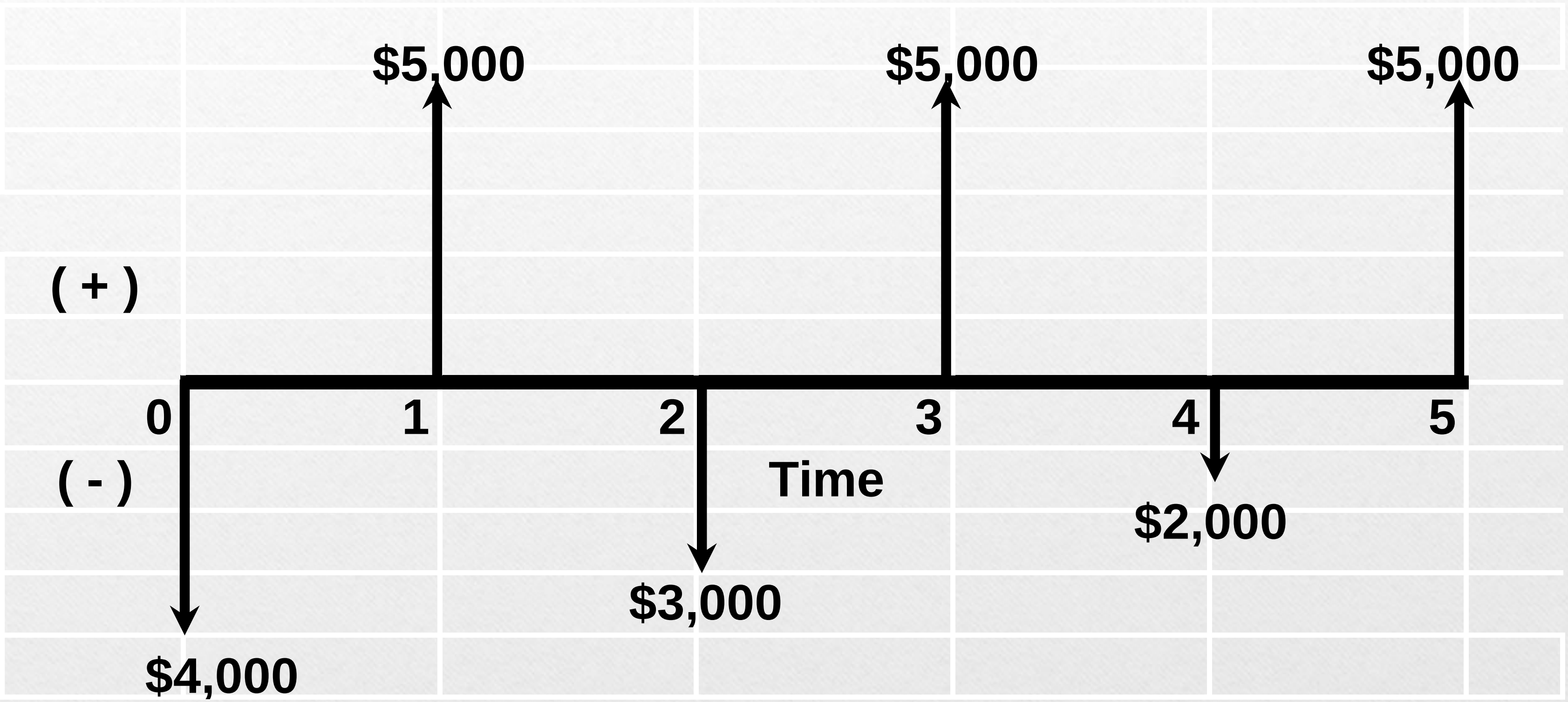
Chapter 2

Time Value of Money (TVOM)

Cash Flow Diagrams



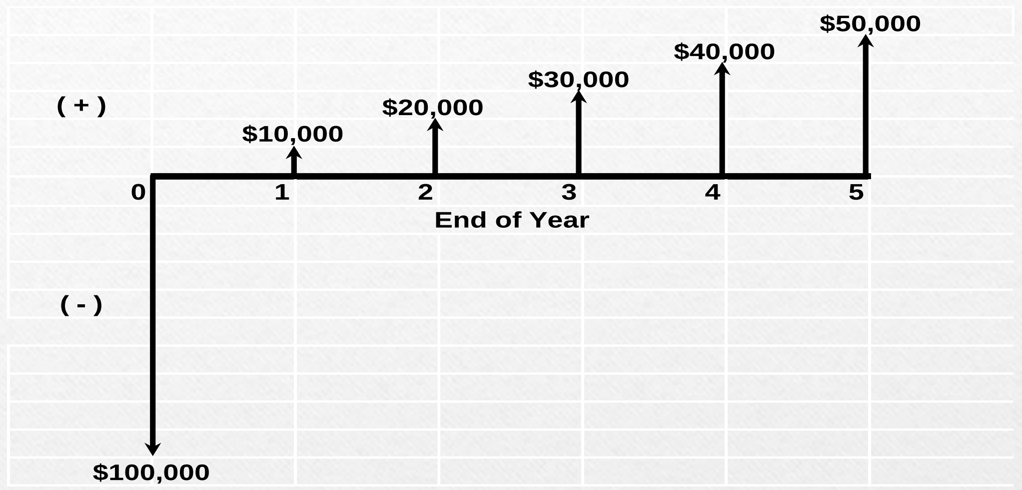
Example 2.1: Cash Flow Profiles for Two Investment Alternatives

(EOY) End of Year	CF(A)	CF(B)	CF(B-A)
0	-$100,000	-$100,000	$0
1	$10,000	$50,000	$40,000
2	$20,000	$40,000	$20,000
3	$30,000	$30,000	$0
4	$40,000	$20,000	-$20,000
5	$50,000	$10,000	-$40,000
Sum	$50,000	$50,000	$0

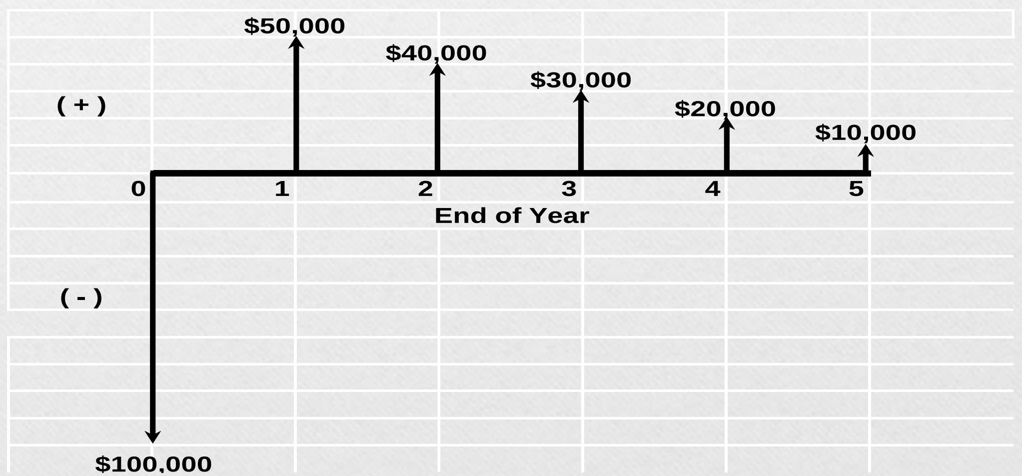
Although the two investment alternatives have the same "bottom line," there are obvious differences. Which would you prefer, A or B? Why?

Example 2.1: (cont.)

Inv. A



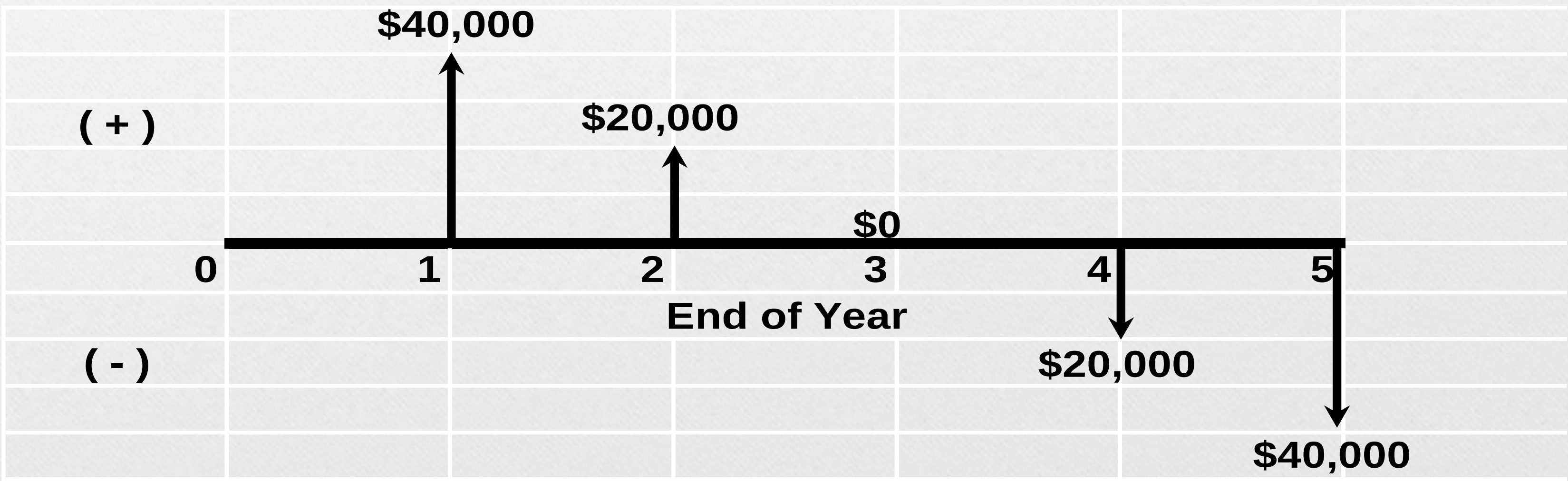
Inv. B



Example 2.1: (cont.)

