Chapter 3

Borrowing, Lending, and Investing

Section 3-7: **Equivalence and Indifference**

Section 3-9: **Variable Interest Rates**

Equivalence

Two cash flow streams are said to be **equivalent** at k% interest if and only if their present worths are equal at k% interest.

Equivalence Example

What uniform series over periods [1,8] is equivalent at 15% to the following cash flow profile?

End of Period	Cash Flow
1	$100
3	$200
4	$100
5	$300

Equivalence Example

What uniform series over periods [1,8] is equivalent at 15% to the following cash flow profile?

End of Period	Cash Flow
1	$100
3	$200
4	$100
5	$300

8-3

Solution:

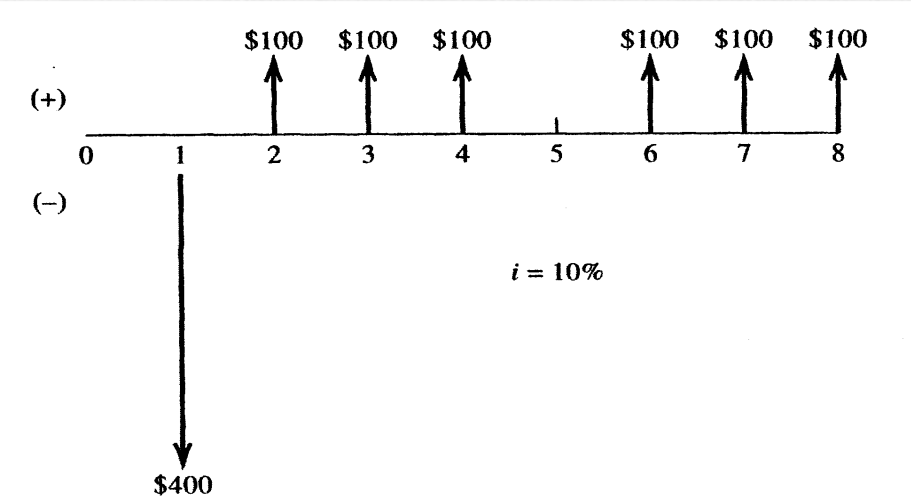
$$A = [100(F|P\ 15\%,7) + 200(F|P\ 15\%,5) + 100(F|P\ 15\%,4) + 300(F|P\ 15\%,3)](A|F\ 15\%,8) = \$94.86$$

Determine the equivalence for each single cash payments with new time and convert all CF to equivalence uniform payment

Example 3.22

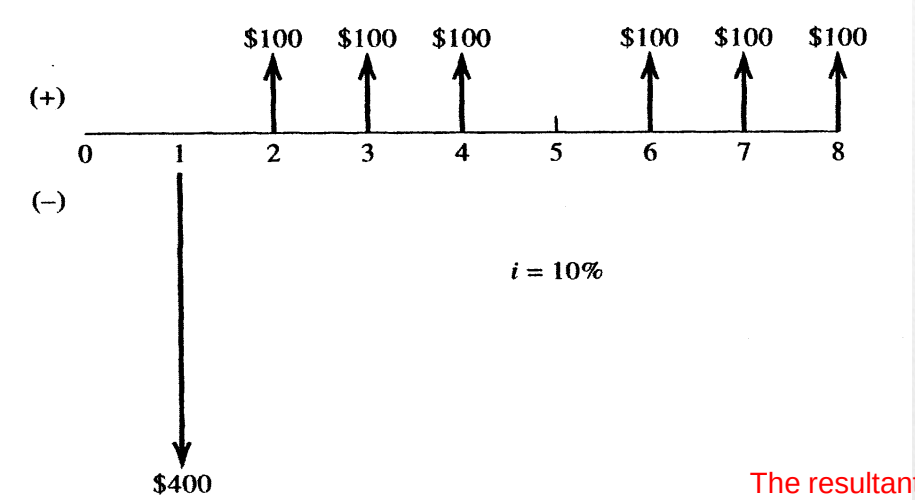
What single sum at t=6 is equivalent at 10% to the following cash flow profile?

