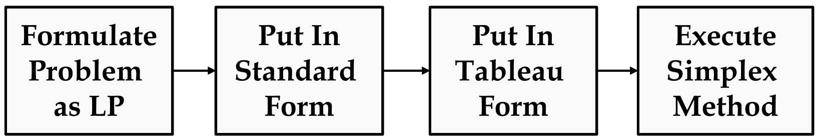
Linear Programming: The Simplex Method

Outlines

- An Overview of the Simplex Method
- Standard Form
- Tableau Form
- Setting Up the Initial Simplex Tableau
- Improving the Solution
- Calculating the Next Tableau
- Solving a Minimization Problem
- Special Cases

Overview of the Simplex Method

- Steps Leading to the Simplex Method



Example: Initial Formulation

- A Minimization Problem

$$\begin{aligned}
\textbf{MIN} \quad & 2x_1 - 3x_2 - 4x_3 \\
\textbf{s. t.} \quad & x_1 + x_2 + x_3 \leq 30 \\
& 2x_1 + x_2 + 3x_3 \geq 60 \\
& x_1 - x_2 + 2x_3 = 20 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}$$

Standard Form

- An LP is in standard form when:
  - All variables are non-negative
  - All constraints are equalities
- Putting an LP formulation into standard form involves:
  - Adding slack variables to "$\leq$" constraints
  - Subtracting surplus variables from "$\geq$" constraints.

Example: Standard Form

- Problem in Standard Form

$$
\begin{aligned}
\textbf{MIN} \quad & 2x_1 - 3x_2 - 4x_3 \\
\textbf{s. t.} \quad & x_1 + x_2 + x_3 + s_1 = 30 \\
& 2x_1 + x_2 + 3x_3 - s_2 = 60 \\
& x_1 - x_2 + 2x_3 = 20 \\
& x_1, x_2, x_3, s_1, s_2 \geq 0
\end{aligned}
$$

Tableau Form

- A set of equations is in tableau form if for each equation:
  - its right hand side (RHS) is non-negative, and
  - there is a basic variable. (A basic variable for an equation is a variable whose coefficient in the equation is +1 and whose coefficient in all other equations of the problem is 0.)
- To generate an initial tableau form:
  - An artificial variable must be added to each constraint that does not have a basic variable.

Example: Tableau Form

- Problem in Tableau Form

$$
\begin{array}{ll}
\textbf{MIN} & 2x_1 - 3x_2 - 4x_3 + 0s_1 - 0s_2 + Ma_2 + Ma_3 \\
\textbf{s. t.} & x_1 + x_2 + x_3 + s_1 = 30 \\
 & 2x_1 + x_2 + 3x_3 - s_2 + a_2 = 60 \\
 & x_1 - x_2 + 2x_3 + a_3 = 20 \\
 & x_1, x_2, x_3, s_1, s_2, a_2, a_3 \geq 0
\end{array}
$$

Simplex Tableau

- The simplex tableau is a convenient means for performing the calculations required by the simplex method.

Setting Up Initial Simplex Tableau

- Step 1: If the problem is a minimization problem, multiply the objective function by -1.
- Step 2: If the problem formulation contains any constraints with negative right-hand sides, multiply each constraint by -1.
- Step 3: Add a slack variable to each $\leq$ constraint.
- Step 4: Subtract a surplus variable and add an artificial variable to each $\geq$ constraint.

Setting Up Initial Simplex Tableau

- Step 5: Add an artificial variable to each = constraint.
- Step 6: Set each slack and surplus variable's coefficient in the objective function equal to zero.
- Step 7: Set each artificial variable's coefficient in the objective function equal to -*M*, where *M* is a very large number.
- Step 8: Each slack and artificial variable becomes one of the basic variables in the initial basic feasible solution.

