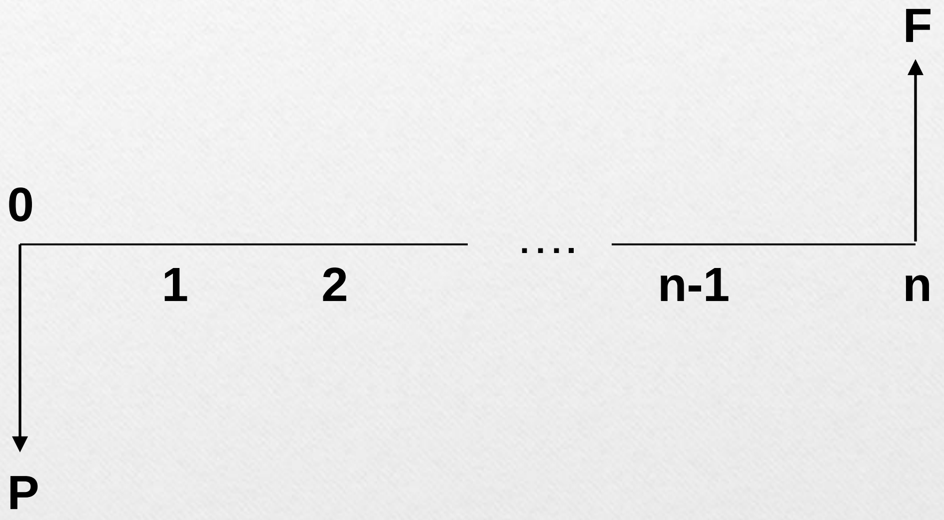
Discounted Cash Flow Formulas



Discounted Cash Flow Formulas

$F = P(1 + i)^n$

$F = P(F|P\ i\%,n)$ **single sum**, **future worth factor**

$P = F(1 + i)^{-n}$

$P = F(P|F\ i\%,n)$ **single sum**, **present worth factor**

Computing the Present Worth of Multiple Cash flows

$$P = \sum_{t=0}^{n} A_t (1+i)^{-t} \qquad \textbf{(2.12)}$$

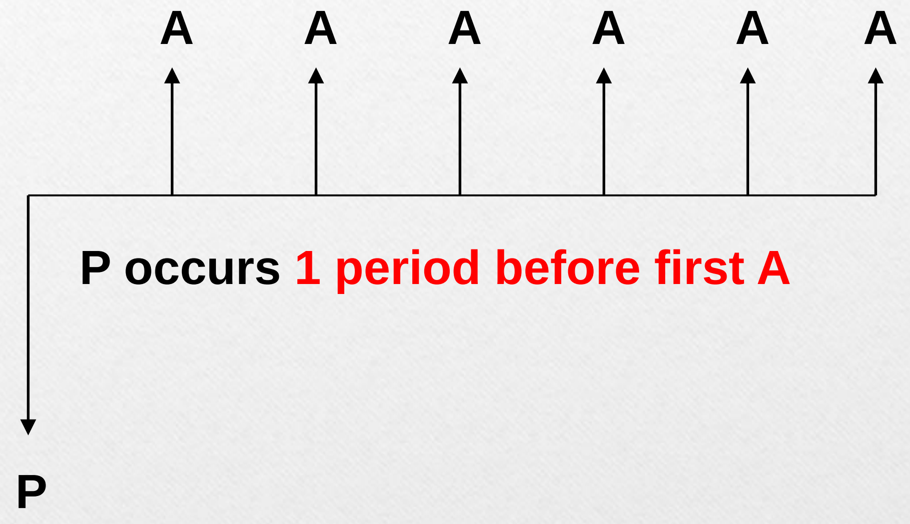
$$P = \sum_{t=0}^{n} A_t (P \mid F\ i\%, t) \qquad \textbf{(2.13)}$$