End of Period	Cash Flow
1	-$400
2-4	+$100
6-8	+$100



Example 3.22

What single sum at t=6 is equivalent at 10% to the following cash flow profile?



Solution:

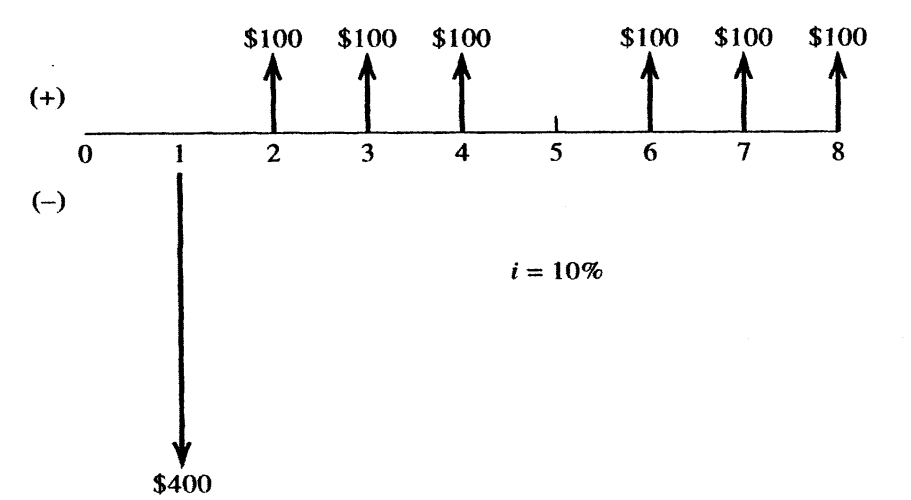
The resultant present value of these pavement will be at t=1

$$PW = -400(P|F\ 10\%,1) + 100\ (P|A\ 10\%,3)\ (P|F\ 10\%,1) + 100\ (P|A\ 10\%,3)\ (P|F\ 10\%,5) = \$\ 16.85$$

$$W_{t=6} = 16.85\ (F|P\ 10\%,6) = \$29.85$$

FW determined Using single sum

Example 3.22 (Alternative Solution)



Solution: Convert all CF to FW (t=8) then again convert the resultant to single sum payment at t=6 (P given F)