Simplex Method

- Step 1: Determine Entering Variable
  - Identify the variable with the most positive value in the $c_j - z_j$ row. (The entering column is called the pivot column.)
- Step 2: Determine Leaving Variable
  - For each positive number in the entering column, compute the ratio of the right-hand side values divided by these entering column values.
  - If there are no positive values in the entering column, STOP; the problem is unbounded.
  - Otherwise, select the variable with the minimal ratio. (The leaving row is called the pivot row.)

Simplex Method

- Step 3: Generate Next Tableau
  - Divide the pivot row by the pivot element (the entry at the intersection of the pivot row and pivot column) to get a new row. We denote this new row as (row *).
  - Replace each non-pivot row *i* with:

    [new row *i*] = [current row *i*] - [(a_{ij}) x (row *)],

    where a_{ij} is the value in entering column *j* of row *i*

Simplex Method

- Step 4: Calculate z_j Row for New Tableau
  - For each column *j*, multiply the objective function coefficients of the basic variables by the corresponding numbers in column *j* and sum them.

Simplex Method

- Step 5: Calculate c_j - z_j Row for New Tableau
  - For each column j, subtract the z_j row from the c_j row.
  - If none of the values in the c_j - z_j row are positive, GO TO STEP 1.
  - If there is an artificial variable in the basis with a positive value, the problem is infeasible. STOP.
  - Otherwise, an optimal solution has been found. The current values of the basic variables are optimal. The optimal values of the non-basic variables are all zero.
  - If any non-basic variable's c_j - z_j value is 0, alternate optimal solutions might exist. STOP.

Example: Simplex Method

- Solve the following problem by the simplex method:

$$
\begin{array}{lrcl}
\text{Max} & 12x_1 + 18x_2 + 10x_3 & & \\
\text{s.t.} & 2x_1 + 3x_2 + 4x_3 & \leq & 50 \\
 & x_1 - x_2 - x_3 & \geq & 0 \\
 & x_2 - 1.5x_3 & \geq & 0 \\
 & x_1, x_2, x_3 & \geq & 0
\end{array}
$$

Example: Simplex Method

- Writing the Problem in Tableau Form

  We can avoid introducing artificial variables to the second and third constraints by multiplying each by -1 (making them $\leq$ constraints). Thus, slack variables s_1, s_2, and s_3 are added to the three constraints.

$$
\begin{aligned}
\textbf{Max}\quad & 12x_1 + 18x_2 + 10x_3 + 0s_1 + 0s_2 + 0s_3 \\
\textbf{s.t.}\quad & 2x_1 + 3x_2 + 4x_3 + s_1 = 50 \\
& -x_1 + x_2 + x_3 + s_2 = 0 \\
& -x_2 + 1.5x_3 + s_3 = 0 \\
& x_1, x_2, x_3, s_1, s_2, s_3 \geq 0
\end{aligned}
$$

Example: Simplex Method

- Initial Simplex Tableau

Basis	c_B	x_1	x_2	x_3	s_1	s_2	s_3		
		12	18	10	0	0	0		
s_1	0	2	3	4	1	0	0	50	
s_2	0	-1	1	1	0	1	0	0	(* row)
s_3	0	0	-1	1.5	0	0	1	0	
z_j		0	0	0	0	0	0	0	
$c_j - z_j$		12	18	10	0	0	0		

Example: Simplex Method

- Iteration 1
  - Step 1: Determine the Entering Variable

    The most positive c_j - z_j = 18. Thus x_2 is the entering variable.
  - Step 2: Determine the Leaving Variable

    Take the ratio between the right hand side and positive numbers in the x_2 column:

    50/3 = 16 2/3

    0/1 = 0 ⟵ minimum

    s_2 is the leaving variable and the 1 is the pivot element.

Example: Simplex Method

- Iteration 1 (continued)
  - Step 3: Generate New Tableau

    Divide the second row by 1, the pivot element. Call the "new" (in this case, unchanged) row the "* row".

    Subtract 3 x (* row) from row 1.

    Subtract -1 x (* row) from row 3.

    New rows 1, 2, and 3 are shown in the upcoming tableau.

