Logic and Computer Design Fundamentals

Chapter 5 – Sequential Circuits

Part 2 - Sequential Circuit Analysis

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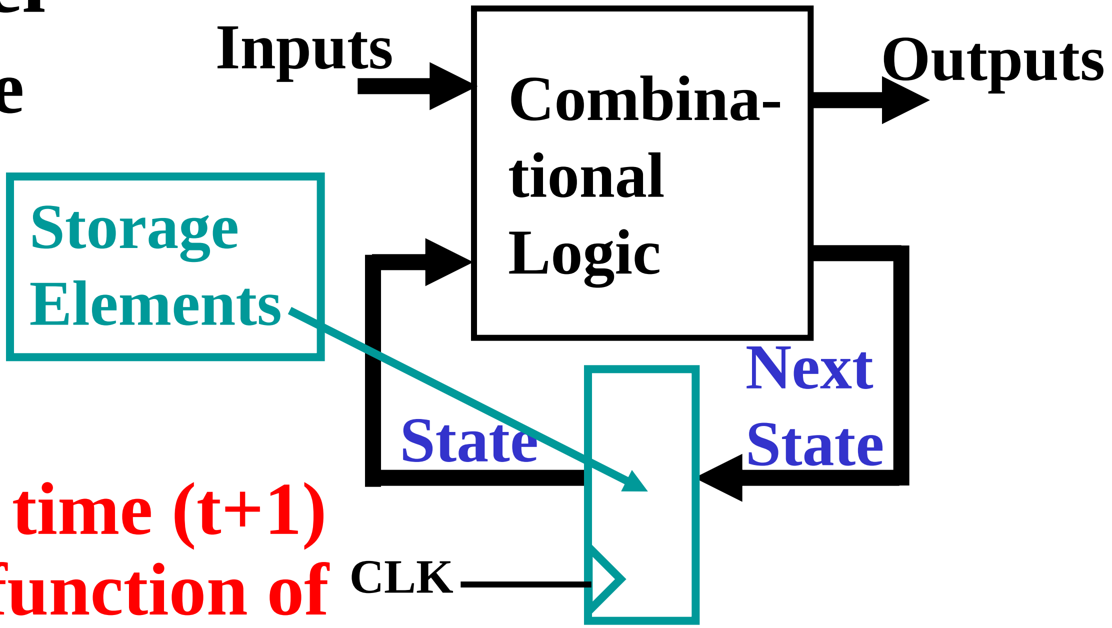
(Hyperlinks are active in View Show mode)

Overview

- Part 1 - Storage Elements
- Part 2 - Sequential Circuit Analysis
  - State tables
  - State diagrams
  - Equivalent states
- Part 3 - Sequential Circuit Design
- Part 4 – State Machine Design

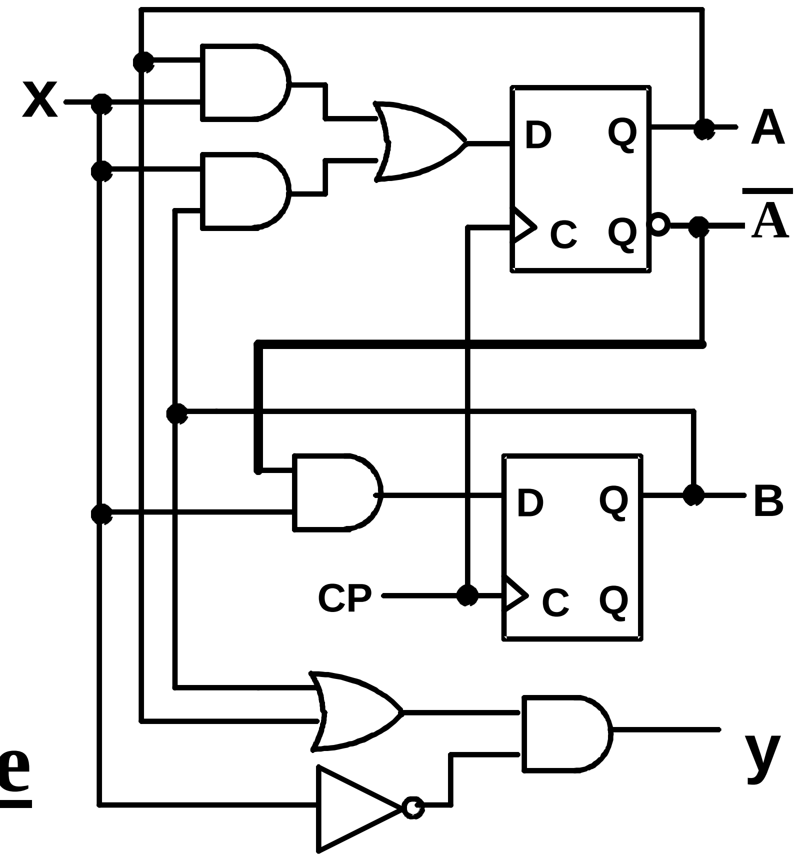
Sequential Circuit Analysis

- **General Model**
  - **Current State at time (t) is stored in an array of flip-flops.**
  - **Next State at time (t+1) is a Boolean function of State and Inputs.**
  - **Outputs at time (t) are a Boolean function of State (t) and (sometimes) Inputs (t).**