Computing the Future worth of Multiple cash Flows

$$F = \sum_{t=1}^{n} A_t (1+i)^{n-t} \qquad (2.15)$$

$$F = \sum_{t=1}^{n} A_t (F \mid P \quad i\%, n-t) \qquad (2.16)$$

DCF Uniform Series Formulas



$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

uniform series, present worth factor

$= A(P|A\ i\%,n)$ $=PV(i\%,n,-A)$

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

to recover a present capital investment

uniform series, capital recovery factor

$= P(A|P\ i\%,n)$ $=PMT(i\%,n,-P)$

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

uniform series, future worth factor

$= A(F|A\ i\%,n)$ $=FV(i\%,n,-A)$

to fund a future capital expense

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

uniform series, sinking fund factor

$= F(A|F\ i\%,n)$ $=PMT(i\%,n,,-F)$

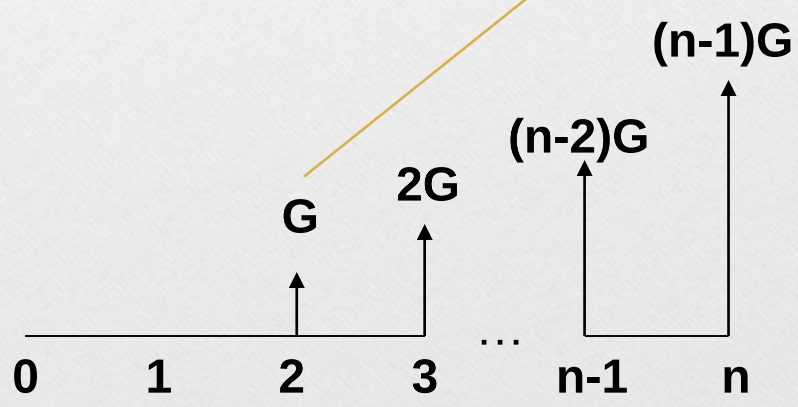
Gradient Series

$$A_t = \begin{cases} 0 & t = 1 \\ A_{t-1} + G & t = 2,\ldots,n \end{cases}$$

or

$$A_t = (t-1)G \qquad t = 1,\ldots,n$$

Start fr n=2



Note: n-1, not n

Converting Gradient Series

Converting gradient series to present worth

$$P = G\left[\frac{1-(1+ni)(1+i)^{-n}}{i^2}\right] \qquad (2.35)$$

$$P = G\left[\frac{(P|A\ i\%,n)-n(P|F\ i\%,n)}{i}\right] \qquad (2.36)$$

$$P = G(P|G\ i\%, n) \qquad (2.37)$$

Converting Gradient Series - II

Converting gradient series to annual worth

$$A = G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

$$A = G\left[\frac{1 - n(A|F\ i\%, n)}{i}\right]$$

$A = G(A|G\ i\%, n)$ **(2.38)**

Converting Gradient Series - III

Converting gradient series to future worth

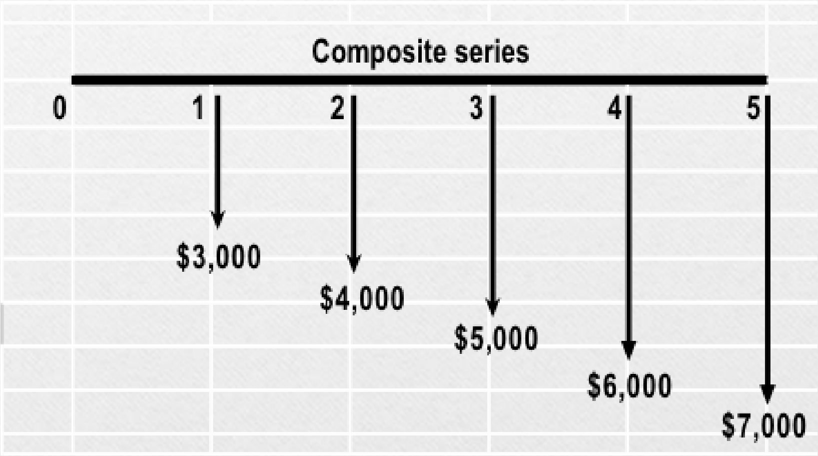
$$F = G\left[\frac{(1+i)^n - (1+ni)}{i^2}\right] \qquad \textbf{(2.39)}$$

$$F = G\left[\frac{(F \mid A\ i\%, n) - n}{i}\right]$$

F = G(F|G i%, n) (not provided in the tables)

