(HW4)

Table 27.1

Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] $(\Omega \cdot m)$	Temperature Coefficient[b] $\alpha[(°C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C.

1. In a particular cathode ray tube, the measured beam current is 30.0 μA. How many electrons strike the tube screen every 40.0 s?

$$I = \frac{\Delta Q}{\Delta t} \qquad \Delta Q = I\Delta t = \left(30.0 \times 10^{-6}\ \text{A}\right)\left(40.0\ \text{s}\right) = 1.20 \times 10^{-3}\ \text{C}$$

$$N = \frac{Q}{e} = \frac{1.20 \times 10^{-3}\ \text{C}}{1.60 \times 10^{-19}\ \text{C/electron}} = \boxed{7.50 \times 10^{15}\ \text{electrons}}$$

11. An aluminum wire having a cross-sectional area of 4.00×10^{-6} m^2 carries a current of 5.00 A. Find the drift speed of the electrons in the wire. The density of aluminum is 2.70 g/cm^3. Assume that one conduction electron is supplied by each atom.

We use $I = nqAv_d$ n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume. We assume a contribution of 1 free electron per atom in the relationship above. For aluminum, which has a molar mass of 27, we know that Avogadro's number of atoms, N_A, has a mass of 27.0 g. Thus, the mass per atom is

$$\frac{27.0 \text{ g}}{N_A} = \frac{27.0 \text{ g}}{6.02 \times 10^{23}} = 4.49 \times 10^{-23} \text{ g/atom}.$$

Thus, $$n = \frac{\text{density of aluminum}}{\text{mass per atom}} = \frac{2.70 \text{ g/cm}^3}{4.49 \times 10^{-23} \text{ g/atom}}$$

$$n = 6.02 \times 10^{22} \text{ atoms/cm}^3 = 6.02 \times 10^{28} \text{ atoms/m}^3 .$$

Therefore, $$v_d = \frac{I}{nqA} = \frac{5.00 \text{ A}}{\left(6.02 \times 10^{28} \text{ m}^{-3}\right)\left(1.60 \times 10^{-19} \text{ C}\right)\left(4.00 \times 10^{-6} \text{ m}^2\right)} = 1.30 \times 10^{-4} \text{ m/s}$$

or, $$v_d = \boxed{0.130 \text{ mm/s}}.$$

15. A 0.900-V potential difference is maintained across a 1.50-m length of tungsten wire that has a cross-sectional area of 0.600 mm^2. What is the current in the wire?

$\Delta V = IR$

and $\quad R = \dfrac{\rho \ell}{A}: \qquad A = (0.600 \text{ mm})^2 \left(\dfrac{1.00 \text{ m}}{1\,000 \text{ mm}} \right)^2 = 6.00 \times 10^{-7} \text{ m}^2$

$$\Delta V = \frac{I \rho \ell}{A}: \qquad I = \frac{\Delta V A}{\rho \ell} = \frac{(0.900 \text{ V})(6.00 \times 10^{-7} \text{ m}^2)}{(5.60 \times 10^{-8} \ \Omega \cdot \text{m})(1.50 \text{ m})}$$

$$I = \boxed{6.43 \text{ A}}$$

17. Suppose that you wish to fabricate a uniform wire out of 1.00 g of copper. If the wire is to have a resistance of $R = 0.500\ \Omega$, and if all of the copper is to be used, what will be (a) the length and (b) the diameter of this wire?

(a) Given $M = \rho_d V = \rho_d A\ell$ where $\rho_d \equiv$ mass density,

we obtain: $A = \dfrac{M}{\rho_d \ell}$. Taking $\rho_r \equiv$ resistivity, $R = \dfrac{\rho_r \ell}{A} = \dfrac{\rho_r \ell}{M/\rho_d \ell} = \dfrac{\rho_r \rho_d \ell^2}{M}$.

Thus, $\ell = \sqrt{\dfrac{MR}{\rho_r \rho_d}} = \sqrt{\dfrac{(1.00\times10^{-3})(0.500)}{(1.70\times10^{-8})(8.92\times10^{3})}}$ $\ell = \boxed{1.82\text{ m}}$.

(b) $V = \dfrac{M}{\rho_d}$, or $\pi r^2 \ell = \dfrac{M}{\rho_d}$.

Thus, $r = \sqrt{\dfrac{M}{\pi \rho_d \ell}} = \sqrt{\dfrac{1.00\times10^{-3}}{\pi(8.92\times10^{3})(1.82)}}$ $r = 1.40\times10^{-4}\text{ m}$.

The diameter is twice this distance: diameter $= \boxed{280\ \mu\text{m}}$.

31. An aluminum wire with a diameter of 0.100 mm has a uniform electric field of 0.200 V/m imposed along its entire length. The temperature of the wire is 50.0°C. Assume one free electron per atom. (a) Use the information in Table 27.1 and determine the resistivity. (b) What is the current density in the wire? (c) What is the total current in the wire? (d) What is the drift speed of the conduction electrons? (e) What potential difference must exist between the ends of a 2.00-m length of the wire to produce the stated electric field?