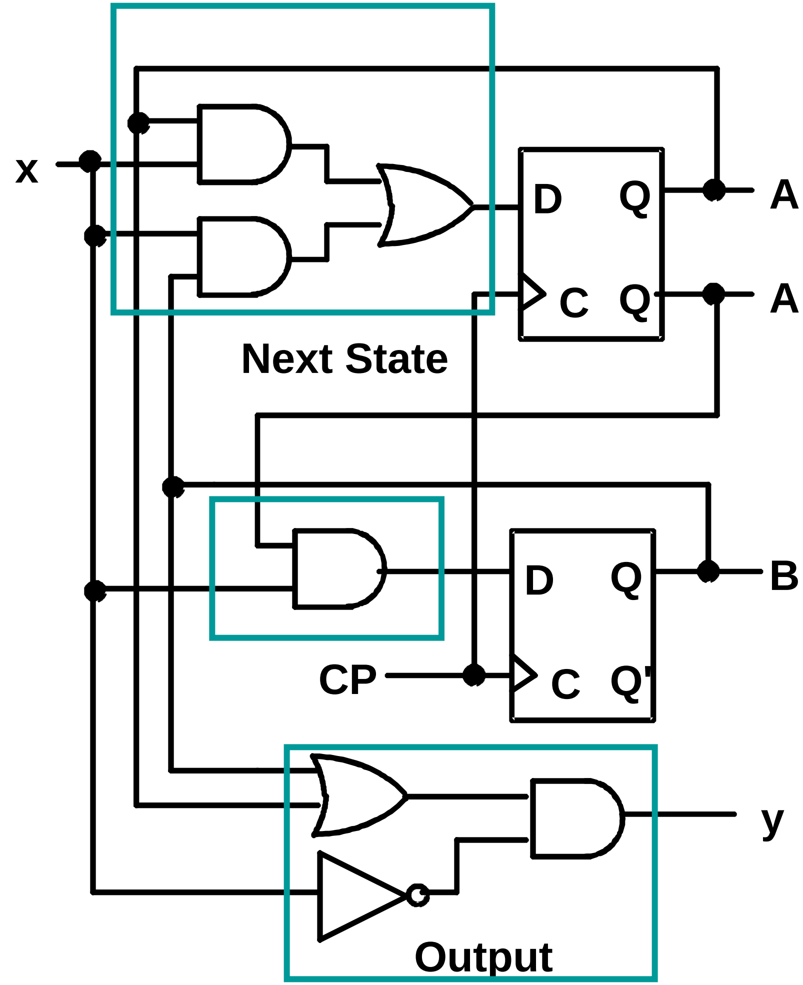
Example 1 (from Fig. 5-15)

- **Input:** x(t)
- **Output:** y(t)
- **State:** (A(t), B(t))
- **What is the Output Function?**
- **What is the Next State Function?**



Example 1 (from Fig. 5-15) (continued)

- **Boolean equations for the functions:**
  - $A(t+1) = A(t)x(t) + B(t)x(t)$
  - $B(t+1) = \bar{A}(t)x(t)$
  - $y(t) = \bar{x}(t)(B(t) + A(t))$



Example 1(from Fig. 5-15) (continued)