Principle #7

Consider only differences in cash flows among investment alternatives

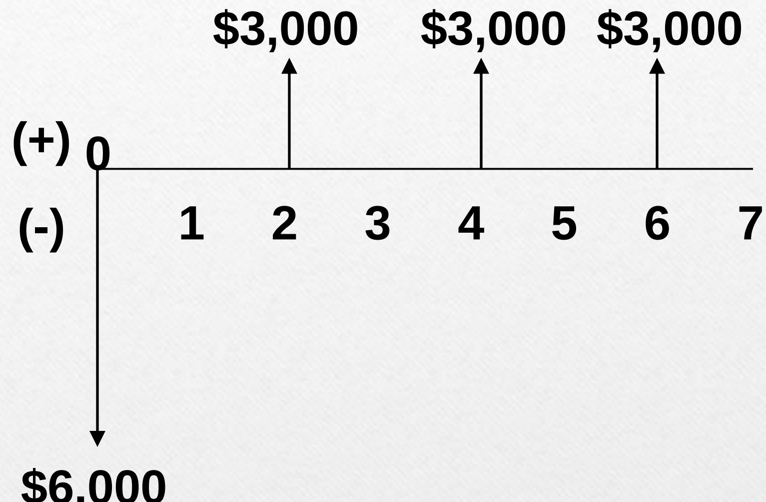


Inv. B – Inv. A

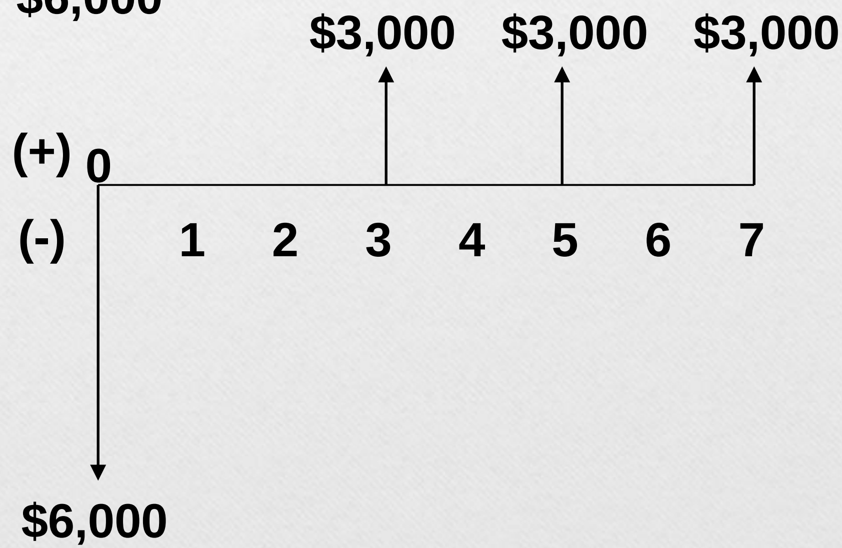
Example 2.2

Which would you choose?

Alternative C



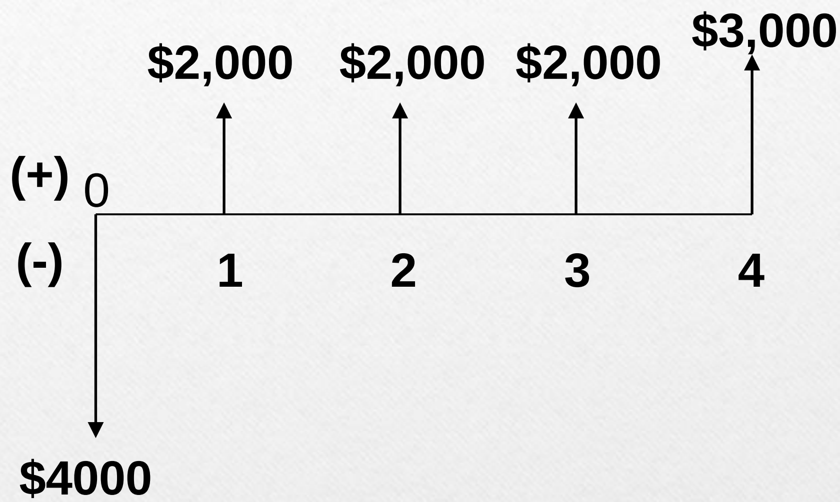
Alternative D

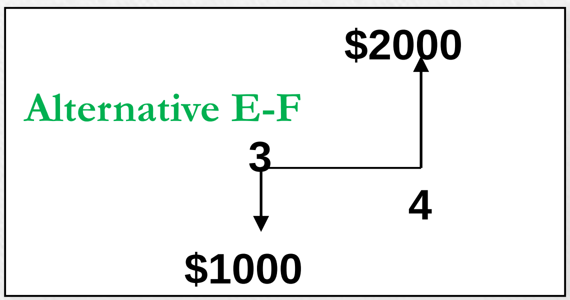


Example 2.3

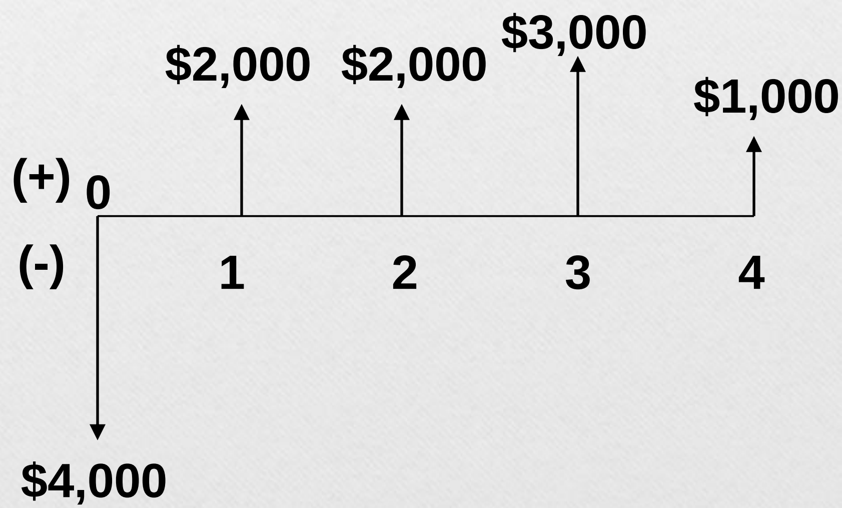
Which would you choose?

Alternative E





Alternative F



Simple interest calculation:

$$F_n = P(1 + in)$$

Compound Interest Calculation:

$$F_n = F_{n-1}(1 + i)$$

Where

P = present value of single sum of money

F_n = accumulated value of P over n periods

i = interest rate per period

n = number of periods

Example 2.7: Simple Interest Calculation

Robert borrows \$4,000 from Susan and agrees to pay \$1,000 plus accrued interest at the end of the first year and \$3,000 plus accrued interest at the end of the fourth year. What should be the size of the payments if 8% simple interest is used?