$$W_{t=6} = [\$100(F|A\ 10\%,7)-\$400(F|P\ 10\%,7)-\$100(F|P\ 10\%,3)](P|F\ 10\%,2)$$

$$W_{t=6} = [\$100(9.48717)-\$400(1.94872)-\$100(1.33100)](0.82645)$$

$$W_{t=6} = \$29.86$$

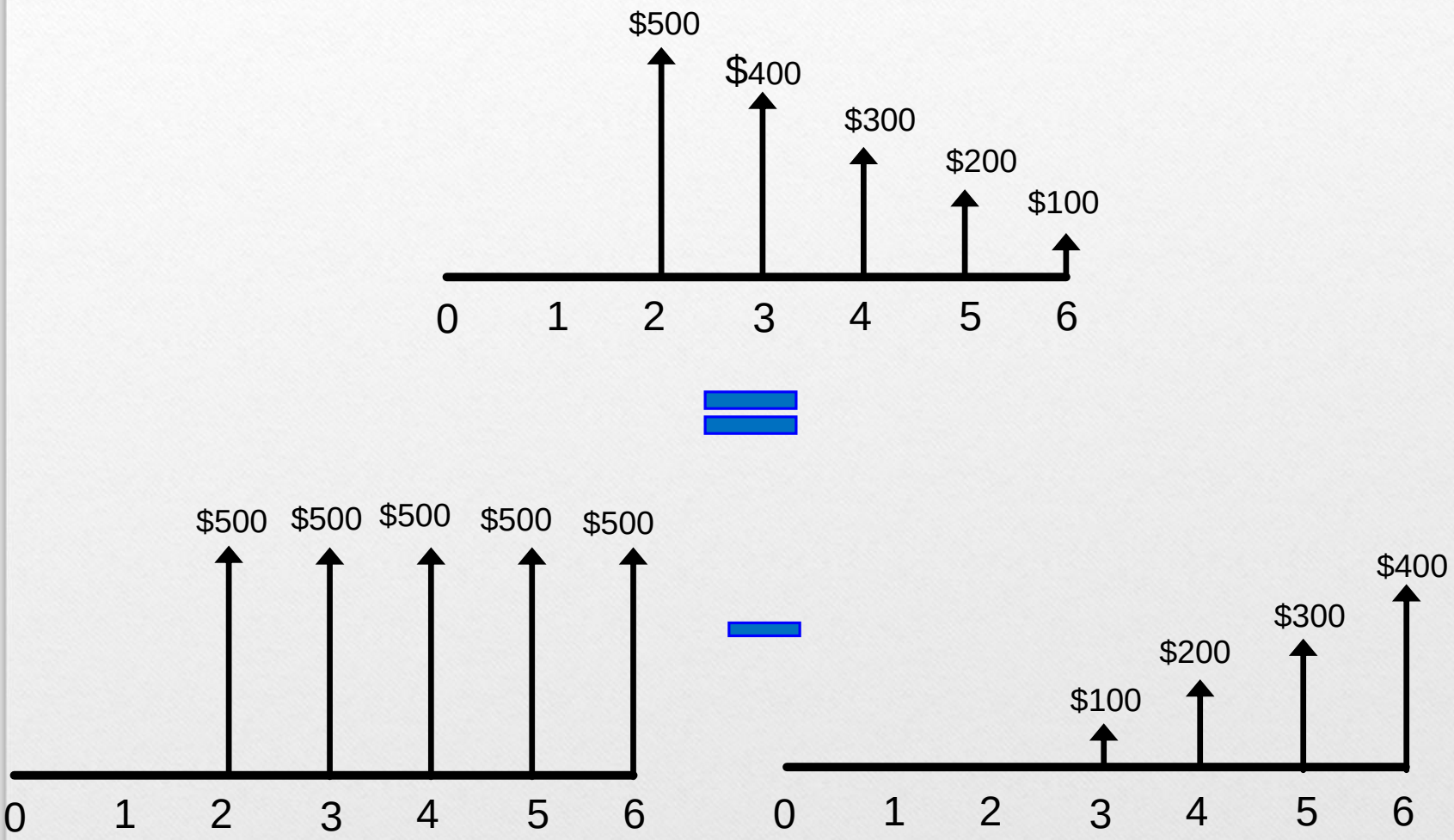
Answer: $29.86

Example 3.23

What uniform series over [1,5] is equivalent to the following cash flow profile if i = 8%?

End of Period	Cash Flow
1	$0
2	$500
3	$400
4	$300
5	$200
6	$100
7	$0

Example 2.29


$500
$400
$300
$200
$100
0 1 2 3 4 5 6
$500 $500 $500 $500 $500
0 1 2 3 4 5 6
$100
$200
$300
$400
0 1 2 3 4 5 6

Example 3.23

What uniform series over [1,5] is equivalent to the following cash flow profile if i = 8%?

Solution:

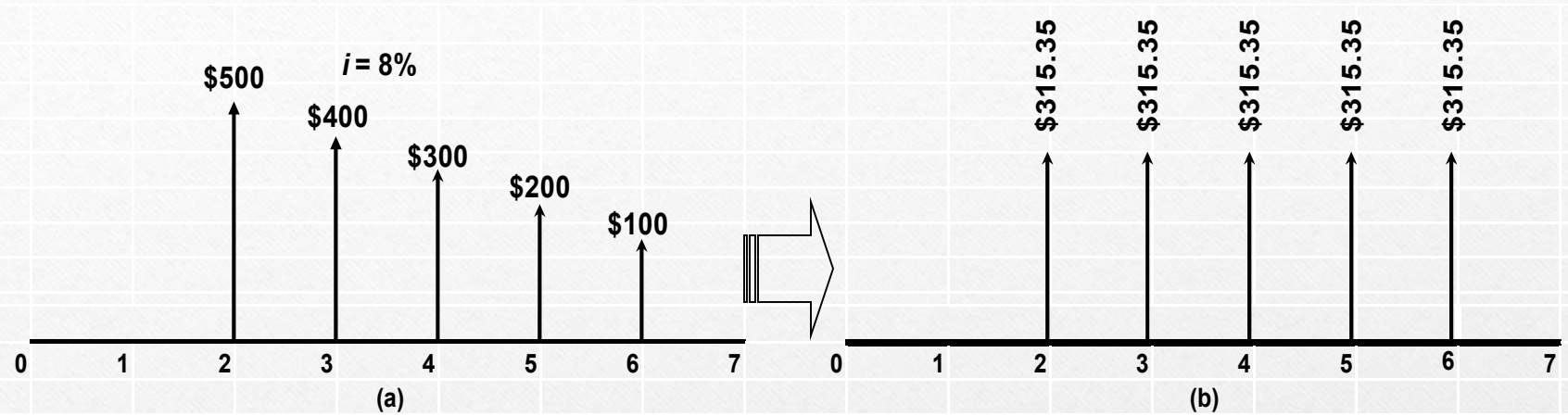
The uniform series equivalent over [2,6] is A = $500 - $100(A|G 8%,5)

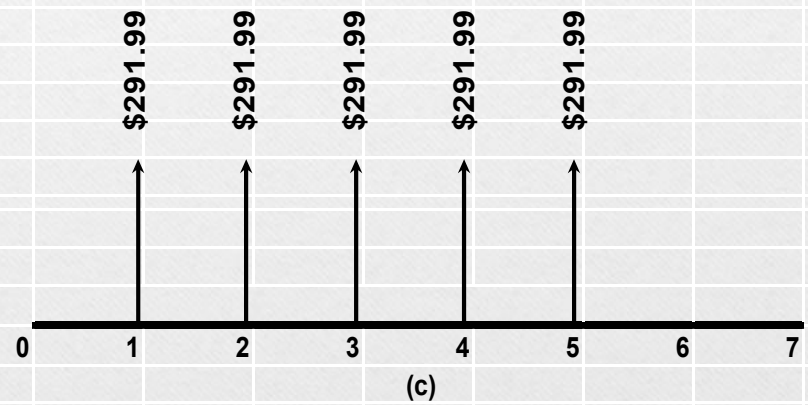
or $500 - $100(1.84647) = $315.35

The uniform series equivalent over [1,5] is A = $315.35(P|F 8%,1) or $315.35(0.92593) = $291.99

Answer: $291.99

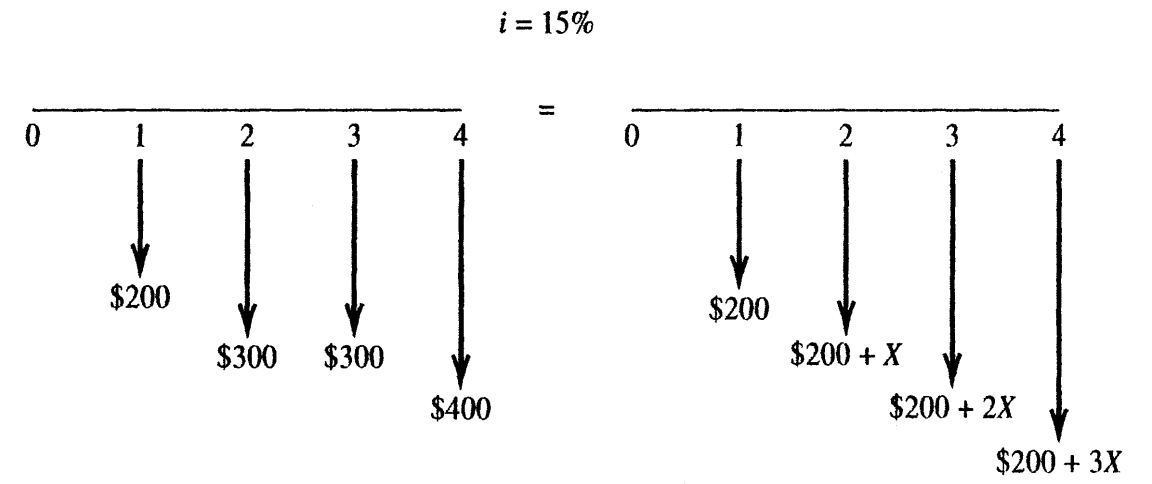
This CF represents future value with respect to t=5