- Where in time are inputs, outputs and states defined?
  - $A(t+1) = A(t)x(t) + B(t)x(t)$
  - $B(t+1) = \bar{A}(t)x(t)$
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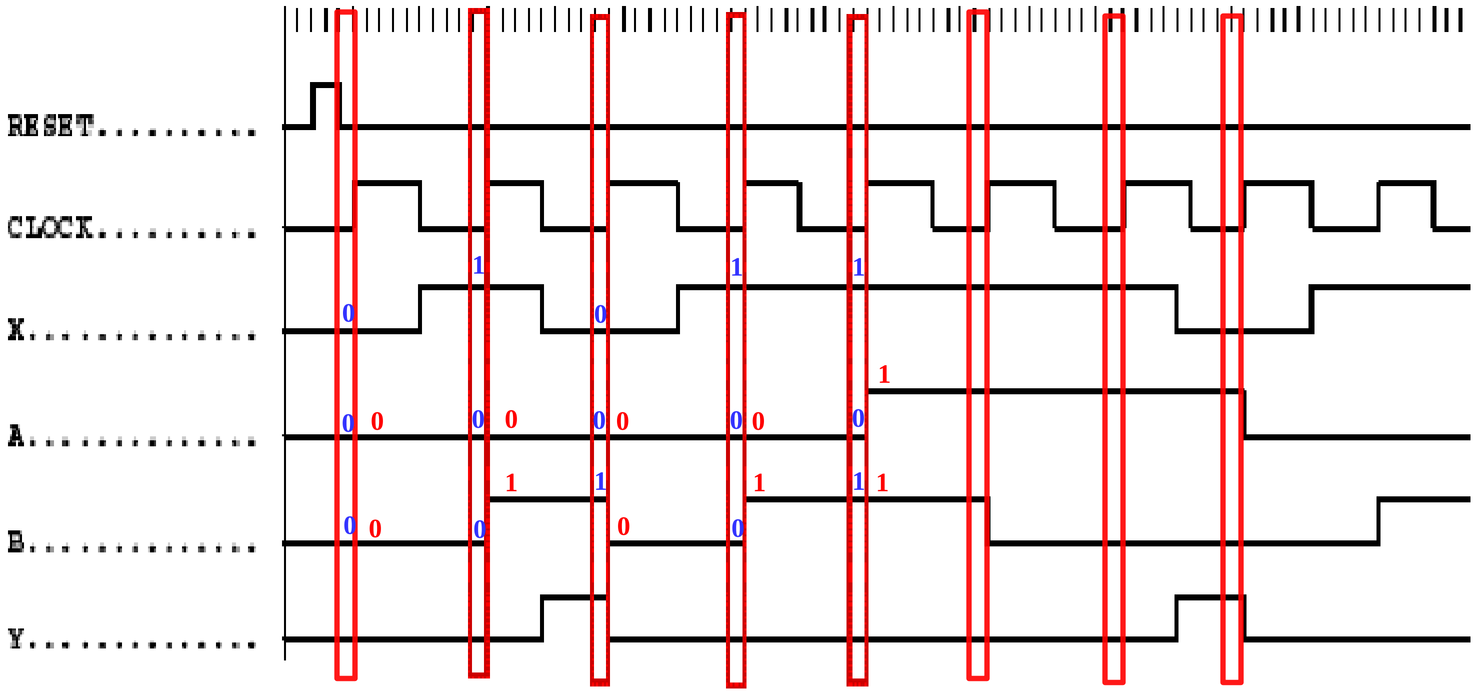
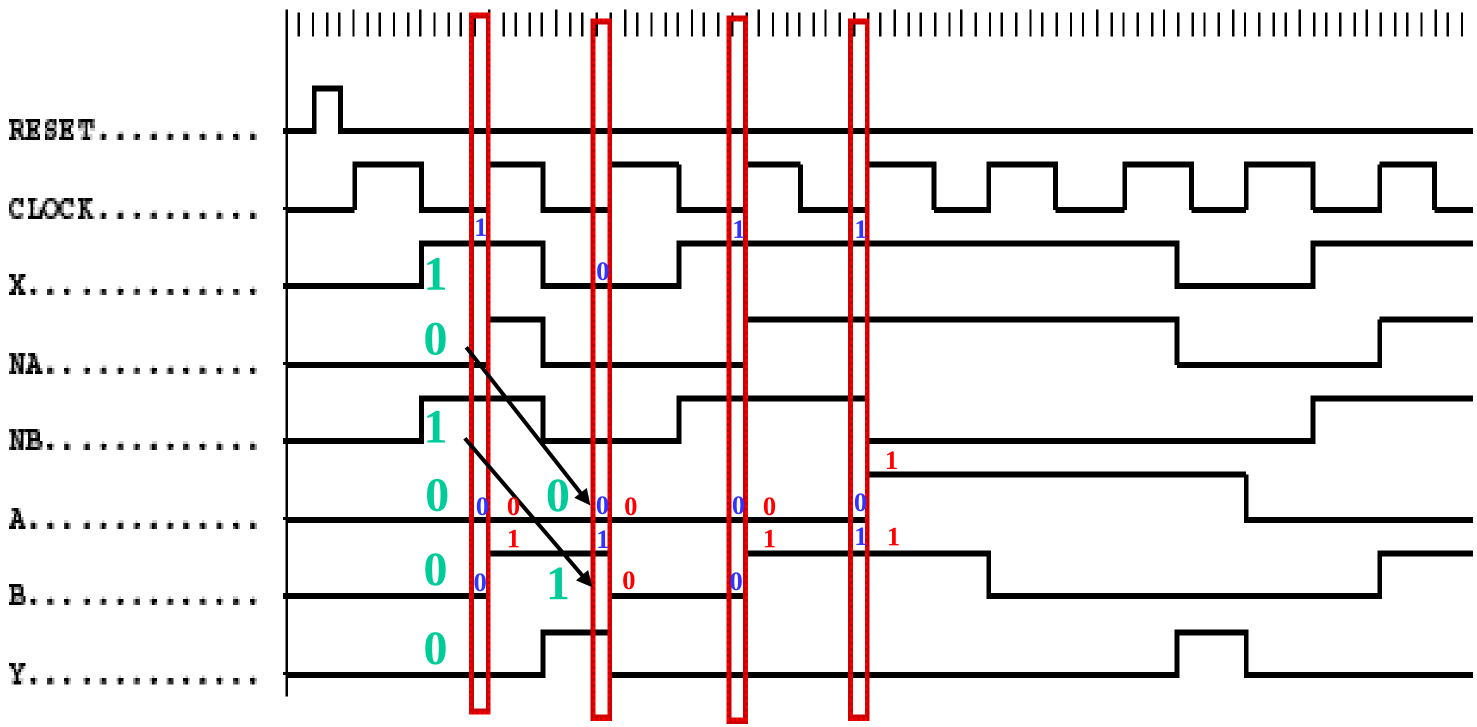