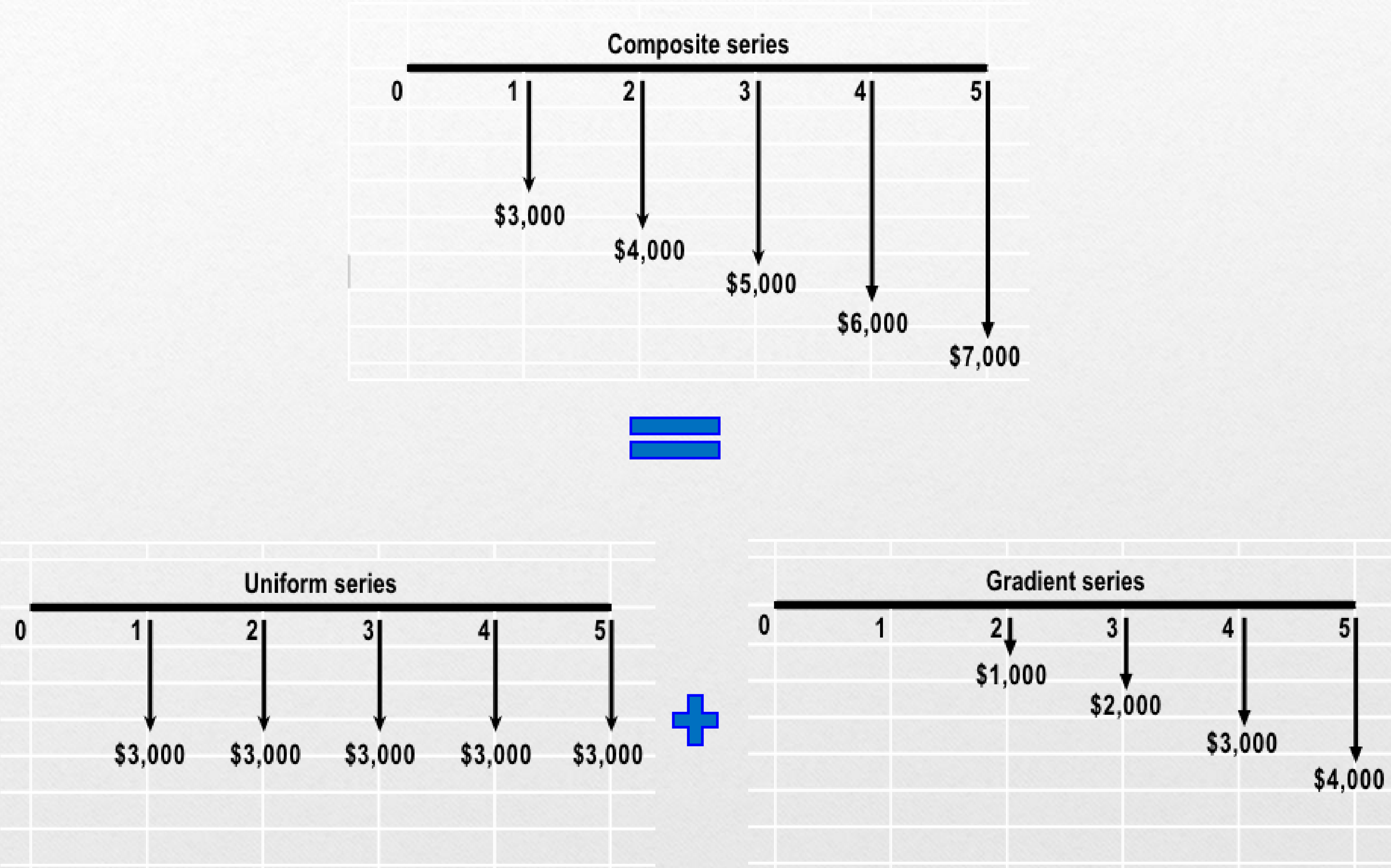
Example 2.28

Maintenance costs for a particular production machine increase by $1,000/year over the 5-yr life of the machine. The initial maintenance cost is $3,000. Using an interest rate of 8% compounded annually, determine the present worth equivalent for the maintenance costs.