$500
i = 8%
$400
$300
$200
$100
0
1
2
3
4
5
6
7
(a)
$315.35
$315.35
$315.35
$315.35
$315.35
0
1
2
3
4
5
6
7
(b)


$291.99
$291.99
$291.99
$291.99
$291.99
0
1
2
3
4
5
6
7
(c)

Example 3.24

Determine the value of X that makes the two CFDs equivalent.



Example 3.24

FW(LHS) = \$200(F|A 15%,4) + \$100(F|A 15%,3) + \$100

FW(RHS) = [\$200 + X(A|G 15%,4)](F|A 15%,4)

Convert Gradient series to uniform and then using single sum convert CF to FW

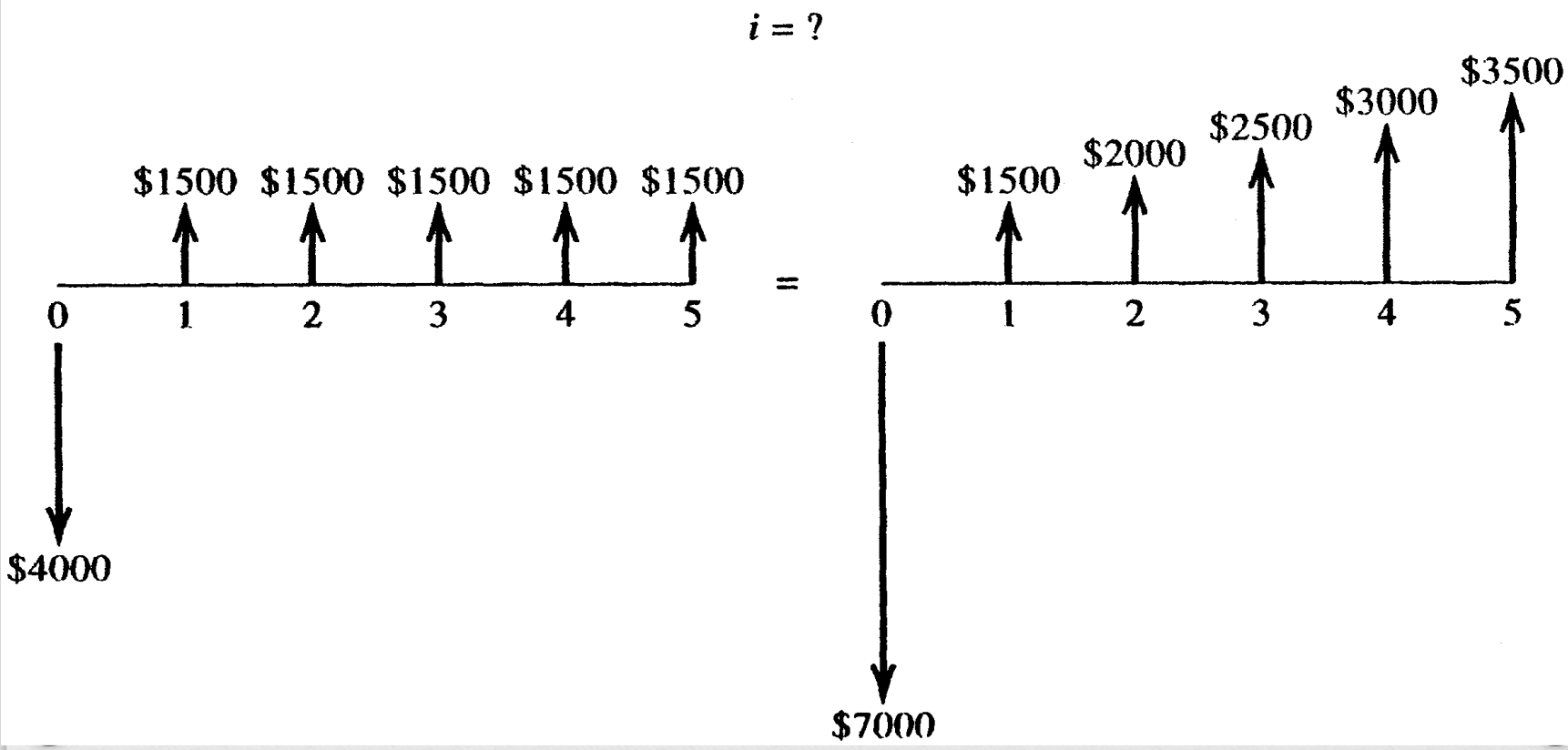
Equating the two and eliminating the common term of \$200(F|A 15%,4),