- **1st payment = \$1,000 + 0.08(\$4,000)**

  = \$1,320

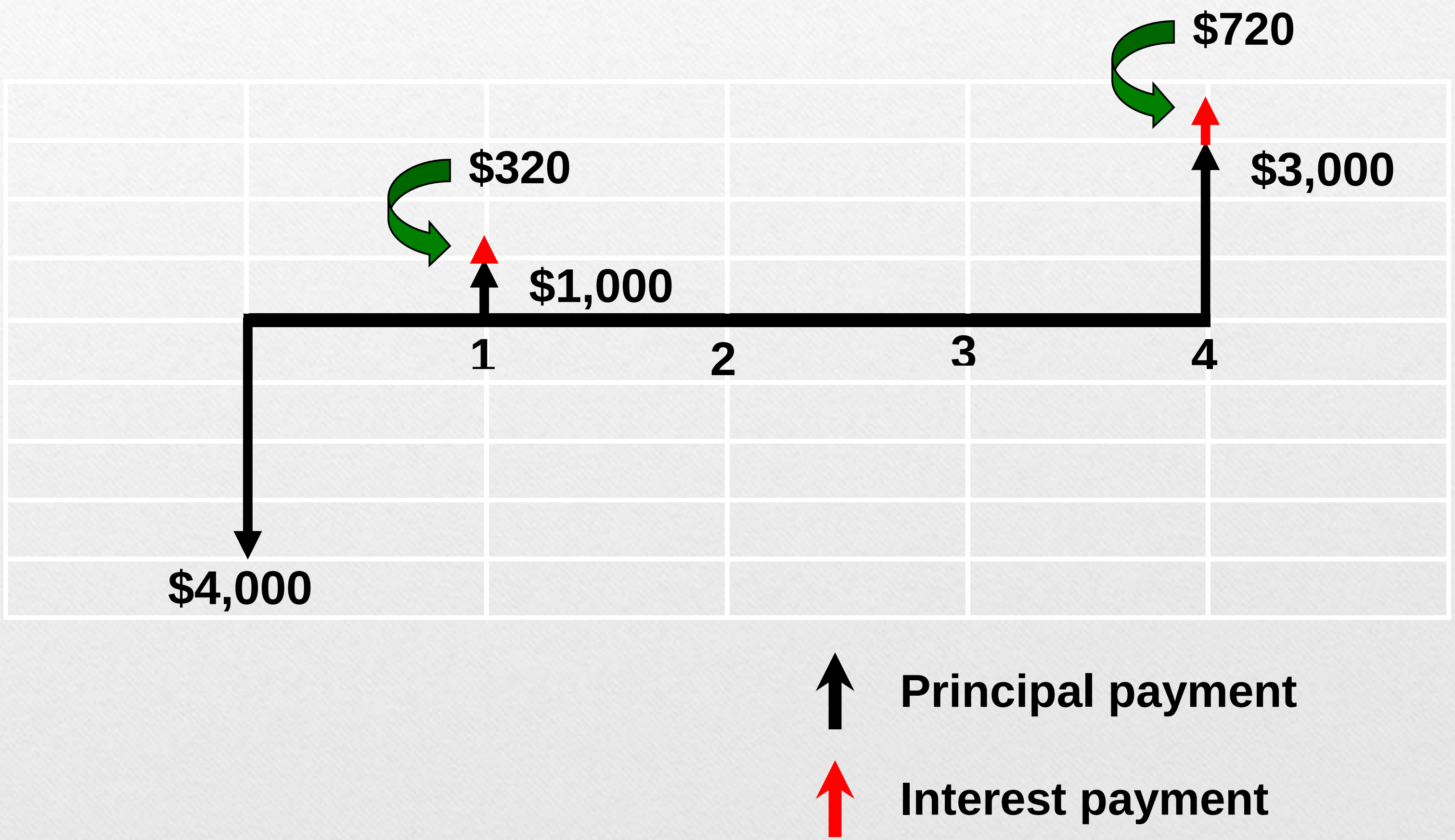
- **2nd payment = \$3,000 + 0.08(\$3,000)(3)**

  = \$3,720

Remaining period after 1st payment

Example 2.7: (Cont.)

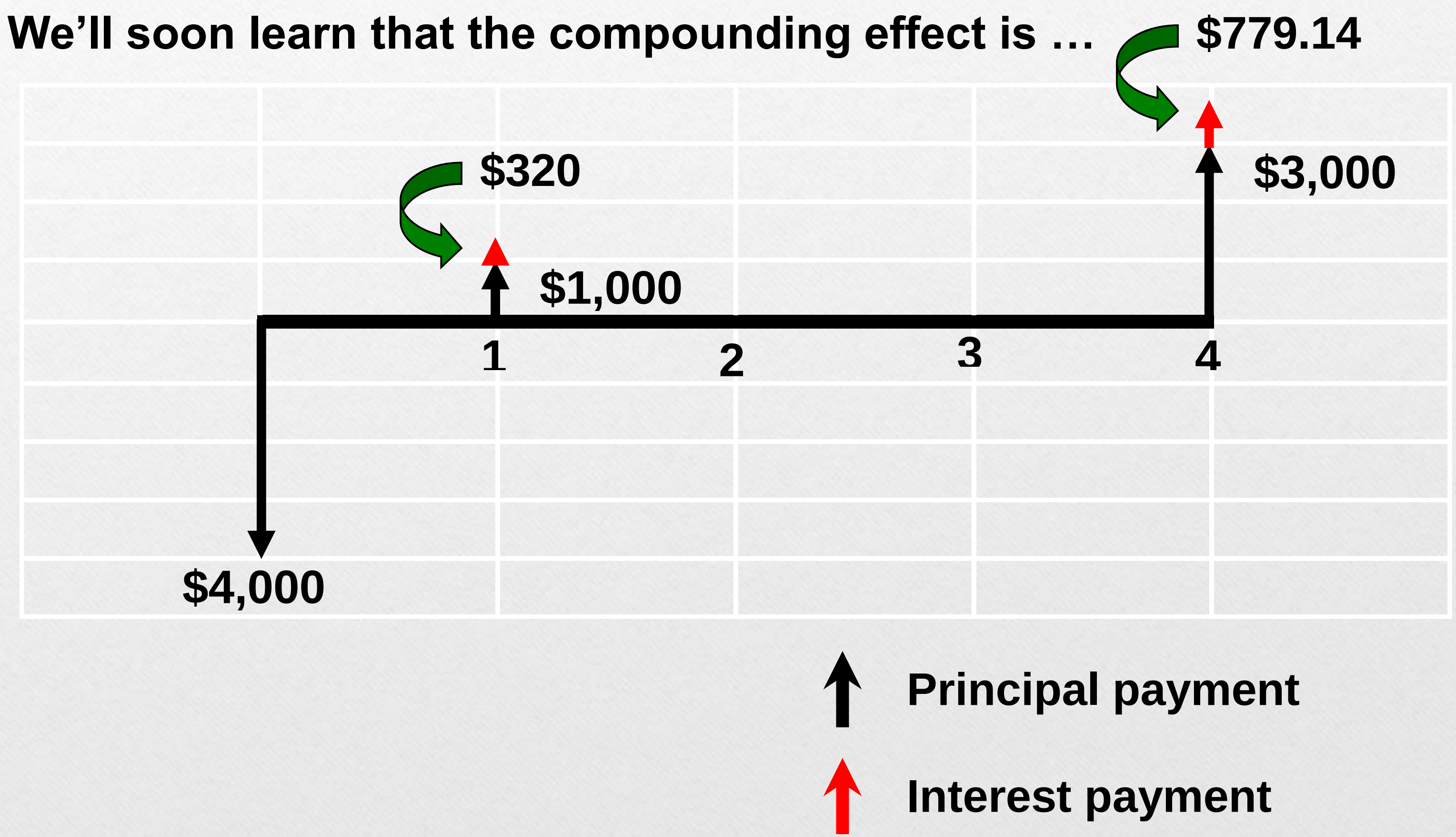
Simple Interest Cash Flow Diagram



RULES
Discounting Cash Flow

1. **Money has time value!**
2. **Cash flows cannot be added unless they occur at the same point(s) in time**
3. **Multiply a cash flow by (1+i) to move it forward one time unit**
4. **Divide a cash flow by (1+i) to move it backward one time unit**

Compound Interest Cash Flow Diagram



Example 2.8: (Lender's Perspective)
Value of $10,000 Investment Growing @ 10% per year

Start of Year	Value of Investment	Interest Earned	End of Year	Value of Investment
1	$10,000.00	$1,000.00	1	$11,000.00
2	$11,000.00	$1,100.00	2	$12,100.00
3	$12,100.00	$1,210.00	3	$13,310.00
4	$13,310.00	$1,331.00	4	$14,641.00
5	$14,641.00	$1,464.10	5	$16,105.10

This means this amount at end of year 5 is equivalent to 10,000 at time zero (present)

Example 2.8: (Borrower's Perspective)
Value of $10,000 Investment Growing @ 10% per year

Year	Unpaid Balance at the Beginning of the Year	Annual Interest	Payment	Unpaid Balance at the End of the Year
1	$10,000.00	$1,000.00	$0.00	$11,000.00
2	$11,000.00	$1,100.00	$0.00	$12,100.00
3	$12,100.00	$1,210.00	$0.00	$13,310.00
4	$13,310.00	$1,331.00	$0.00	$14,641.00
5	$14,641.00	$1,464.10	$16,105.10	$0.00

Compounding of Money

Beginning of Period	Amount Owed at Beginning (PW)	Interest Earned	End of Period	Amount Owed at End (FW)
1	P	Pi	1	P(1+i)
2	P(1+i)	P(1+i)i	2	$P(1+i)^2$
3	$P(1+i)^2$	$P(1+i)^2 i$	3	$P(1+i)^3$
4	$P(1+i)^3$	$P(1+i)^3 i$	4	$P(1+i)^4$
5	$P(1+i)^4$	$P(1+i)^4 i$	5	$P(1+i)^5$
:	:	:	:	:
n-1	$P(1+i)^{n-2}$	$P(1+i)^{n-2} i$	n-1	$P(1+i)^{n-1}$
n	$P(1+i)^{n-1}$	$P(1+i)^{n-1} i$	n	$P(1+i)^n$

Discounted Cash Flow Formulas

$$F = P(1+i)^n \qquad (2.8)$$

$$F = P\,(F|P\ i\%,\ n)$$

Vertical line means "given"

$$P = F(1+i)^{-n} \qquad (2.9)$$

$$= F/(1+i)^n$$

$$P = F\,(P|F\ i\%,\ n)$$

Excel® DCF Worksheet Functions

$F = P(1+i)^{n}$ (2.1)