Increasing gradient

Example 2.28



Composite series
0
1
2
3
4
5
$3,000
$4,000
$5,000
$6,000
$7,000
Uniform series
0
1
2
3
4
5
$3,000
$3,000
$3,000
$3,000
$3,000
Gradient series
0
1
2
3
4
5
$1,000
$2,000
$3,000
$4,000

Example 2.28

Maintenance costs for a particular production machine increase by $1,000/year over the 5-yr life of the machine. The initial maintenance cost is $3,000. Using an interest rate of 8% compounded annually, determine the present worth equivalent for the maintenance costs.

n in uniform series = number of payment

n in gradient series = number of payment +1

P = $3,000(P|A 8%,5) + $1,000(P|G 8%,5)

P = $3,000(3.99271) + $1,000(7.372.43) = $19,350.56

OR

Convert all CF to uniform payments then convert it using single sum to PW

P = ($3,000 + $1,000(A|G 8%,5))(P|A 8%,5)

P = ($3,000 + $1,000(1.846.47))(3.99271) = $19,350.55 or

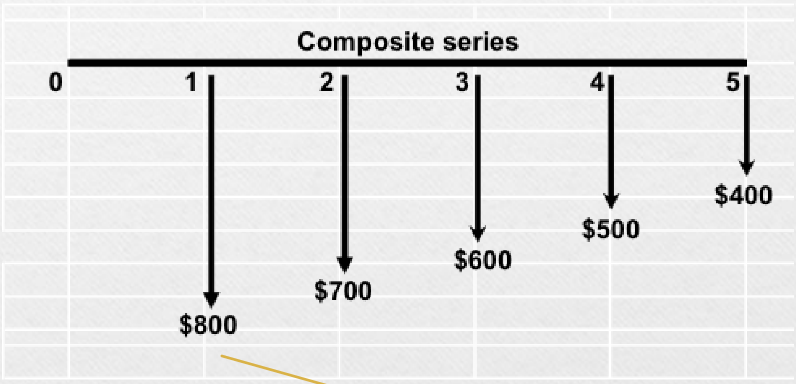
P =1000*NPV(8%,3,4,5,6,7) = $19,350.56

C11 =FV(8%,5,-C10)

End of Year (n)	Cash Flow (CF)	
0	$0	
1	$3,000	
2	$4,000	
3	$5,000	
4	$6,000	
5	$7,000	
P =	$19,350.56	=NPV(8%,C4:C8)
A =	$4,846.47	=PMT(8%,5,-C9)
F =	$28,432.31	=FV(8%,5,-C10)