Example 1(from Fig. 5-15) (continued)

- **Where in time are inputs, outputs and states defined?**

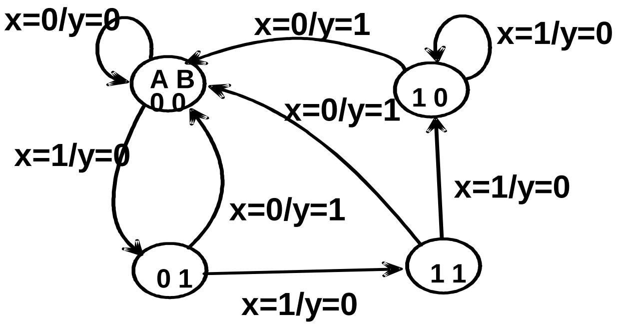


State Table Characteristics

- ***State table*** **– a multiple variable table with the following four sections:**
  - ***Present State*** **– the values of the state variables for each allowed state.**
  - ***Input*** **– the input combinations allowed.**
  - ***Next-state*** **– the value of the state at time (t+1) based on the present state and the input.**
  - ***Output*** **– the value of the output as a function of the present state and (sometimes) the input.**
- **From the viewpoint of a truth table:**
  - **the inputs are Input, Present State**
  - **and the outputs are Output, Next State**

Example 1: State Table (from Fig. 5-15)

- The state table can be filled in using the next state and output equations:
  - $A(t+1) = A(t)x(t) + B(t)x(t)$
  - $B(t+1) = \overline{A}(t)x(t)$
  - $y(t) = \overline{x}(t)(B(t) + A(t))$



Present State		Input	Next State		Output
A(t)	B(t)	x(t)	A(t+1)	B(t+1)	y(t)
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

Example 1: Alternate State Table

- **2-dimensional table that matches well to a K-map. Present state rows and input columns in Gray code order.**
  - $A(t+1) = A(t)x(t) + B(t)x(t)$
  - $B(t+1) = \overline{A}(t)x(t)$
  - $y(t) = \overline{x}(t)(B(t) + A(t))$

Present State	Next State		Output	
	x(t)=0	x(t)=1	x(t)=0	x(t)=1
A(t) B(t)	A(t+1)B(t+1)	A(t+1)B(t+1)	y(t)	y(t)
0 0	0 0	0 1	0	0
0 1	0 0	1 1	1	0
1 0	0 0	1 0	1	0
1 1	0 0	1 0	1	0

