahead!

1. True or False: Present worth analysis is the most popular *DCF* measure of economic worth.
2. True or False: Unless non-monetary considerations dictate otherwise, choose the mutually exclusive investment alternative that has the greatest present worth, regardless of the lives of the alternatives.
3. True or False: When using present worth analysis to evaluate the economic viability of mutually exclusive alternatives, use a common period of time in the comparison.
4. True or False: If $PW > 0$ and $MARR = 20\%$, then $DPBP < 5$ years.
5. True or False: $DPBP \geq PBP$.
6. True or False: If $CW > 0$, then $PW > 0$.
7. True or False: If $PW(A) > PW(B)$, then $CW(A) > CW(B)$, $DPBP(A) < DPBP(B)$, and $PBP(A) < PBP(B)$.
8. True or False: PW , FW , AW , CW , and B/C are ranking methods; therefore, the alternative having the greatest PW , FW , AW , CW , or B/C should be recommended.
9. True or False: Either ranking or incremental analysis can be used with all four “worth” methods (PW , FW , AW , and CW).
10. True or False: The “do nothing” alternative always has negligible incremental costs and revenues.

Pit Stop #5— Open Road Ahead!

1. True or False: Present worth analysis is the most popular *DCF* measure of economic worth. **TRUE**
2. True or False: Unless non-monetary considerations dictate otherwise, choose the mutually exclusive investment alternative that has the greatest present worth, regardless of the lives of the alternatives. **FALSE**
3. True or False: When using present worth analysis to evaluate the economic viability of mutually exclusive alternatives, use a common period of time in the comparison. **TRUE**
4. True or False: If $PW > 0$ and $MARR = 20\%$, then $DPBP < 5$ years. **FALSE**
5. True or False: $DPBP \geq PBP$. **TRUE**
6. True or False: If $CW > 0$, then $PW > 0$. **TRUE**
7. True or False: If $PW(A) > PW(B)$, then $CW(A) > CW(B)$, $DPBP(A) < DPBP(B)$, and $PBP(A) < PBP(B)$. **FALSE (NOT ALWAYS)**
8. True or False: PW , FW , AW , CW , and B/C are ranking methods; therefore, the alternative having the greatest PW , FW , AW , CW , or B/C should be recommended. **FALSE**
9. True or False: Either ranking or **incremental analysis** can be used with all four “worth” methods (PW , FW , AW , and CW). **TRUE**
10. True or False: The “do nothing” alternative always has negligible incremental costs and revenues. **FALSE**