Example: Simplex Method

- Iteration 1 (continued)
  - Step 4: Calculate z_j Row for New Tableau

    The new z_j row values are obtained by multiplying the c_B column by each column, element by element and summing.

    For example, $z_1 = 5(0) + -1(18) + -1(0) = -18$.

Example: Simplex Method

- Iteration 1 (continued)
  - Step 5: Calculate $c_j - z_j$ Row for New Tableau

    The new c_j-z_j row values are obtained by subtracting z_j value in a column from the c_j value in the same column.

    For example, c_1-z_1 = 12 - (-18) = 30.

Example: Simplex Method

- Iteration 1 (continued) - New Tableau

		x_1	x_2	x_3	s_1	s_2	s_3		
Basis	c_B	**12**	**18**	**10**	**0**	**0**	**0**		
s_1	**0**	**5**	**0**	**1**	**1**	**-3**	**0**	**50**	**(* row)**
x_2	**18**	**-1**	**1**	**1**	**0**	**1**	**0**	**0**	
s_3	**0**	**-1**	**0**	**2.5**	**0**	**1**	**1**	**0**	
z_j		**-18**	**18**	**18**	**0**	**18**	**0**	**0**	
$c_j - z_j$		**30**	**0**	**-8**	**0**	**-18**	**0**		

Example: Simplex Method

- Iteration 2
  - Step 1: Determine the Entering Variable
    The most positive $c_j - z_j = 30$. x_1 is the entering variable.
  - Step 2: Determine the Leaving Variable
    Take the ratio between the right hand side and positive numbers in the x_1 column:

    10/5 = 2 ← minimum

    There are no ratios for the second and third rows because their column elements (-1) are negative.

    Thus, s_1 (corresponding to row 1) is the leaving variable and 5 is the pivot element.

Example: Simplex Method

- Iteration 2 (continued)
  - Step 3: Generate New Tableau

    Divide row 1 by 5, the pivot element. (Call this new row 1 the "* row").

    Subtract (-1) x (* row) from the second row.

    Subtract (-1) x (* row) from the third row.
  - Step 4: Calculate z_j Row for New Tableau

    The new z_j row values are obtained by multiplying the c_B column by each column, element by element and summing.

    For example, $z_3 = .2(12) + 1.2(18) + .2(0) = 24$.

Example: Simplex Method

- Iteration 2 (continued)
  - Step 5: Calculate $c_j - z_j$ Row for New Tableau

    The new c_j-z_j row values are obtained by subtracting z_j value in a column from the c_j value in the same column.

    For example, c_3-z_3 = 10 - (24) = -14.

    Since there are no positive numbers in the $c_j - z_j$ row, this tableau is optimal. The optimal solution is: $x_1 = 10$; $x_2 = 10$; $x_3 = 0$; $s_1 = 0$; $s_2 = 0$ $s_3 = 10$, and the optimal value of the objective function is 300.

Example: Simplex Method

- Iteration 2 (continued) – Final Tableau

Basis	c_B	x_1 12	x_2 18	x_3 10	s_1 0	s_2 0	s_3 0		
x_1	12	1	0	.2	.2	-.6	0	10	(* row)
x_2	18	0	1	1.2	.2	.4	0	10	
s_3	0	0	0	2.7	.2	.4	1	10	
z_j		12	18	24	6	0	0	300	
$c_j - z_j$		0	0	-14	-6	0	0		

Special Cases

- Infeasibility
- Unboundedness
- Alternative Optimal Solution
- Degeneracy

Infeasibility

- Infeasibility is detected in the simplex method when an artificial variable remains positive in the final tableau.