State Diagrams

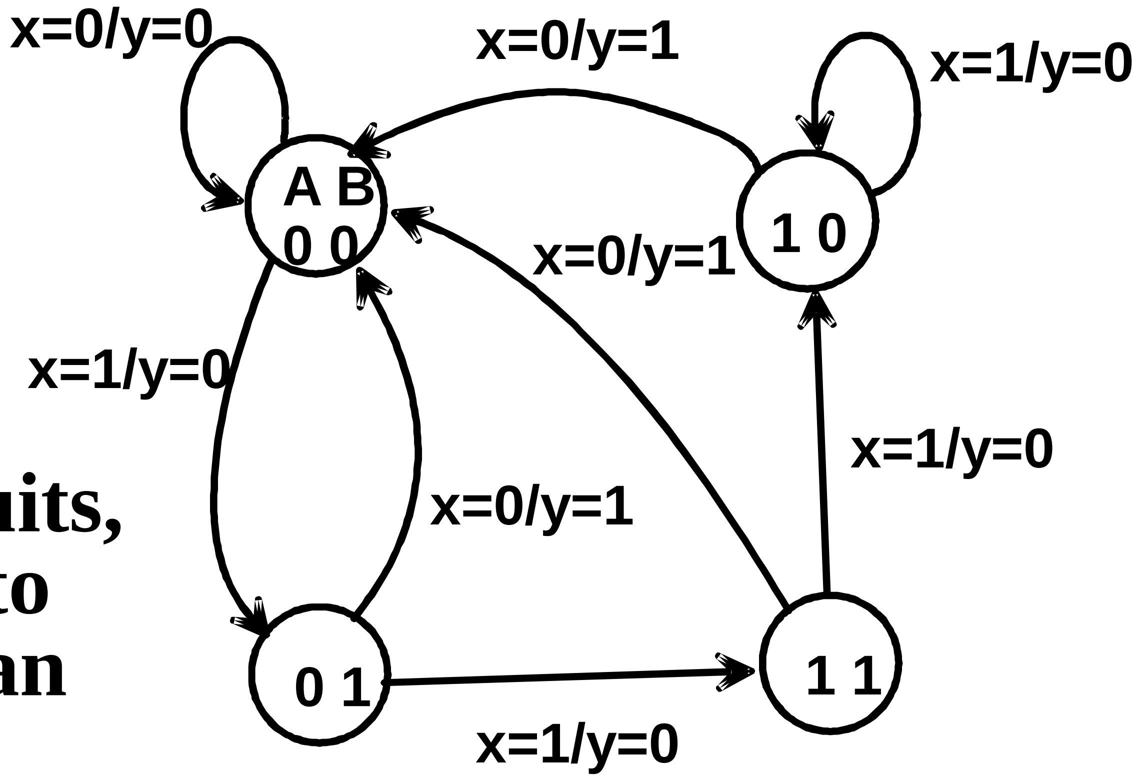
- **The sequential circuit function can be represented in graphical form as a state diagram with the following components:**
  - **A circle with the state name in it for each state**
  - **A directed arc from the Present State to the Next State for each state transition**
  - **A label on each directed arc with the Input values which causes the state transition, and**
  - **A label:**
    - **On each circle with the output value produced, or**
    - **On each directed arc with the output value produced.**

State Diagrams

- **Label form:**
  - **On <u>circle</u> with output included:**
    - **state/output**
    - **Moore type output depends only on state**
  - **On <u>directed arc</u> with the <u>output</u> included:**
    - **input/output**
    - **Mealy type output depends on state and input**

Example 1: State Diagram

- Which type?
- Diagram gets confusing for large circuits
- For small circuits, usually easier to understand than the state table

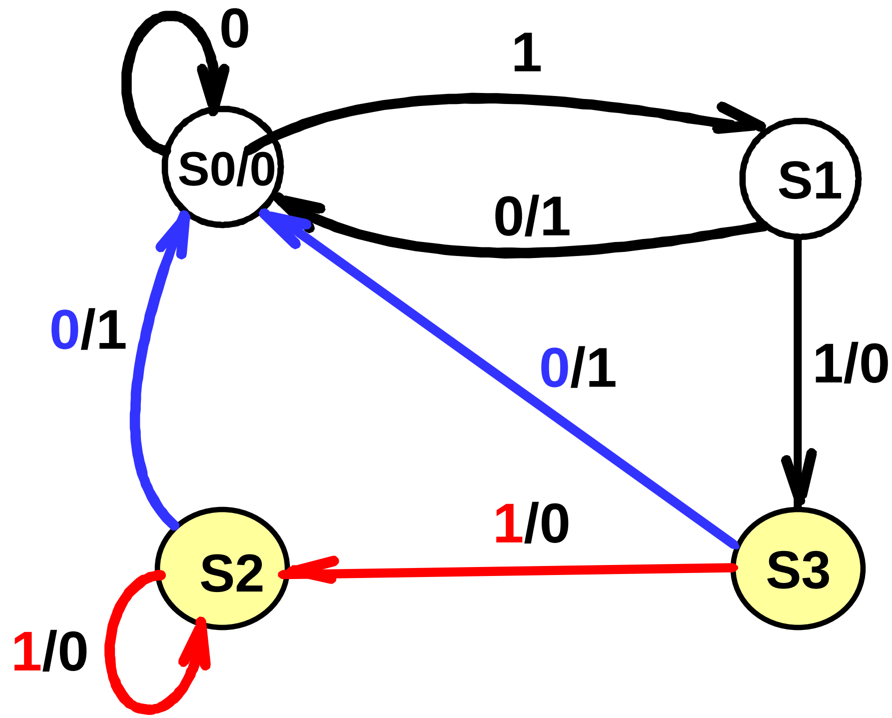


Equivalent State Definitions

- **Two states are *equivalent* if their response for each possible input sequence is an identical output sequence.**
- **Alternatively, two states are *equivalent* if their outputs produced for each input symbol is identical and their next states for each input symbol are the same or equivalent.**