\$100(3.47250) + \$100 = X(1.32626)(4.99338)

Solving for X give a value of \$67.53.

Example 3.25

For what interest rate are the two cash flow diagrams equivalent?

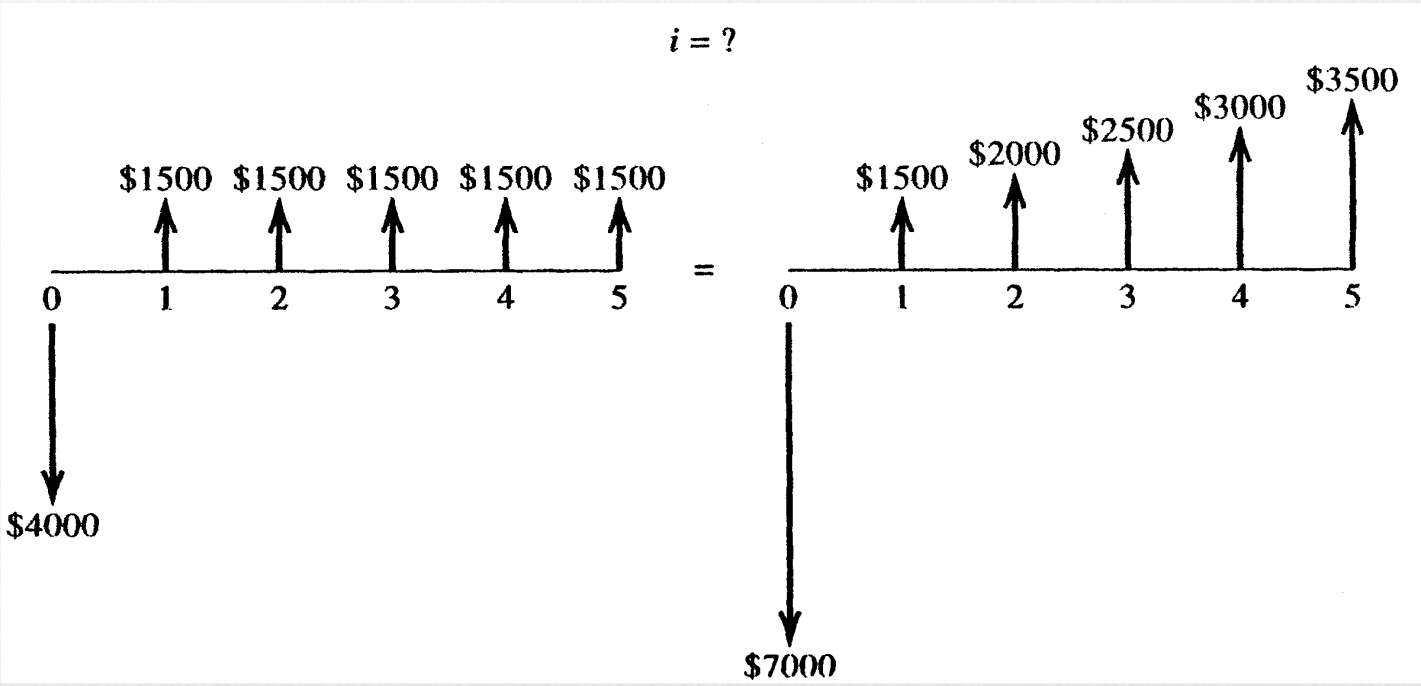


Example 3.25 (Continued)

-\$4000(A | P i%,5) + \$1500 =

-\$7000(A | P i%,5) + \$1500 + \$500(A | G i%,5)

i ≈ 13.8641% (by interpolation)



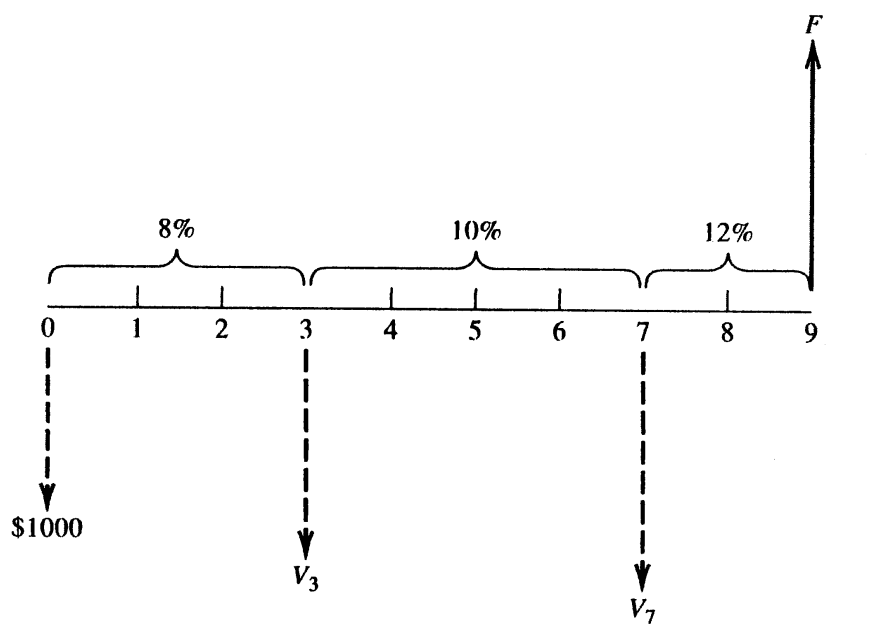
Variable Interest Rates

Consider the case in which different interest rates apply for different time periods. Let A_t denote the magnitude of the cash flow at the end of time period t, t = 1, …, n. Let i_s denote the interest rate during time period s, s = 1, …, t. The present worth of $\{A_t\}$ is given by

$$P = \sum_{t=1}^{n} A_t \prod_{s=1}^{t} (1+i_s)^{-1}$$

Example 3.30

You deposit $1000 in a fund paying 8% annual interest; after 3 years the fund increases its interest rate to 10%; after 4 years of paying 10% interest the fund begins paying 12%. How much will be in the fund 9 years after the initial deposit?



Example 3.30

You deposit $1000 in a fund paying 8% annual interest; after 3 years the fund increases its interest rate to 10%; after 4 years of paying 10% interest the fund begins paying 12%. How much will be in the fund 9 years after the initial deposit?