Example: Infeasibility

- LP Formulation

$$
\begin{aligned}
\textbf{MAX} \quad & 2x_1 + 6x_2 \\
\textbf{s. t.} \quad & 4x_1 + 3x_2 \leq 12 \\
& 2x_1 + x_2 \geq 8 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Example: Infeasibility

- Final Tableau

		x_1	x_2	s_1	s_2	a_2	
Basis	C_B	**2**	**6**	**0**	**0**	**-M**	
x_1	**2**	**1**	**3/4**	**1/4**	**0**	**0**	**3**
a_2	**-M**	**0**	**-1/2**	**-1/2**	**-1**	**1**	**2**
	z_j	**2**	**(1/2)M +3/2**	**(1/2)M +1/2**	M	**-M**	**-2M +6**
	c_j - z_j	**0**	**-(1/2)M +9/2**	**-(1/2)M -1/2**	**-M**	**0**	

Example: Infeasibility

In the previous slide we see that the tableau is the final tableau because all c_j - $z_j \leq 0$. However, an artificial variable is still positive, so the problem is infeasible.

Unboundedness

- A linear program has an unbounded solution if all entries in an entering column are non-positive.

Example: Unboundedness

- LP Formulation

$$\begin{aligned} \textbf{MAX} \quad & 2x_1 + 6x_2 \\ \textbf{s. t.} \quad & 4x_1 + 3x_2 \geq 12 \\ & 2x_1 + x_2 \geq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Example: Unboundedness

- Final Tableau

Basis	c_B	x_1 3	x_2 4	s_1 0	s_2 0	
x_2	4	3	1	0	-1	8
s_1	0	2	0	1	-1	3
	z_j	12	4	0	-4	32
	$c_j - z_j$	-9	0	0	4	

Example: Unboundedness

In the previous slide we see that $c_4 - z_4 = 4$ (is positive), but its column is all non-positive. This indicates that the problem is unbounded.

Alternative Optimal Solution

- A linear program has <u>alternate optimal solutions</u> if the final tableau has a c_j - z_j value equal to 0 for a non-basic variable.

Example: Alternative Optimal Solution

- Final Tableau

		x_1	x_2	x_3	s_1	s_2	s_3	s_4	
Basis	c_B	**2**	**4**	**6**	**0**	**0**	**0**	**0**	
s_3	**0**	**0**	**0**	**2**	**4**	**-2**	**1**	**0**	**8**
x_2	**4**	**0**	**1**	**2**	**2**	**-1**	**0**	**0**	**6**
x_1	**2**	**1**	**0**	**-1**	**1**	**2**	**0**	**0**	**4**
s_4	**0**	**0**	**0**	**1**	**3**	**2**	**0**	**1**	**12**
	z_j	**2**	**4**	**6**	**10**	**0**	**0**	**0**	**32**
	$c_j - z_j$	**0**	**0**	**0**	**-10**	**0**	**0**	**0**	

Example: Alternative Optimal Solution

In the previous slide we see that the optimal solution is:

$x_1 = 4$, $x_2 = 6$, $x_3 = 0$, and $z = 32$

Note that x_3 is non-basic and its $c_3 - z_3 = 0$. This 0 indicates that if x_3 were increased, the value of the objective function would not change.

Another optimal solution can be found by choosing x_3 as the entering variable and performing one iteration of the simplex method. The new tableau on the next slide shows an alternative optimal solution is:

$x_1 = 7$, $x_2 = 0$, $x_3 = 3$, and $z = 32$

Example: Alternative Optimal Solution

- New Tableau

		x_1	x_2	x_3	s_1	s_2	s_3	s_4	
Basis	c_B	**2**	**4**	**6**	**0**	**0**	**0**	**0**	
s_3	**0**	**0**	**-1**	**0**	**2**	**-1**	**1**	**0**	**2**
x_3	**6**	**0**	**.5**	**1**	**1**	**- .5**	**0**	**0**	**3**
x_1	**2**	**1**	**.5**	**0**	**2**	**1.5**	**0**	**0**	**7**
s_4	**0**	**0**	**- .5**	**0**	**2**	**2.5**	**0**	**1**	**9**
	z_j	**2**	**4**	**6**	**10**	**0**	**0**	**0**	**32**
	$c_j - z_j$	**0**	**0**	**0**	**-10**	**0**	**0**	**0**	

Degeneracy

- A degenerate solution to a linear program is one in which at least one of the basic variables equals 0.
- This can occur at formulation or if there is a tie for the minimizing value in the ratio test to determine the leaving variable.
- When degeneracy occurs, an optimal solution may have been attained even though some $c_j - z_j > 0$.
- Thus, the condition that $c_j - z_j \leq 0$ is sufficient for optimality, but not necessary.