$F = P(F|P\ i\%, n)$

F =FV(i%,n,,-P)

$P = F(1+i)^{-n}$ (2.3)

$P = F(P|F\ i\%, n)$

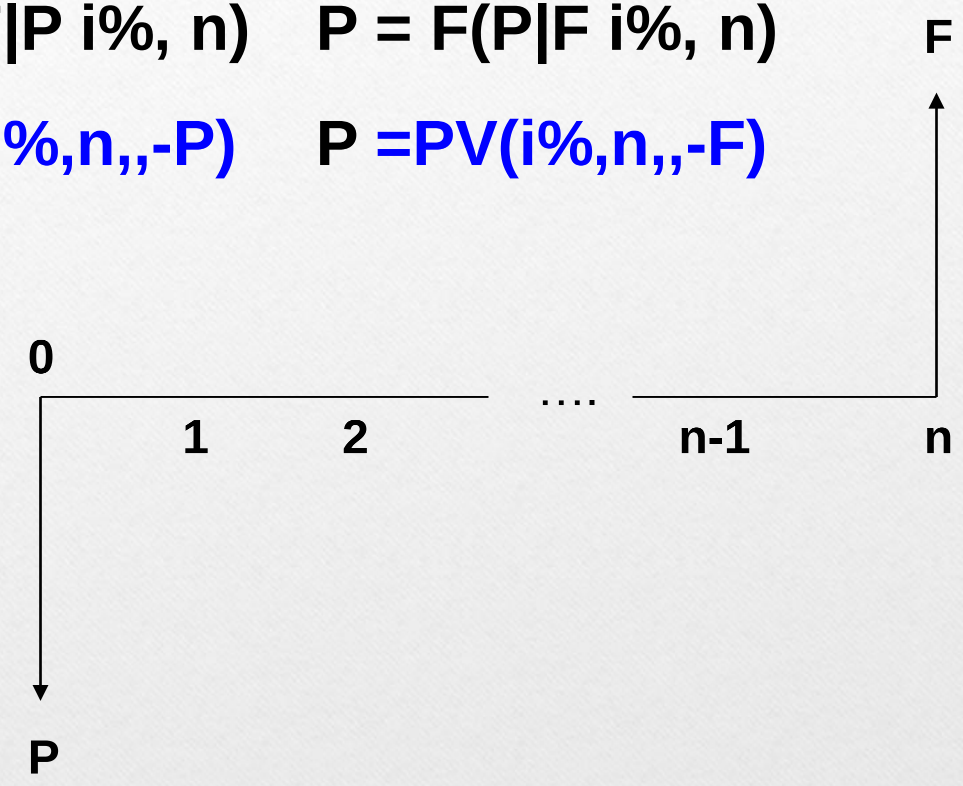
P =PV(i%,n,,-F)

$F = P(1+i)^n$ $F = P(F\|P\ i\%,n)$ F =FV(i%,n,,-P)	single sum, **future worth** factor
$P = F(1+i)^{-n}$ $P = F(P\|F\ i\%,n)$ P =PV(i%,n,,-F)	single sum, **present worth** factor

$F = P(1+i)^{n}$ $\quad$ $P = F(1+i)^{-n}$

$F = P(F|P\ i\%, n)$ $\quad$ $P = F(P|F\ i\%, n)$

F =FV(i%,n,,-P) $\quad$ P =PV(i%,n,,-F)



P occurs n periods before F

(F occurs n periods after P)

Relationships among *P*, *F*, and *A*

P occurs at the same time as A_0, i.e., at t = 0

F occurs at the same time as A_n, i.e., at t = *n*

Discounted Cash Flow (DCF) Methods

- DCF values are tabulated in the Appendixes
- Financial calculators can be used
- Financial spreadsheet software is available, e.g., Excel® financial functions include
  - PV, NPV, PMT, FV
  - IRR, MIRR, RATE
  - NPER

Example 2.9

Dia St. John borrows $1,000 at 12% compounded annually. The loan is to be repaid after 5 years. How much must she repay in 5 years?

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Dia St. John borrows $1,000 at 12% compounded annually. The loan is to be repaid after 5 years. How much must she repay in 5 years?

$F = P(F|P\ i\%, n)$

$F = \$1,000(F|P\ 12\%,5)$

$F = \$1,000(1.12)^5$

$F = \$1,000(1.76234)$

$F = \$1,762.34$

Example 2.9

Dia St. John borrows $1,000 at 12% compounded annually. The loan is to be repaid after 5 years. How much must she repay in 5 years?

F = P(F|P i, n)

F = $1,000(F|P 12%,5)

F = $1,000(1.12)^5

F = $1,000(1.76234)

F = $1,762.34

F =FV(12%,5,,-1000)

F = $1,762.34

Example 2.10

How long does it take for money to double in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest?

Example 2.10

How long does it take for money to double in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest?

I can think of six ways to solve this problem:

1) **Solve using the Rule of 72**
2) **Use the interest tables; look for F|P factor equal to 2.0**
3) **Solve numerically; n = log(2)/log(1+i)**
4) **Solve using Excel® NPER function: =NPER(i%,,-1,2)**
5) **Solve using Excel® GOAL SEEK tool**
6) **Solve using Excel® SOLVER tool**

Example 2.10

How long does it take for money to double in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest?

RULE OF 72

Divide 72 by interest rate to determine how long it takes for money to double in value.

(Quick, but not always accurate.)

Example 2.10

How long does it take for money to double in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest?

Rule of 72 solution

(a) 72/2 = 36 yrs

(b) 72/3 = 24 yrs

(c) 72/4 = 18 yrs

(d) 72/6 = 12 yrs

(e) 72/8 = 9 yrs

(f) 72/12 = 6 yrs

Example 2.10

How long does it take for money to double in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest?

Using interest tables & interpolating

(a) 34.953 yrs

(b) 23.446 yrs

(c) 17.669 yrs

(d) 11.893 yrs

(e) 9.006 yrs

(f) 6.111 yrs

Example 2.10

How long does it take for money to double in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest?

Mathematical solution

(a) log 2/log 1.02 = 35.003 yrs

(b) log 2/log 1.03 = 23.450 yrs

(c) log 2/log 1.04 = 17.673 yrs

(d) log 2/log 1.06 = 11.896 yrs

(e) log 2/log 1.08 = 9.006 yrs

(f) log 2/log 1.12 = 6.116 yrs

Example 2.10

How long does it take for money to double in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest?

Using the Excel® NPER function

(a) n =NPER(2%,,-1,2) = 35.003 yrs

(b) n =NPER(3%,,-1,2) = 23.450 yrs

(c) n =NPER(4%,,-1,2) = 17.673 yrs

(d) n =NPER(6%,,-1,2) = 11.896 yrs

(e) n =NPER(8%,,-1,2) = 9.006 yrs

(f) n =NPER(12%,,-1,2) = 6.116 yrs

Identical solution to that obtained mathematically

Example 2.10

How long does it take for money to double in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest?

Using the Excel® GOAL SEEK tool

(a) n =34.999 yrs

(b) n =23.448 yrs

(c) n =17.672 yrs

(d) n =11.895 yrs

(e) n =9.008 yrs

(f) n =6.116 yrs

Solution obtained differs from that obtained mathematically; red digits differ

i%	n	(F|P i%,n)	Excel's FV Function
2%	34.99911185231	1.99985437960	=FV(A2,B2,,-1)
3%	23.44819333654	1.99990666057	=FV(A3,B3,,-1)
4%	17.67238866717	1.99995301273	=FV(A4,B4,,-1)
6%	11.89466421507	1.99988383488	=FV(A5,B5,,-1)
8%	9.00760138602	2.00017440810	=FV(A6,B6,,-1)
12%	6.11628834874	2.00000747394	=FV(A7,B7,,-1)

Goal Seek

Set cell: C7

To value: 2

By changing cell: B7

OK | Cancel

Example 2.10

How long does it take for money to double in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest?