Example 2.29

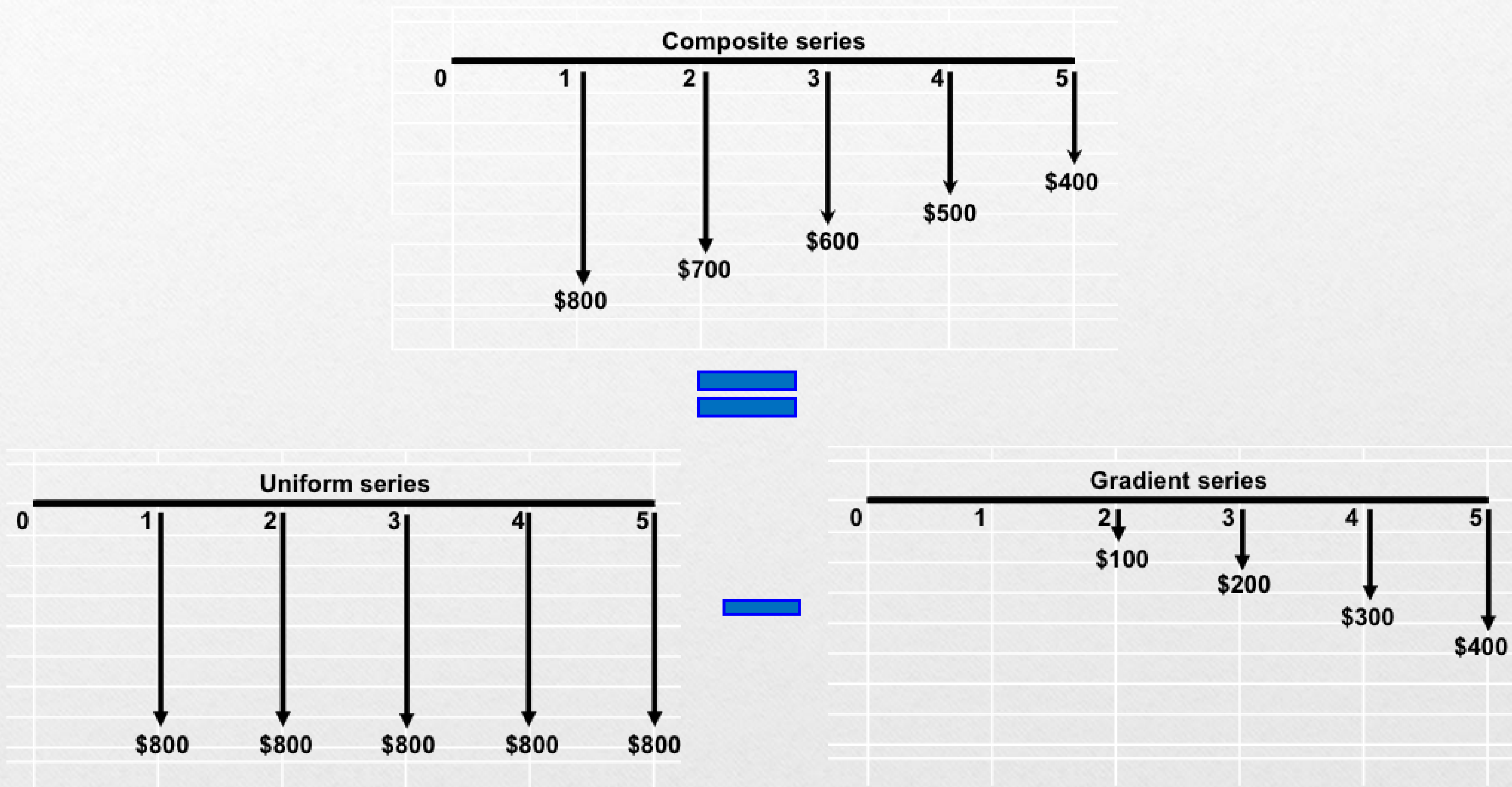
Amanda Dearman made 5 annual deposits into a fund that paid 8% compound annual interest. Her first deposit was $800; each successive deposit was $100 less than the previous deposit. How much was in the fund immediately after the 5th deposit?



Decreasing gradient

To apply Gradient rule first G should begin fr second year

Example 2.29



Composite series
0
1
2
3
4
5
$800
$700
$600
$500
$400
Uniform series
0
1
2
3
4
5
$800
$800
$800
$800
$800
Gradient series
0
1
2
3
4
5
$100
$200
$300
$400

Example 2.29

Amanda Dearman made 5 annual deposits into a fund that paid 8% compound annual interest. Her first deposit was $800; each successive deposit was $100 less than the previous deposit. How much was in the fund immediately after the 5th deposit?

A = $800 - $100(A|G 8%,5) = $800 - $100(1.84647) = $615.35

F = $615.35(F|A 8%,5) = $615.35(5.86660) = $3,610.01

Convert all CF to uniform payments then convert it using single sum to FW

But why not to use F= 800 (F|A 8%,5) -100 (F|G 8%,5)

Example 2.29

Amanda Dearman made 5 annual deposits into a fund that paid 8% compound annual interest. Her first deposit was $800; each successive deposit was $100 less than the previous deposit. How much was in the fund immediately after the 5th deposit?

A = $800 - $100(A|G 8%,5) = $800 - $100(1.84647) = $615.35

F = $615.35(F|A 8%,5) = $615.35(5.86660) = $3,6101.01

F =(FV(8%,5,,-NPV(8%,800,700,600,500,400))

F = $3,610.03 (*Using excel worksheet*)

Chapter 2 tables and figures (10-21-08) [Compatibility Mode] - Microsoft Excel

C9 =NPV(8%,C4:C8)

End of Year (n)	Cash Flow (CF)	
0	$0	
1	$800	
2	$700	
3	$600	
4	$500	
5	$400	
P =	$2,456.93	=NPV(8%,C4:C8)
F =	$3,610.03	=FV(8%,5,,-C9)