THE REVISED SIMPLEX METHOD

CONTENTS

42

Matrix Notation

Maximize $\sum_{j=1,n} c_j x_j$

subject to

$$\sum_{j=1,n} a_{ij}\, x_j \leq b_i \quad \text{for all } i = 1, 2, \ldots, m$$
$$x_j \geq 0 \quad \text{for all } j = 1, 2, \ldots, n$$

Add the slack variables:

$$x_{n+i} = b_i - \sum_{j=1,n} a_{ij}\, x_j \quad \text{for all } i = 1, 2, \ldots, m$$

Problem in the matrix notation:

Maximize cx

subject to

$$Ax = b$$
$$x \geq 0$$

43

Matrix Notation

Maximize cx

subject to

$$Ax = b$$
$$x \geq 0$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & 1 & & \\ a_{21} & a_{22} & \cdots & a_{2n} & & 1 & \\ \vdots & \vdots & & \vdots & & & \vdots \\ a_{m1} & a_{m2} & & a_{mn} & & & 1 \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} \quad c = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ x_{n+1} \\ \vdots \\ x_{n+m} \end{bmatrix}$$

44

The Primal Simplex Method

B : The set of indices corresponding to basic variables

N : The set of indices corresponding to nonbasic variables

$x = \begin{bmatrix} x_B \\ x_N \end{bmatrix}$ $c = \begin{bmatrix} c_B \\ c_N \end{bmatrix}$ $A = \begin{bmatrix} B & N \end{bmatrix}$

The linear program in the matrix form:

Maximize $z = c_B x_B + c_N x_N$

subject to

$$B x_B + N x_N = b$$
$$x_B \geq 0, x_N \geq 0$$

45

The Primal Simplex Method (contd.)

Constraint Matrix:

$B x_B + N x_N = b$ **or** $B x_B = b - N x_N$

Let x_B define a basis, then

$$x_B = B^{-1} b - B^{-1} N x_N$$

where B is an invertible mxm matrix (that is, whose columns are linearly independent).

Objective Function:

$$z = c_B x_B + c_N x_N$$
$$z = c_B (B^{-1} b - B^{-1} N x_N) + c_N x_N$$
$$z = c_B B^{-1} b + (c_N - c_B B^{-1} N) x_N$$

46

The Primal Simplex Method (contd.)

Basis in the Matrix Notation:

$$\mathbf{z = c_B B^{-1} b + (c_N - c_B B^{-1} N)\, x_N}$$

$$\mathbf{x_B = B^{-1} b - B^{-1} N x_N}$$

Basic Solution associated with this Basis:

$$\mathbf{x_N = 0}$$
$$\mathbf{x_B = B^{-1} b}$$

Computing Simplex Multipliers in Matrix Notation

The simplex multipliers π must be such that $z_j - c_j$:

$\bar{c}_j = c_j - \sum_{i=1,m} a_{ij}\, \pi_i = 0$ for each basic variable x_j

Alternatively,

$c_j = \sum_{i=1,m} a_{ij}\, \pi_i$ for each basic variable x_j

or

$c_B = \pi B$

or

$\pi = c_B B^{-1}$

Summary of Formulas w.r.t. a Basis

Suppose that B is a set of basic variables at some iteration. Let B denote the associated basis (columns of A in the basis). Then we can obtain the simplex tableau for this iteration using the following formulas:

x_j column in the constraints	$= B^{-1}a_j$
Right-hand side of the tableau	$= B^{-1}b$
The $z_j - c_j$	$= c_B B^{-1} a_j - c_j$
Right-hand side of row 0	$= c_B B^{-1} b$
Simplex multipliers π	$= c_B B^{-1}$