Using the Excel® SOLVER tool

(a) n =35.003 yrs

(b) n =23.450 yrs

(c) n =17.673 yrs

(d) n =11.896 yrs

(e) n =9.006 yrs

(f) n =6.116 yrs

Solution differs from mathematical solution, but at the 6th to 10th decimal place

Chapter 2 tables and figures (07-18-08).xls [Compatibility Mode] - Microsoft Excel

C7 =FV(A7,B7,,-1)

	A	B	C	D
1	*i* %	*n*	(*F*|*P* *i* %,*n*)	**Excel's FV Function**
2	2%	35.00278878391	2.00000000109640	**=FV(A2,B2,,-1)**
3	3%	23.44976171081	1.99999376922840	**=FV(A3,B3,,-1)**
4	4%	17.67298990119	2.00000173831020	**=FV(A4,B4,,-1)**
5	6%	11.89566104644	2.00000000058340	**=FV(A5,B5,,-1)**
6	8%	9.00646833967	1.99999999641560	**=FV(A6,B6,,-1)**
7	12%	6.11625537418	1.99999999994970	**=FV(A7,B7,,-1)**

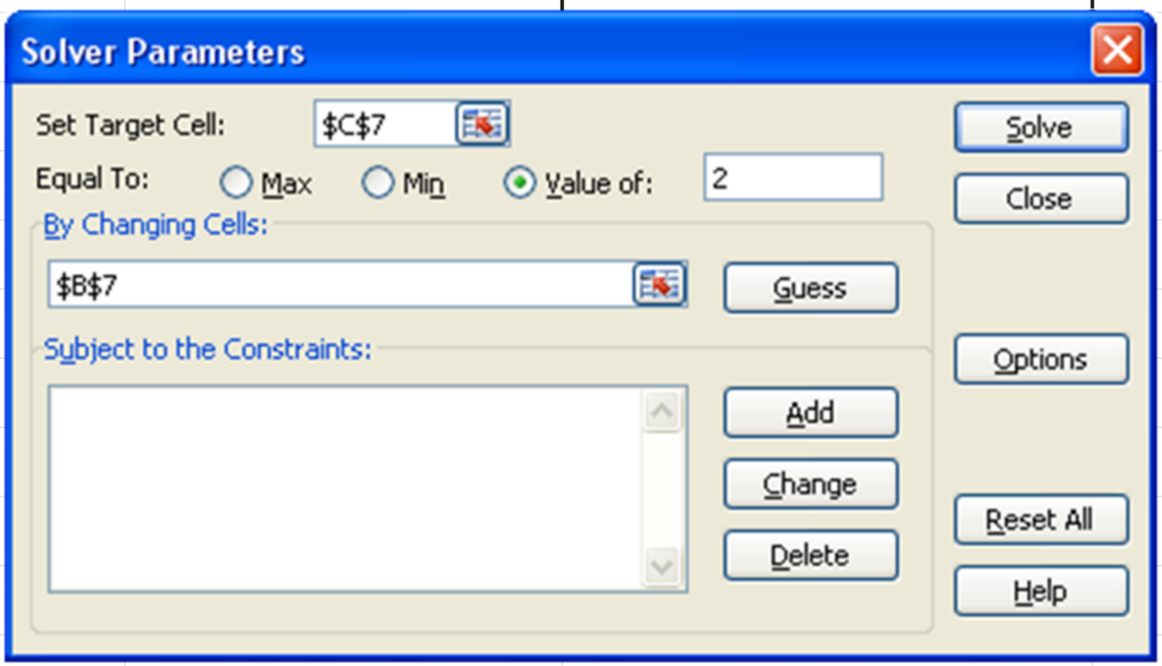


Figure 2.3 Figure 2.4 Figure 2.5 Figure 2.6 Figure 2.7 Figure 2.8 Figure 2.9 Figure 2.10 Figure 2.11 **Figure 2.12** Figure 2.13

F|P Example

How long does it take for money to triple in value, if you earn (a) 4%, (b) 6%, (c) 8%, (d) 10%, (e) 12%, (f) 15%, (g) 18% interest?

F|P Example

How long does it take for money to <u>triple</u> in value, if you earn (a) 4%, (b) 6%, (c) 8%, (d) 10%, (e) 12%, (f) 15%, (g) 18% interest?

1st option log equation

2nd option by using interest tables

3rd option using different excel equation solving tools

F|P Example

How long does it take for money to triple in value, if you earn (a) 4%, (b) 6%, (c) 8%, (d) 10%, (e) 12%, (f) 15%, (g) 18% interest?

(a) n =NPER(4%,,-1,3) = 28.011

(b) n =NPER(6%,,-1,3) = 18.854

(c) n =NPER(8%,,-1,3) = 14.275

(d) n =NPER(10%,,-1,3) = 11.527

(e) n =NPER(12%,,-1,3) = 9.694

(f) n =NPER(15%,,-1,3) = 7.861

(g) n =NPER(18%,,-1,3) = 6.638

Example 2.11

How much must you deposit, today, in order to accumulate $10,000 in 4 years, if you earn 5% compounded annually on your investment? P =PV(5%,4,,-10000)

P = $8227.02

Example 2.11

How much must you deposit, today, in order to accumulate $10,000 in 4 years, if you earn 5% compounded annually on your investment?

P = F(P|F i, n)

P = $10,000(P|F 5%,4)

P = $10,000(0.82270)= 8,227.00

OR

P = $10,000$(1.05)^{-4}$

P = $8,227.00

P =PV(5%,4,,-10000)

P = $8227.02

Example 2.11

How much must you deposit, today, in order to accumulate $10,000 in 4 years, if you earn 5% compounded annually on your investment?

P = F(P|F i, n)

P = $10,000(P|F 5%,4)

P = $10,000(1.05)$^{-4}$

P = $10,000(0.82270)

P = $8,227.00

P =PV(5%,4,,-10000)

P = $8,227.02

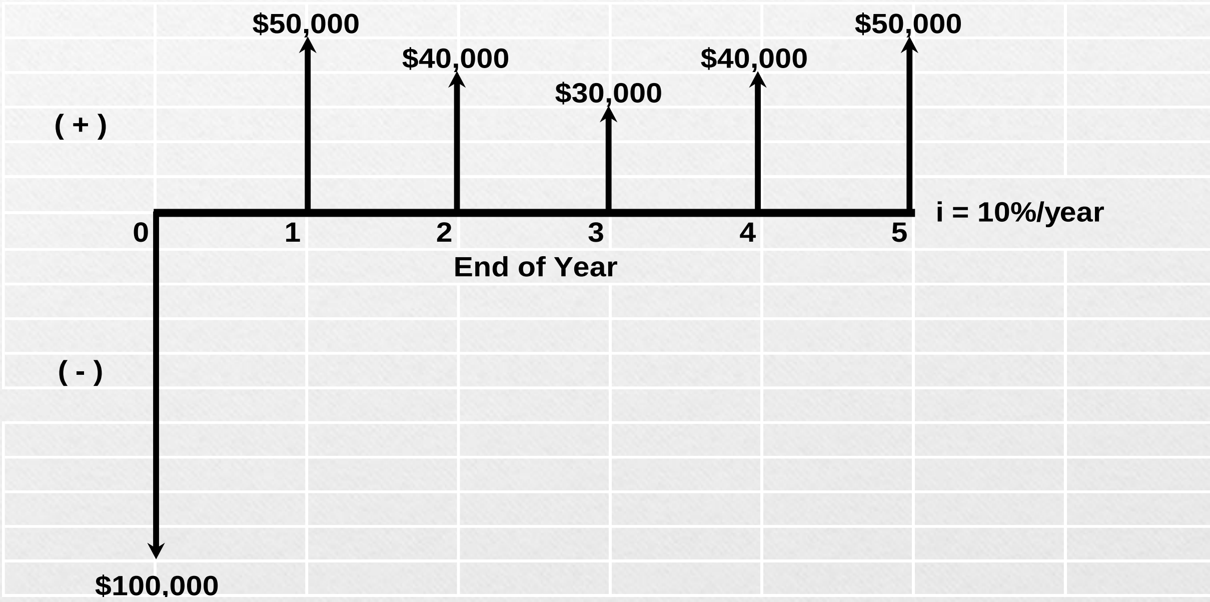
Computing the Present Worth of Multiple Cash flows

$$P = \sum_{t=0}^{n} A_t (1+i)^{-t} \quad (2.12)$$

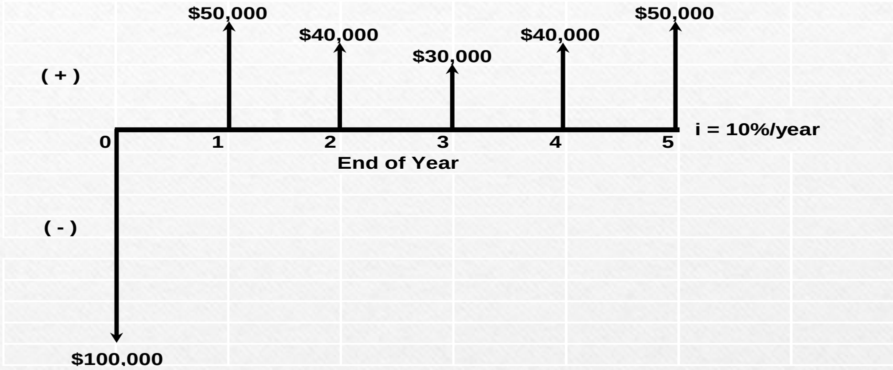
$$P = \sum_{t=0}^{n} A_t (P|F\, i\%, t) \quad (2.13)$$

Example 2.12

Determine the present worth equivalent of the CFD shown below, using an interest rate of 10% compounded annually.



Example 2.12



End of Year (n)	Cash Flow (CF)	(*P*|*F* 10%,n)	Present Worth	PV(10%,n,,-CF)	(*F*|*P* 10%,5-n)	Future Worth	FV(10%,5-n,,-CF)
0	-$100,000	1.00000	-$100,000.00	-$100,000.00	1.61051	-$161,051.00	-$161,051.00
1	$50,000	0.90909	$45,454.50	$45,454.55	1.46410	$73,205.00	$73,205.00
2	$40,000	0.82645	$33,058.00	$33,057.85	1.33100	$53,240.00	$53,240.00
3	$30,000	0.75131	$22,539.30	$22,539.44	1.21000	$36,300.00	$36,300.00
4	$40,000	0.68301	$27,320.40	$27,320.54	1.10000	$44,000.00	$44,000.00
5	$50,000	0.62092	$31,046.00	$31,046.07	1.00000	$50,000.00	$50,000.00
SUM			$59,418.20	$59,418.45		$95,694.00	$95,694.00

P =NPV(10%,50000,40000,30000,40000,50000)-100000

= $59,418.45

Example 2.13 & 2.16

Determine the present worth equivalent of the following series of cash flows. Use an interest rate of 6% per interest period.