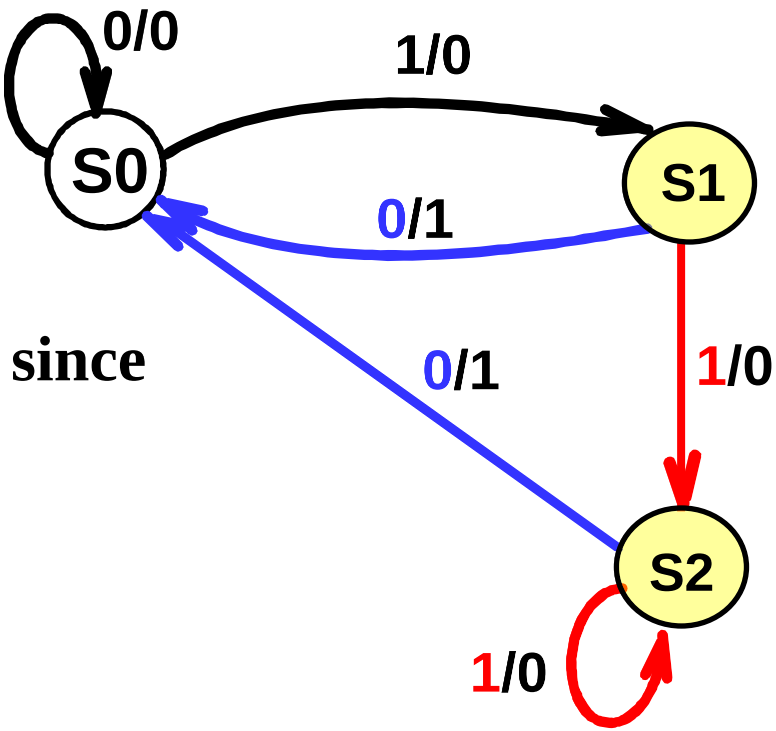
Equivalent State Example

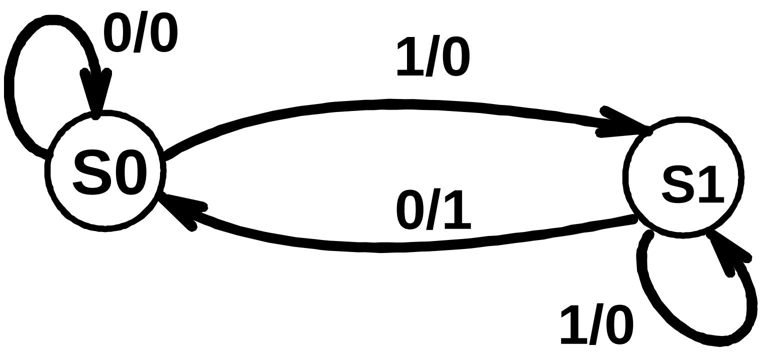
- Text Figure 5-17(a):
- For states S3 and S2,
  - the output for input 0 is 1 and input 1 is 0, and
  - the next state for input 0 is S0 and for input 1 is S2.
  - By the alternative definition, states S3 and S2 are equivalent.



Equivalent State Example

- Replacing S3 and S2 by a single state gives state diagram:
- Examining the new diagram, states S1 and S2 are equivalent since
  - their outputs for input 0 is 1 and input 1 is 0, and
  - their next state for input 0 is S0 and for input 1 is S2,
- Replacing S1 and S2 by a single state gives state diagram:



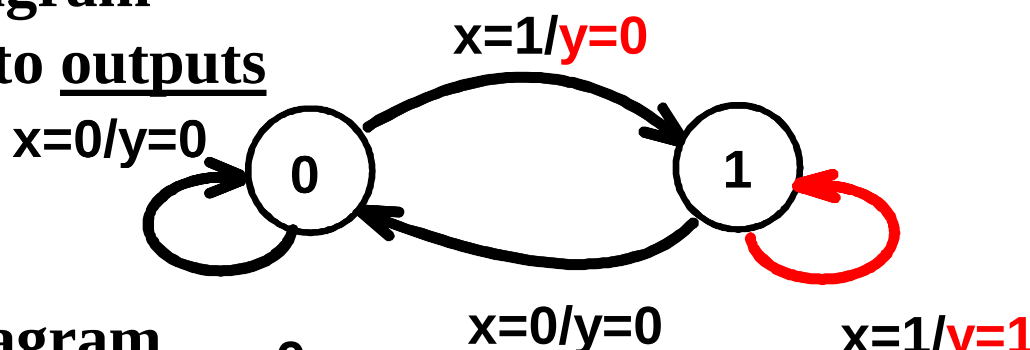


Moore and Mealy Models

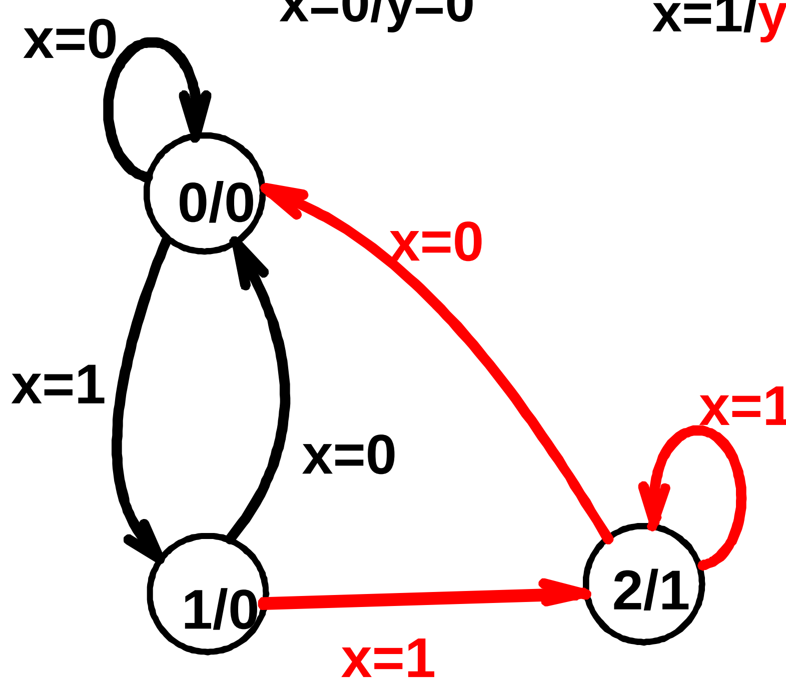
- **Sequential Circuits or Sequential Machines are also called *Finite State Machines* (FSMs). Two formal models exist:**
  - **Moore Model**
    - **Named after E.F. Moore**
    - **Outputs are a function ONLY of states**
    - **Usually specified on the states.**
  - **Mealy Model**
    - **Named after G. Mealy**
    - **Outputs are a function of inputs AND states**
    - **Usually specified on the state transition arcs.**

Moore and Mealy Example Diagrams

- **Mealy Model State Diagram** maps <u>inputs and state</u> to <u>outputs</u>



- **Moore Model State Diagram** maps <u>states</u> to <u>outputs</u>



Moore and Mealy Example Tables

- **Moore Model** state table maps state to outputs

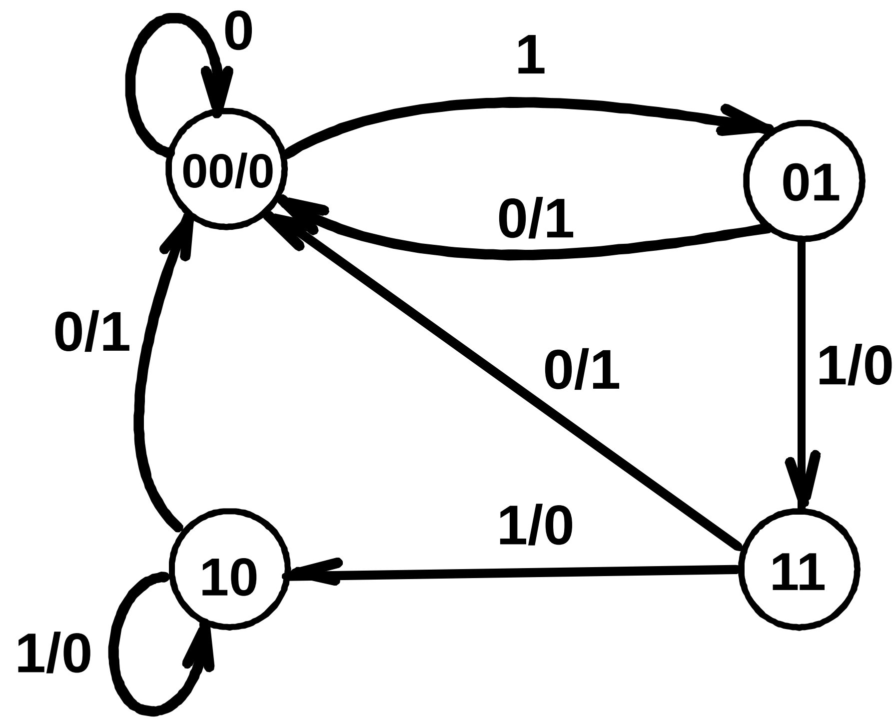
Present State	Next State x=0	Next State x=1	Output
0	0	1	0
1	0	2	0
2	0	2	1