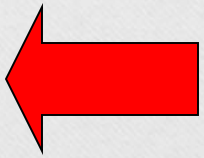
Solution:

let V_t = value of fund at time t

$V_3 = \$1000.00(F|P\ 8\%,3) = \$1259.71$

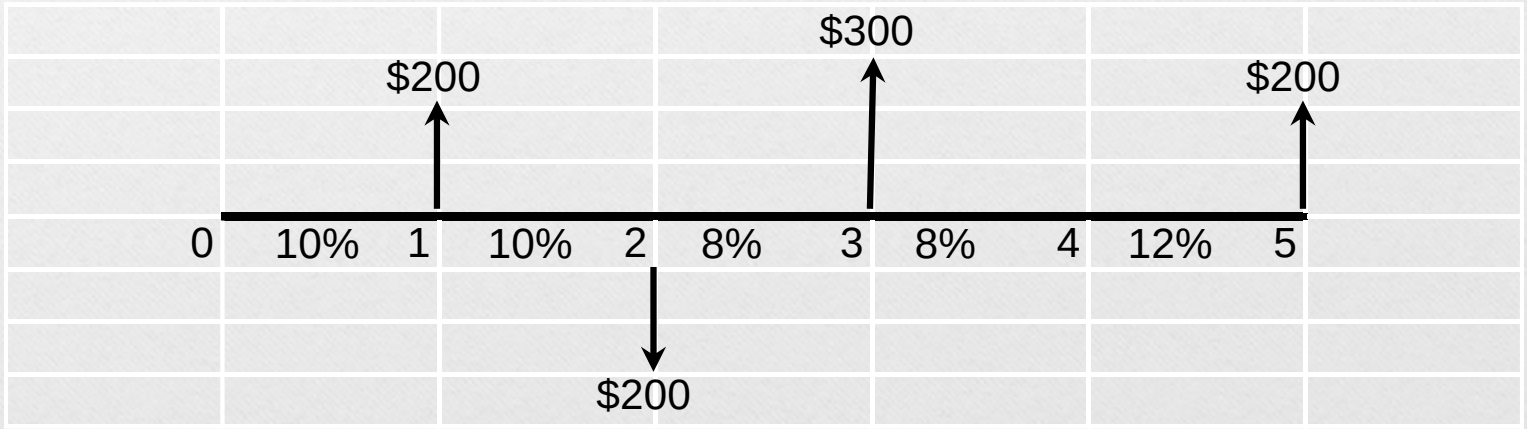
$V_7 = \$1259.71(F|P\ 10\%,4) = \$1844.34$

$V_9 = \$1844.34(F|P\ 12\%,2) = \$2313.54$



Example 3.31

Consider a cash flow profile in which $200 is received at t=1, spent at t=2, and received at t=5, and $300 is received at t=3. Suppose the interest rate is 10% the first 2 periods, 8% the next two periods, and is 12% the 5th period. What are the equivalent present worth, future worth, and uniform series for the cash flow profile? [note: t denotes end of period t]



Example 3.31

Solution:

$$P = \$200(P|F\ 10\%,1) - \$200(P|F\ 10\%,2) +$$

$$\$300(P|F\ 8\%,1)(P|F\ 10\%,2) +$$

$$\$200(P|F\ 12\%,1)(P|F\ 8\%,2)(P|F\ 10\%,2)$$

$$P = \$372.63$$

Example 3.31 (Continued)

$$F = \$200 + \$300(F|P\ 8\%,1)(F|P\ 12\%,1) -$$

$$\$200(F|P\ 8\%,2)(F|P\ 12\%,1) +$$

$$\$200(F|P\ 10\%,1)(F|P\ 8\%,2)(F|P\ 12\%,1)$$

$$F = \$589.01$$

Example 3.31 (Continued)

To solve for the uniform series equivalent, notice

$$F = A[1+(F|P\ 12\%,1)+(F|P\ 8\%,1)(F|P\ 12\%,1)+$$

$$(F|P\ 8\%,2)(F|P\ 12\%,1)+$$

$$(F|P\ 10\%,1)(F|P\ 8\%,2)(F|P\ 12\%,1)]$$

$$= A[1+1.12+1.08(1.12)+1.1664(1.12)$$

$$+1.1(1.08)(1.12)] \quad = \$589.01$$

$$\$589.01 = 6.073A$$

$$A = \$589.01/6.073 = \$96.99$$