End of Period	Cash Flow
0	$0
1	$300
2	$0
3	-$300
4	$200
5	$0
6	$400
7	$0
8	$200

P = \$300(P|F 6%,1)- \$300(P|F 6%,3)+\$200(P|F 6%,4)+\$400(P|F 6%,6)

+\$200(P|F 6%,8) = \$597.02

P =NPV(6%,300,0,-300,200,0,400,0,200)

P =\$597.02

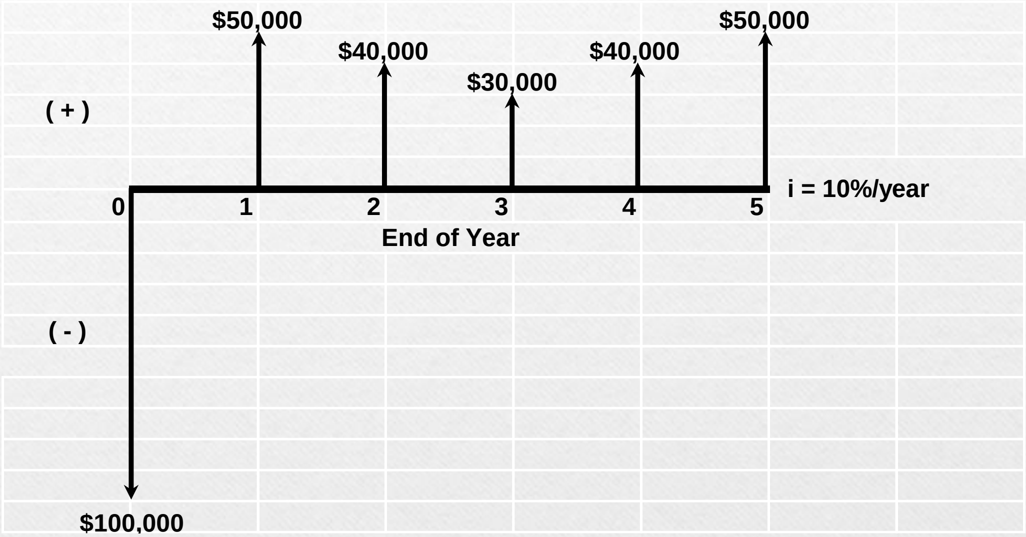
Computing the Future worth of Multiple cash Flows

$$F = \sum_{t=1}^{n} A_t (1+i)^{n-t} \qquad \textbf{(2.15)}$$

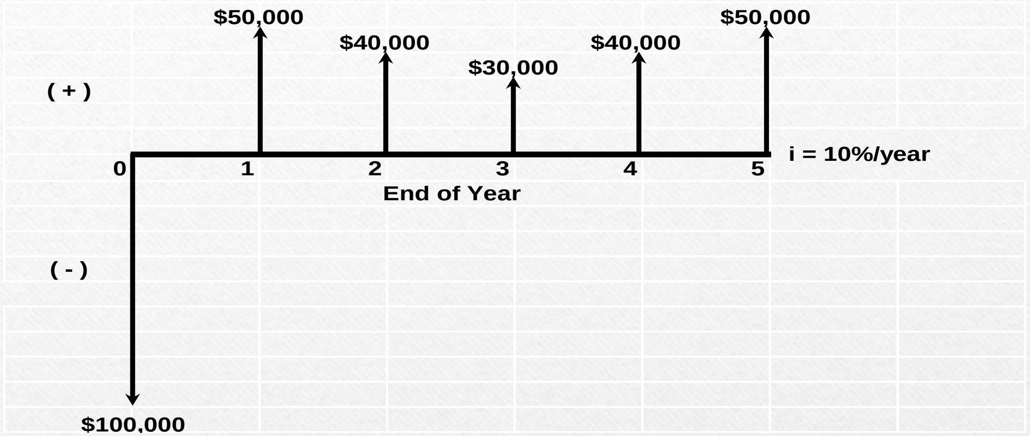
$$F = \sum_{t=1}^{n} A_t (F \mid P \quad i\%, n-t) \qquad \textbf{(2.16)}$$

Example 2.15

Determine the future worth equivalent of the CFD shown below, using an interest rate of 10% compounded annually.



Example 2.15



End of Year (n)	Cash Flow (CF)	(P|F 10%,n)	Present Worth	PV(10%,n,,-CF)	(F|P 10%,5-n)	Future Worth	FV(10%,5-n,,-CF)
0	-$100,000	1.00000	-$100,000.00	-$100,000.00	1.61051	-$161,051.00	-$161,051.00
1	$50,000	0.90909	$45,454.50	$45,454.55	1.46410	$73,205.00	$73,205.00
2	$40,000	0.82645	$33,058.00	$33,057.85	1.33100	$53,240.00	$53,240.00
3	$30,000	0.75131	$22,539.30	$22,539.44	1.21000	$36,300.00	$36,300.00
4	$40,000	0.68301	$27,320.40	$27,320.54	1.10000	$44,000.00	$44,000.00
5	$50,000	0.62092	$31,046.00	$31,046.07	1.00000	$50,000.00	$50,000.00
SUM			$59,418.20	$59,418.45		$95,694.00	$95,694.00

F =10000*FV(10%,5,,-NPV(10%,5,4,3,4,5)+10)

= $95,694.00

Example 2.14 & 2.16

Determine the future worth equivalent of the following series of cash flows. Use an interest rate of 6% per interest period.

End of Period	Cash Flow
0	$0
1	$300
2	$0
3	-$300
4	$200
5	$0
6	$400
7	$0
8	$200

F = $300(F|P 6%,7)-$300(F|P 6%,5)

+$200(F|P 6%,4)+$400(F|P 6%,2)+$200

F = $951.59

F =FV(6%,8,,-NPV(6%,300,0,-300,200,0,400,0,200))

F =$951.56

(The 3¢ difference in the answers is due to round-off error in the tables in Appendix A.)

End of Year (n)	Cash Flow (CF)		
0	$0		
1	$300		
2	$0		
3	-$300		
4	$200		
5	$0		
6	$400		
7	$0		
8	$200		
P =	$597.02		=NPV(6%,C4:C11)
F =	$951.56		=FV(6%,8,,-C12)

Some Common Cash Flow Series

- Uniform Series

$A_t = A \qquad t = 1,\ldots,n$

- Gradient Series

$A_t = 0 \qquad t = 1$

$= A_{t-1}+G \quad t = 2,\ldots,n$

$= (t-1)G \quad t = 1,\ldots,n$

- Geometric Series

$A_t = A \qquad t = 1$

$= A_{t-1}(1+j) \quad t = 2,\ldots,n$

$= A_1(1+j)^{t-1} \quad t = 1,\ldots,n$

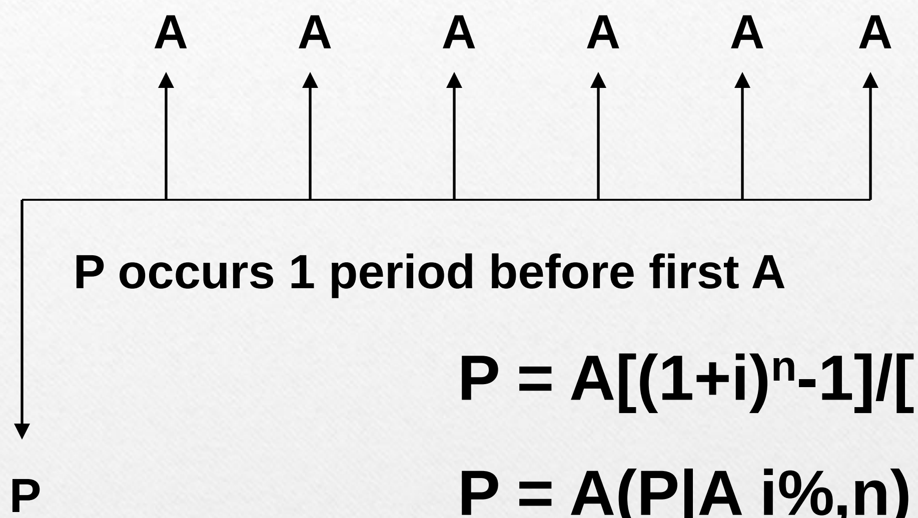
Relationships among P, F, and A

- P occurs at the same time as A_0, i.e., at
t = 0 (one period before the first A in a uniform series)

- F occurs at the same time as A_n, i.e., at
t = n (the same time as the last A in a uniform series)