(a) $$\rho = \rho_0\left[1+\alpha(T-T_0)\right] = \left(2.82\times10^{-8}\ \Omega\cdot\text{m}\right)\left[1+3.90\times10^{-3}(30.0^\circ)\right] = \boxed{3.15\times10^{-8}\ \Omega\cdot\text{m}}$$

(b) $$J = \frac{E}{\rho} = \frac{0.200\ \text{V/m}}{3.15\times10^{-8}\ \Omega\cdot\text{m}} = \boxed{6.35\times10^{6}\ \text{A/m}^2}$$

(c) $$I = JA = J\left(\frac{\pi d^2}{4}\right) = \left(6.35\times10^{6}\ \text{A/m}^2\right)\left[\frac{\pi\left(1.00\times10^{-4}\ \text{m}\right)^2}{4}\right] = \boxed{49.9\ \text{mA}}$$

(d) $$n = \frac{6.02\times10^{23}\ \text{electrons}}{\left[26.98\ \text{g}/\left(2.70\times10^{6}\ \text{g/m}^3\right)\right]} = 6.02\times10^{28}\ \text{electrons/m}^3$$

$$v_d = \frac{J}{ne} = \frac{\left(6.35\times10^{6}\ \text{A/m}^2\right)}{\left(6.02\times10^{28}\ \text{electrons/m}^3\right)\left(1.60\times10^{-19}\ \text{C}\right)} = \boxed{659\ \mu\text{m/s}}$$

(e) $$\Delta V = E\ell = (0.200\ \text{V/m})(2.00\ \text{m}) = \boxed{0.400\ \text{V}}$$

35. The temperature of a sample of tungsten is raised while a sample of copper is maintained at 20.0°C. At what temperature will the resistivity of the tungsten be four times that of the copper?

$\rho = \rho_0(1 + \alpha\Delta T)$ or $$\Delta T_W = \frac{1}{\alpha_W}\left(\frac{\rho_W}{\rho_{0W}} - 1\right)$$

Require that $\rho_W = 4\rho_{0_{Cu}}$ so that $$\Delta T_W = \left(\frac{1}{4.50\times10^{-3}/°C}\right)\left(\frac{4(1.70\times10^{-8})}{5.60\times10^{-8}} - 1\right) = 47.6°C.$$

Therefore, $$T_W = 47.6°C + T_0 = \boxed{67.6°C}.$$

39. What is the required resistance of an immersion heater that increases the temperature of 1.50 kg of water from 10.0°C to 50.0°C in 10.0 min while operating at 110 V?

The heat that must be added to the water is

$$Q = mc\Delta T = (1.50\text{ kg})(4\,186\text{ J/kg°C})(40.0\text{°C}) = 2.51 \times 10^5\text{ J}.$$

Thus, the power supplied by the heater is

$$\mathscr{P} = \frac{W}{\Delta t} = \frac{Q}{\Delta t} = \frac{2.51 \times 10^5\text{ J}}{600\text{ s}} = 419\text{ W}$$

and the resistance is $R = \dfrac{(\Delta V)^2}{\mathscr{P}} = \dfrac{(110\text{ V})^2}{419\text{ W}} = \boxed{28.9\ \Omega}$.

41. Suppose that a voltage surge produces 140 V for a moment. By what percentage does the power output of a 120-V, 100-W lightbulb increase? Assume that its resistance does not change.

$$\frac{\mathcal{P}}{\mathcal{P}_0} = \frac{(\Delta V)^2/R}{(\Delta V_0)^2/R} = \left(\frac{\Delta V}{\Delta V_0}\right)^2 = \left(\frac{140}{120}\right)^2 = 1.361$$

$$\Delta\% = \left(\frac{\mathcal{P} - \mathcal{P}_0}{\mathcal{P}_0}\right)(100\%) = \left(\frac{\mathcal{P}}{\mathcal{P}_0} - 1\right)(100\%) = (1.361 - 1)100\% = \boxed{36.1\%}$$

51. A certain toaster has a heating element made of Nichrome wire. When the toaster is first connected to a 120-V source (and the wire is at a temperature of 20.0°C), the initial current is 1.80 A. However, the current begins to decrease as the heating element warms up. When the toaster reaches its final operating temperature, the current drops to 1.53 A. (a) Find the power delivered to the toaster when it is at its operating temperature. (b) What is the final temperature of the heating element?

At operating temperature,

(a) $\mathcal{P} = I\Delta V = (1.53\ \text{A})(120\ \text{V}) = \boxed{184\ \text{W}}$

(b) Use the change in resistance to find the final operating temperature of the toaster.

$R = R_0(1 + \alpha\Delta T)$ $\qquad \dfrac{120}{1.53} = \dfrac{120}{1.80}\left[1 + \left(0.400 \times 10^{-3}\right)\Delta T\right]$

$\Delta T = 441°\text{C}$ $\qquad T = 20.0°\text{C} + 441°\text{C} = \boxed{461°\text{C}}$