49

Example of the Revised Simplex Method

Maximize $4x_1 + 3x_2$

subject to

$$x_1 - x_2 \leq 1$$
$$2x_1 - x_2 \leq 3$$
$$x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

$$A = \begin{bmatrix} 1 & -1 & 1 & & \\ 2 & -1 & & 1 & \\ 0 & 1 & & & 1 \end{bmatrix}$$

B = {3, 4, 5} **and** **N = {1, 2}**

$$B = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \quad N = \begin{bmatrix} 1 & -1 \\ 2 & -1 \\ 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

50

Iteration 1

Step 1. The current primal solution x_B:

$$x_B = \bar{b} = B^{-1}b = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

Step 2. The current dual solution π:

$$y = c_B B^{-1} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$d_j = c_j - z_j = c_j - \sum_{i=1,m} a_{ij}\, y_i = [4 \quad 3]$ **Entering variable = x_1**

Iteration 1 (contd.)

Step 3. Determine A'^1.

$$A'^1 = B^{-1}A^1 = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

Step 4. Perform the minimum ratio test.

$$B = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \quad b' = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \quad A'^1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

Minimum ratio = 1/1 = 1

Leaving variable = x_3

Iteration 2

B = {1, 4, 5} and N = {3, 2}

$$[A,I] = \begin{bmatrix} 1 & -1 & 1 & & \\ 2 & -1 & & 1 & \\ 0 & 1 & & & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & & \\ 2 & 1 & \\ 0 & & 1 \end{bmatrix} \qquad N = \begin{bmatrix} 1 & -1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}$$

53

Iteration 2 (contd.)

Step 1. The current primal solution x_B:

$$x_B = b' = B^{-1}b = \begin{bmatrix} 1 & & \\ 2 & 1 & \\ 0 & & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 & & \\ -2 & 1 & \\ 0 & & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

Step 2. Determine the simplex multipliers y and d_j for the nonbasic variables: $y = c_B B^{-1} = [4 \;\; 0 \;\; 0]\, B^{-1}$

$d_j = c_j - z_j = c_j - \sum_{i=1,m} a_{ij}\, y_i = [-4 \quad 7]$, $\quad z_3 - c_3 = -4$, $\quad z_2 - c_2 = -7$

Entering variable = x_2

54

Iteration 2 (contd.)

Step 3. Determine A'^2 .

$$A'^2 = B^{-1}A^2 = \begin{bmatrix} 1 & & \\ 2 & 1 & \\ 0 & & 1 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Step 4. Perform the minimum ratio test.

$$B = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} \quad b' = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix} \quad A'^2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Minimum ratio = 1/1 = 1

Leaving variable = x_4

55

Revised Simplex Method

Step 1. Obtain the initial primal feasible basis B. Determine the corresponding feasible solution $\bar{b} = B^{-1}b$

Step 2. Obtain the corresponding simplex multipliers $\pi = c_B B^{-1}$. Check the optimality of the current BFS. If the current basis is optimal, then STOP.

Step 3. If the current BFS is not optimal, identify the entering variable x_k (that is, $z_k - c_k = \sum_{i=1,m} a_{ik}\pi_i - c_k > 0$).

Step 4. Obtain the column $\bar{a}_k = B^{-1}a_k$ and perform the minimum ratio test to determine the leaving variable x_l.

Step 5. Update the basis B (or B^{-1}) and go to Step 2.

56

(Original) Simplex Method

Step 1. Obtain the initial feasible basis.

Step 2. Check the optimality of the current basis (that is, $z_j - c_j \leq 0$ for each $j \in N$). If optimal, STOP.

Step 3. If the current basis is not optimal, identify the entering variable x_k (that is, $z_k - c_k > 0$).

Step 4. Perform the minimum ratio test to determine the leaving variable x_l.

Step 5. Perform a pivot operation to update the basis and go to Step 2.