- Be careful in using the formulas we develop

DCF Uniform Series Formulas



$$P = A[(1+i)^n-1]/[i(1+i)^n]$$

$$P = A(P|A\ i\%,n)$$

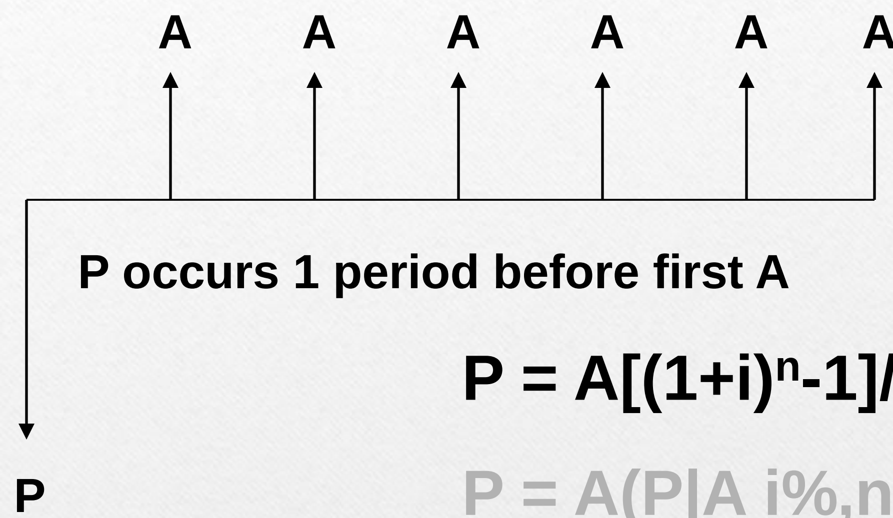
P = [=PV(i%,n,-A)]

$$A = Pi(1+i)^n/[(1+i)^n-1]$$

$$A = P(A|P\ i\%,n)$$

A = [=PMT(i%,n,-P)]

DCF Uniform Series Formulas



$$P = A[(1+i)^n-1]/[i(1+i)^n]$$

P = A(P|A i%,n)

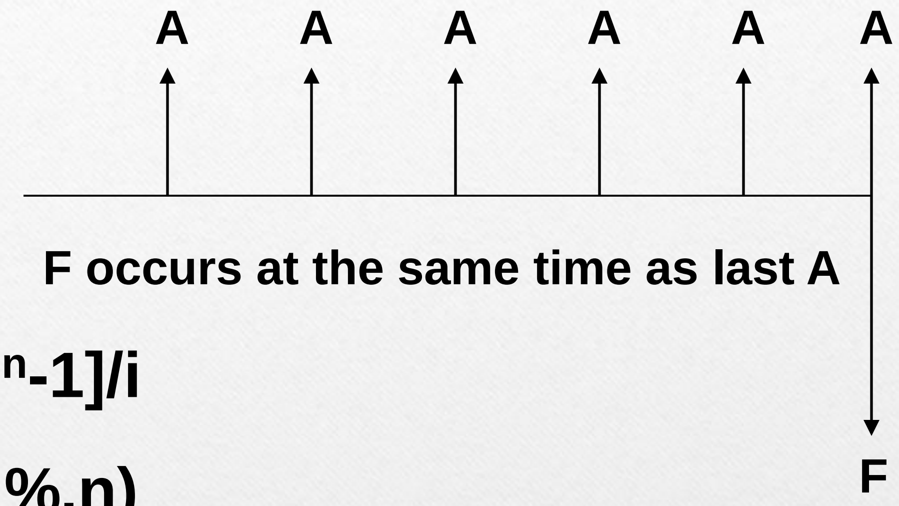
P =PV(i%,n,-A)

$$A = Pi(1+i)^n/[(1+i)^n-1]$$

A = P(A|P i%,n)

A =PMT(i%,n,-P)

DCF Uniform Series Formulas



$F = A[(1+i)^n-1]/i$

$F = A(F|A\ i\%,n)$

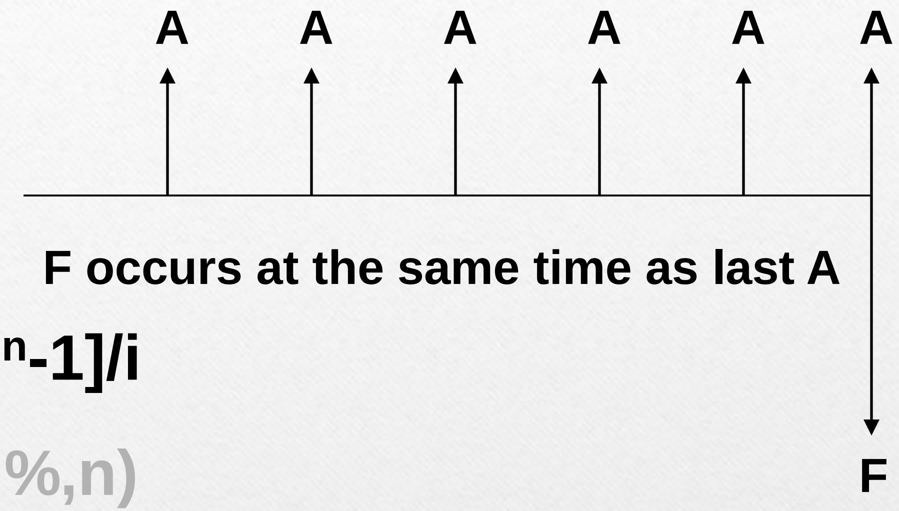
F = [=FV(i%,n,-A)]

$A = Fi/[(1+i)^n-1]$

$A = F(A|F\ i\%,n)$

A = [=PMT(i%,n,,-F)]

DCF Uniform Series Formulas



$F = A[(1+i)^n-1]/i$

F = A(F|A i%,n)

F =FV(i%,n,-A)

$A = Fi/[(1+i)^n-1]$

A = F(A|F i%,n)

A =PMT(i%,n,,-F)

Uniform Series of Cash Flows
Discounted Cash Flow Formulas

$$P = A(P|A\ i\%,n) = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \qquad (2.22)$$

$$A = P(A|P\ i\%,n) = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \qquad (2.25)$$

P occurs one period before the first A

$$F = A(F|A\ i\%,n) = A\left[\frac{(1+i)^n - 1}{i}\right] \qquad (2.28)$$

$$A = F(A|F\ i\%,n) = F\left[\frac{i}{(1+i)^n - 1}\right] \qquad (2.30)$$

F occurs at the same time as the last A

Example 2.17

Troy Long deposits a single sum of money in a savings account that pays 5% compounded annually. How much must he deposit in order to withdraw $2,000/yr for 5 years, with the first withdrawal occurring 1 year after the deposit?

P = $2000(P|A 5%,5)

P = $2000(4.32948) = $8658.96

P =PV(5%,5,-2000)

P = $8658.95

Example 2.17

Troy Long deposits a single sum of money in a savings account that pays 5% compounded annually. How much must he deposit in order to withdraw $2,000/yr for 5 years, with the first withdrawal occurring 1 year after the deposit?

P = $2,000(P|A 5%,5)

P = $2,000(4.32948) = $8,658.96

P =PV(5%,5,-2000)

P = $8658.95

Example 2.17

Troy Long deposits a single sum of money in a savings account that pays 5% compounded annually. How much must he deposit in order to withdraw $2,000/yr for 5 years, with the first withdrawal occurring 1 year after the deposit?

P = $2,000(P|A 5%,5)

P = $2,000(4.32948) = $8,658.96

P =PV(5%,5,-2000)

P = $8,658.95

Example 2.18

Troy Long deposits a single sum of money in a savings account that pays 5% compounded annually. How much must he deposit in order to withdraw \$2,000/yr for 5 years, *with the first withdrawal occurring 3 years after the deposit?*

P = \$2000(P|A 5%,5)(P|F 5%,2)

P = \$2000(4.32948)(0.90703) = \$7853.94

P =PV(5%,2,,-PV(5%,5,-2000))

P = \$7853.93

Example 2.18

Troy Long deposits a single sum of money in a savings account that pays 5% compounded annually. How much must he deposit in order to withdraw $2,000/yr for 5 years, *with the first withdrawal occurring 3 years after the deposit?*

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P = $2,000(4.32948)(0.90703) = $7,853.94

P =PV(5%,2,,-PV(5%,5,-2000))

P = $7853.93

Example 2.18

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P = $2,000(4.32948)(0.90703) = $7,853.94

P =PV(5%,2,,-PV(5%,5,-2000))

P = $7,853.93

Example 2.19

Rachel Townsley invests \$10,000 in a fund that pays 8% compounded annually. If she makes 10 equal annual withdrawals from the fund, how much can she withdraw if the first withdrawal occurs 1 year after her investment?

A = \$10,000(A|P 8%,10)

A = \$10,000(0.14903) = \$1490.30

A =PMT(8%,10,-10000)

A = \$1490.29

Example 2.19

Rachel Townsley invests $10,000 in a fund that pays 8% compounded annually. If she makes 10 equal annual withdrawals from the fund, how much can she withdraw if the first withdrawal occurs 1 year after her investment?

A = $10,000(A|P 8%,10)

A = $10,000(0.14903) = $1,490.30

A =PMT(8%,10,-10000)

A = $1490.29

Example 2.19

Rachel Townsley invests $10,000 in a fund that pays 8% compounded annually. If she makes 10 equal annual withdrawals from the fund, how much can she withdraw if the first withdrawal occurs 1 year after her investment?