Figure 2.26

$$P = G\left[\frac{1 - (1 + ni)(1 + i)^{-n}}{i^2}\right]$$

gradient series, **present worth factor**

$= G(P|G\ i\%,n)$

$$A = G\left[\frac{(1 + i)^n - (1 + ni)}{i[(1 + i)^n - 1]}\right]$$

gradient-to-uniform series conversion factor

$= G(A|G\ i\%,n)$

$$F = G\left[\frac{(1 + i)^n - (1 + ni)}{i^2}\right]$$

gradient series, **future worth factor**

$= A(F|G\ i\%,n)$

Geometric Series

$A_t = A_{t-1}(1+j) \qquad t = 2,\ldots,n$

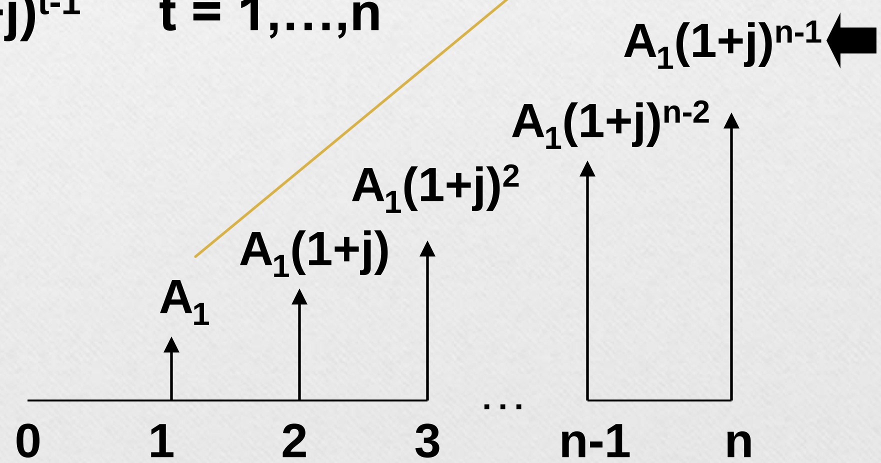
n in Geometric series = number of payments

or

First payment unlike the gradient series

$A_t = A_1(1+j)^{t-1} \qquad t = 1,\ldots,n$

Note: n-1 not n



Converting Geometric Series – I

Converting a geometric series to a present worth

$$P = A_1\left[\frac{1-(1+j)^n(1+i)^{-n}}{i-j}\right] \qquad i \neq j$$ **(2.42)**

$$P = nA_1/(1+i) \qquad i = j$$ **(2.42)**

$$P = A_1(P \mid A_1 \; i\%, j\%, n)$$ **(2.44)**

$$P = A_1\left[\frac{1-(F \mid P\ j\%, n)(P \mid F\ i\%, n)}{i-j}\right] \qquad i \neq j \quad j > 0$$ **(2.43)**

Converting Geometric Series – II

Converting a geometric series to a future worth

$$F = A_1\left[\frac{(1+i)^n - (1+j)^n}{i-j}\right] \qquad i \neq j$$

(2.45)

$$F = nA_1(1+i)^{n-1} \qquad i = j$$

$$F = A_1\left[\frac{(F \mid P\ i\%, n) - (F \mid P\ j\%, n)}{i-j}\right] \qquad i \neq j \quad j > 0$$

$$F = A_1(F \mid A_1\ i\%, j\%, n)$$

Note: $(F \mid A_1\ i\%, j\%, n) = (F \mid A_1\ j\%, i\%, n)$ **Notice the symmetry**

Example 2.30

A firm is considering purchasing a new machine. It will have maintenance costs that increase 8% per year. An initial maintenance cost of $1,000 is expected. Using a 10% interest rate, what present worth cost is equivalent to the cash flows for maintenance of the machine over its 15-year expected life?

Example 2.30

A firm is considering purchasing a new machine. It will have maintenance costs that increase 8% per year. An initial maintenance cost of $1,000 is expected. Using a 10% interest rate, what present worth cost is equivalent to the cash flows for maintenance of the machine over its 15-year expected life?

A_1 = $1,000, i = 10%, j = 8%, n = 15, P = ?

P = $1,000(P|$A_1$10%,8%,15) = $1,000(12.03040) = $12,030.40

P =1000*NPV(10%,1,1.08,1.08^2,1.08^3,1.08^4,1.08^5,

1.08^6,1.08^7,1.08^8,1.08^9,1.08^10,1.08^11,1.08^12,

1.08^13,1.08^14) = $12,030.40

Example 2.30

A firm is considering purchasing a new machine. It will have maintenance costs that increase 8% per year. An initial maintenance cost of $1,000 is expected. Using a 10% interest rate, what present worth cost is equivalent to the cash flows for maintenance of the machine over its 15-year expected life?

A_1 = $1,000, i = 10%, j = 8%, n = 15, P = ?

P = $1,000(P|$A_1$10%,8%,15) = $1,000(12.03040) = $12,030.40

P =1000*NPV(10%,1,1.08,1.08^2,1.08^3,1.08^4,1.08^5,
1.08^6,1.08^7,1.08^8,1.08^9,1.08^10,1.08^11,1.08^12,
1.08^13,1.08^14)

P = $12,030.40

End of Year (n)	Cash Flow (CF)
0	$0
1	$1,000
2	$1,080
3	$1,166
4	$1,260
5	$1,360
6	$1,469
7	$1,587
8	$1,714
9	$1,851
10	$1,999
11	$2,159
12	$2,332
13	$2,518
14	$2,720
15	$2,937
P =	$12,030.40

=C14*1.08

=NPV(10%,C4:C18)

Example 2.31

Mattie Bookhout deposits her annual bonus in a savings account that pays 8% compound annual interest. Her annual bonus is expected to increase by 10% each year. If her initial deposit is $500, how much will be in her account immediately after her 10th deposit?

A_1 = \$500, i = 8%, j = 10%, n = 10, F = ?

$F = \$500(F|A_1\ 8\%,10\%,10) = \$500(21.74087)$

F = \$10,870.44 Excel)

Chapter 2 tables and figures (10-20-08) [Compatibility Mode] - Microsoft Excel

C14 =NPV(8%,C4:C13)

End of Year (n)	Cash Flow (CF)	
0	$0	
1	$500	
2	$550	
3	$605	
4	$666	=C6*1.1
5	$732	
6	$805	
7	$886	
8	$974	
9	$1,072	
10	$1,179	
P =	$5,035.12	=NPV(8%,C4:C13)
F =	$10,870.44	=FV(8%,10,,-C14)

Figure 2.24 | Figure 2.25 | Figure 2.26 | Figure 2.27 | Figure 2.28 | Figure 2.29

Example 2.32

Julian Stewart invested \$100,000 in a limited partnership in a natural gas drilling project. His net revenue the 1st year was \$25,000. Each year, thereafter, his revenue decreased 10%/yr. Based on a 12% TVOM, what is the present worth of his investment over a 20-year period?

A_1 = \$25,000, i = 12%, j = -10%, n = 20, P = ?

$P = -\$100,000 + \$25,000(P|A_1\ 12\%,-10\%,20)$

$P = -\$100,000 + \$25,000[1 – (0.90)^{20}(1.12)^{-20}]/(0.12 + 0.10)$

$P = \$12,204.15$

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C15 =C14*0.9

End of Year (n)	Cash Flow (CF)
0	-$100,000
1	$25,000
2	$22,500
3	$20,250
4	$18,225
5	$16,403
6	$14,762
7	$13,286
8	$11,957
9	$10,762
10	$9,686
11	$8,717
12	$7,845
13	$7,061
14	$6,355
15	$5,719
16	$5,147
17	$4,633
18	$4,169
19	$3,752
20	$3,377
P =	$12,204.15

$7,845 → **=C14*0.9**

$12,204.15 → **=NPV(12%,C4:C23)+C3**

Figure 2.24 | Figure 2.25 | Figure 2.26 | Figure 2.27 | Figure 2.28 | Figure 2.29 | **Figure 2.30**

$$P = A_1 \left[\frac{1 - (1 + j)^n (1 + i)^{-n}}{i - j} \right] \qquad i \neq j$$

geometric series, present worth factor

$$P = nA_1/(1 + i) \qquad i = j$$

$$P = A_1(P|A_1\ i\%, j\%, n)$$

$$F = A_1 \left[\frac{(1 + i)^n - (1 + j)^n}{i - j} \right] \qquad i \neq j$$

geometric series, future worth factor

$$F = nA_1(1 + i)^{n-1} \qquad i = j$$

$$F = A_1(F|A_1\ i\%, j\%, n)$$