A = $10,000(A|P 8%,10)

A = $10,000(0.14903) = $1,490.30

A =PMT(8%,10,-10000)

A = $1,490.29

Example 2.22 *(note the skipping)*

Suppose Rachel delays the first withdrawal for 2 years. How much can be withdrawn each of the 10 years?

A = \$10,000(F|P 8%,2)(A|P 8%,10)

A = \$10,000(1.16640)(0.14903)

A = \$1738.29

A =PMT(8%,10-FV(8%,2,,-10000))

A = \$1738.29

Example 2.22

Suppose Rachel delays the first withdrawal for 2 years. How much can be withdrawn each of the 10 years?

A = $10,000(F|P 8%,2)(A|P 8%,10)

A = $10,000(1.16640)(0.14903)

A = $1,738.29

A =PMT(8%,10-FV(8%,2,,-10000))

A = $1738.29

Example 2.22

Suppose Rachel delays the first withdrawal for 2 years. How much can be withdrawn each of the 10 years?

A = $10,000(F|P 8%,2)(A|P 8%,10)

A = $10,000(1.16640)(0.14903)

A = $1,738.29

A =PMT(8%,10,-FV(8%,2,,-10000))

A = $1,738.29

Example 2.20

A firm borrows $2,000,000 at 12% annual interest and pays it back with 10 equal annual payments. What is the payment?

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A firm borrows $2,000,000 at 12% annual interest and pays it back with 10 equal annual payments. What is the payment?

A = $2,000,000(A | P 12%,10)

A = $2,000,000(0.17698)

A = $353,960

Example 2.20

A firm borrows $2,000,000 at 12% annual interest and pays it back with 10 equal annual payments. What is the payment?

A = $2,000,000(A|P 12%,10)

A = $2,000,000(0.17698)

A = $353,960

A =PMT(12%,10,-2000000)

A = $353,968.33

Example 2.21

Suppose the firm pays back the loan over 15 years in order to obtain a 10% interest rate. What would be the size of the annual payment?

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Suppose the firm pays back the loan over 15 years in order to obtain a 10% interest rate. What would be the size of the annual payment?

$A = \$2{,}000{,}000(A|P\ 10\%,15)$

$A = \$2{,}000{,}000(0.13147)$

$A = \$262{,}940$

Example 2.21

Suppose the firm pays back the loan over 15 years in order to obtain a 10% interest rate. What would be the size of the annual payment?

A = \$2,000,000(A|P 10%,15)

A = \$2,000,000(0.13147)

A = \$262,940

A =PMT(10%,15,-2000000)

A = \$262,947.55

Extending the loan period 5 years reduced the payment by \$91,020.78

Example 2.23

Luis Jimenez deposits \$1,000/yr in a savings account that pays 6% compounded annually. How much will be in the account immediately after his 30th deposit?

F = \$1000(F|A 6%,30)

F = \$1000(79.05819) = \$79,058.19

F =FV(6%,30,-1000)

A = \$78,058.19

Example 2.23

Luis Jimenez deposits $1,000/yr in a savings account that pays 6% compounded annually. How much will be in the account immediately after his 30th deposit?

F = $1,000(F|A 6%,30)

F = $1,000(79.05819) = $79,058.19

F =FV(6%,30,-1000)

A = $78,058.19

Example 2.23

Luis Jimenez deposits $1,000/yr in a savings account that pays 6% compounded annually. How much will be in the account immediately after his 30th deposit?

F = $1,000(F|A 6%,30)

F = $1,000(79.05819) = $79,058.19

F =FV(6%,30,-1000)

A = $79,058.19

Example 2.24

Andrew Brewer invests $5,000/yr and earns 6% compounded annually. How much will he have in his investment portfolio after 15 yrs? 20 yrs? 25 yrs? 30 yrs? (What if he earns 3%/yr?)

F = $5000(F|A 6%,15) = $5000(23.27597) = $116,379.85

F = $5000(F|A 6%,20) = $5000(36.78559) = $183,927.95

F = $5000(F|A 6%,25) = $5000(54.86451) = $274,322.55

F = $5000(F|A 6%,30) = $5000(79.05819) = $395,290.95

F = $5000(F|A 3%,15) = $5000(18.59891) = $92,994.55

F = $5000(F|A 3%,20) = $5000(26.87037) = $134,351.85

F = $5000(F|A 3%,25) = $5000(36.45926) = $182,296.30

F = $5000(F|A 3%,30) = $5000(47.57542) = $237,877.10

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Twice the time at half the rate is best! $(1 + i)^n$

Example 2.24

Andrew Brewer invests $5,000/yr and earns 6% compounded annually. How much will he have in his investment portfolio after 15 yrs? 20 yrs? 25 yrs? 30 yrs? (What if he earns 3%/yr?)

F =FV(6%,15,-5000) = $116,379.85

F =FV(6%,20,-5000) = $183,927.96

F =FV(6%,25,-5000) = $274,322.56

F =FV(6%,30,-5000) = $395,290.93

F =FV(3%,15,-5000) = $92,994.57

F =FV(3%,20,-5000) = $134,351.87

F =FV(3%,25,-5000) = $182,296.32

F =FV(3%,30,-5000) = $237,877.08

Example 2.24

Andrew Brewer invests $5,000/yr and earns 6% compounded annually. How much will he have in his investment portfolio after 15 yrs? 20 yrs? 25 yrs? 30 yrs? (What if he earns 3%/yr?)

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F =FV(3%,20,-5000) = $134,351.87

F =FV(3%,25,-5000) = $182,296.32

F =FV(3%,30,-5000) = $237,877.08

Twice the time at half the rate is best! $(1 + i)^n$

Example 2.25

If Coby Durham earns 7% on his investments, how much must he invest annually in order to accumulate $1,500,000 in 25 years?

A = $1,500,000(A|F 7%,25)

A = $1,500,000(0.01581)

A = $23,715

A =PMT(7%,25,,-1500000)

A = $23,715.78

Example 2.25

If Coby Durham earns 7% on his investments, how much must he invest annually in order to accumulate $1,500,000 in 25 years?

A = $1,500,000(A|F 7%,25)

A = $1,500,000(0.01581)

A = $23,715

A =PMT(7%,25,,-1500000)

A = $23,715.78

Example 2.25

If Coby Durham earns 7% on his investments, how much must he invest annually in order to accumulate $1,500,000 in 25 years?

A = $1,500,000(A|F 7%,25)

A = $1,500,000(0.01581)

A = $23,715

A =PMT(7%,25,,-1500000)

A = $23,715.78

Example 2.26

If Crystal Wilson earns 10% on her investments, how much must she invest annually in order to accumulate $1,000,000 in 40 years?

A = $1,000,000(A|F 10%,40)

A = $1,000,000(0.0022594)

A = $2259.40

A =PMT(10%,40,,-1000000)

A = $2259.41

Example 2.26

If Crystal Wilson earns 10% on her investments, how much must she invest annually in order to accumulate $1,000,000 in 40 years?

A = $1,000,000(A|F 10%,40)

A = $1,000,000(0.0022594)

A = $2,259.40

A =PMT(10%,40,,-1000000)

A = $2259.41

Example 2.26

If Crystal Wilson earns 10% on her investments, how much must she invest annually in order to accumulate \$1,000,000 in 40 years?

A = \$1,000,000(A|F 10%,40)

A = \$1,000,000(0.0022594)

A = \$2,259.40

A =PMT(10%,40,,-1000000)

A = \$2,259.41

Example 2.27

$500,000 is spent for a SMP machine in order to reduce annual expenses by $92,500/yr. At the end of a 10-year planning horizon, the SMP machine is worth $50,000. Based on a 10% TVOM, a) what single sum at t = 0 is equivalent to the SMP investment? b) what single sum at t = 10 is equivalent to the SMP investment? c) what uniform annual series over the 10-year period is equivalent to the SMP investment?

Example 2.27 (Solution)

P = -\$500,000 + \$92,500(P|A 10%,10) + \$50,000(P|F 10%,10)

P = -\$500,000 + \$92,500(6.14457) + \$50,000(0.38554)

P = \$87,649.73

P =PV(10%,10,-92500,-50000)-500000

P = \$87,649.62(Chapter 5)

F = -\$500,000(F|P 10%,10) + \$92,500(F|A 10%,10) + \$50,000

F = -\$500,000(2.59374) + \$92,500(15.93742) + \$50,000

F = \$227,341.40

F =FV(10%,10,-92500,500000)+50000

F = \$227,340.55 (Chapter 6)

Example 2.27 (Solution)

A = -$500,000(A|P 10%,10) + $92,500 + $50,000(A|F 10%,10)

A = -$500,000(0.16275) + $92,500 + $50,000(0.06275)

A = $14,262.50

A =PMT(10%,10,500000,-50000)+92500

A = $14,264.57 (Chapter 7)

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

uniform series, present worth factor

= $A(P|A\ i\%,n)$ =PV($i\%,n,-A$)

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

uniform series, capital recovery factor

= $P(A|P\ i\%,n)$ =PMT($i\%,n,-P$)

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

uniform series, future worth factor

= $A(F|A\ i\%,n)$ =FV($i\%,n,-A$)

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

uniform series, sinking fund factor

= $F(A|F\ i\%,n)$ =PMT($i\%,n,,-F$)