- **Mealy Model** state table maps inputs and state to outputs

Present State	Next State x=0	Next State x=1	Output x=0	Output x=1
0	0	1	0	0
1	0	1	0	1

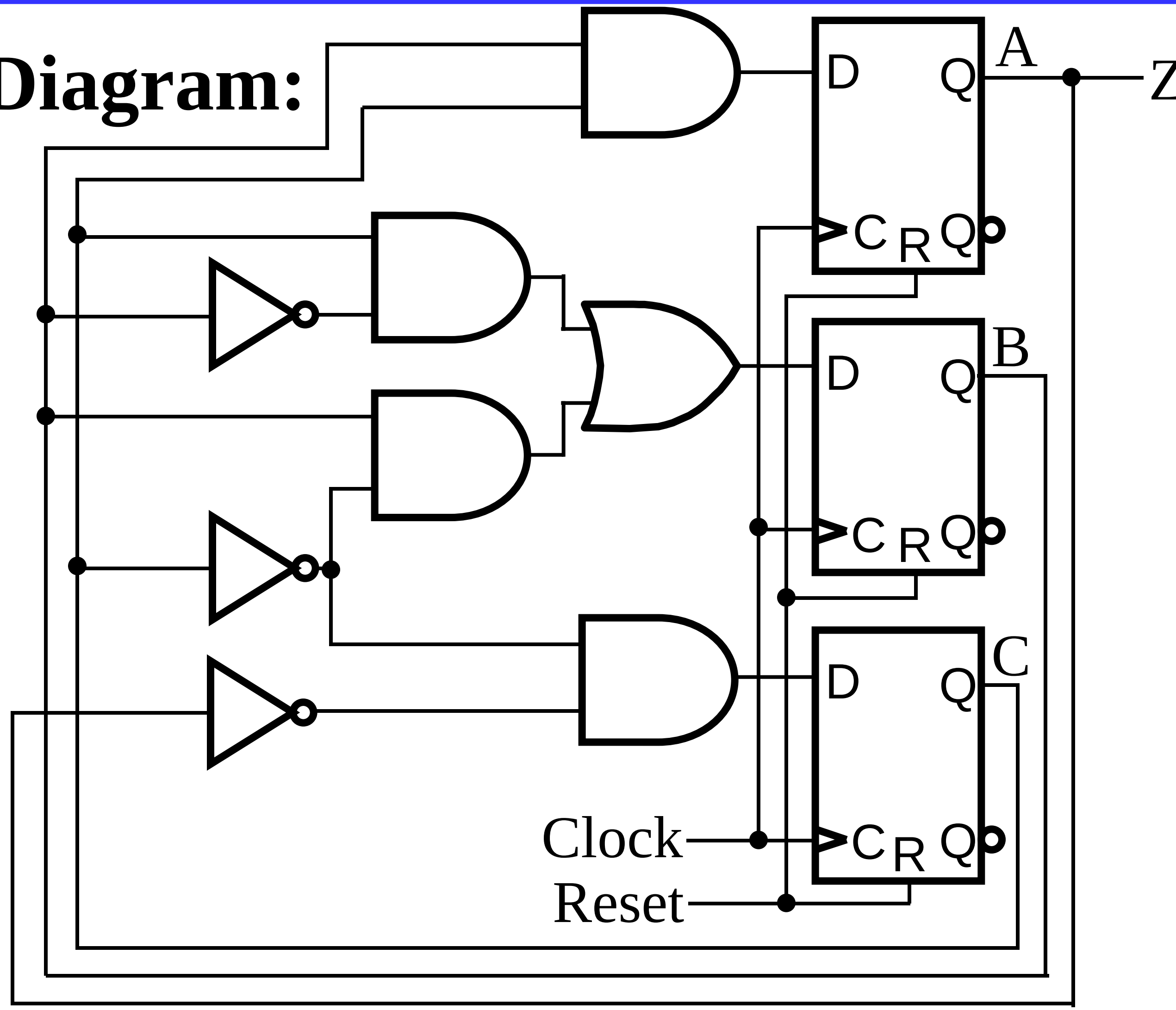
Mixed Moore and Mealy Outputs

- In real designs, some outputs may be Moore type and other outputs may be Mealy type.
- Example: Figure 5-17(a) can be modified to illustrate this
  - State 00: Moore
  - States 01, 10, and 11: Mealy
- Simplifies output specification



Example 2: Sequential Circuit Analysis

- **Logic Diagram:**



Example 2: Flip-Flop Input Equations

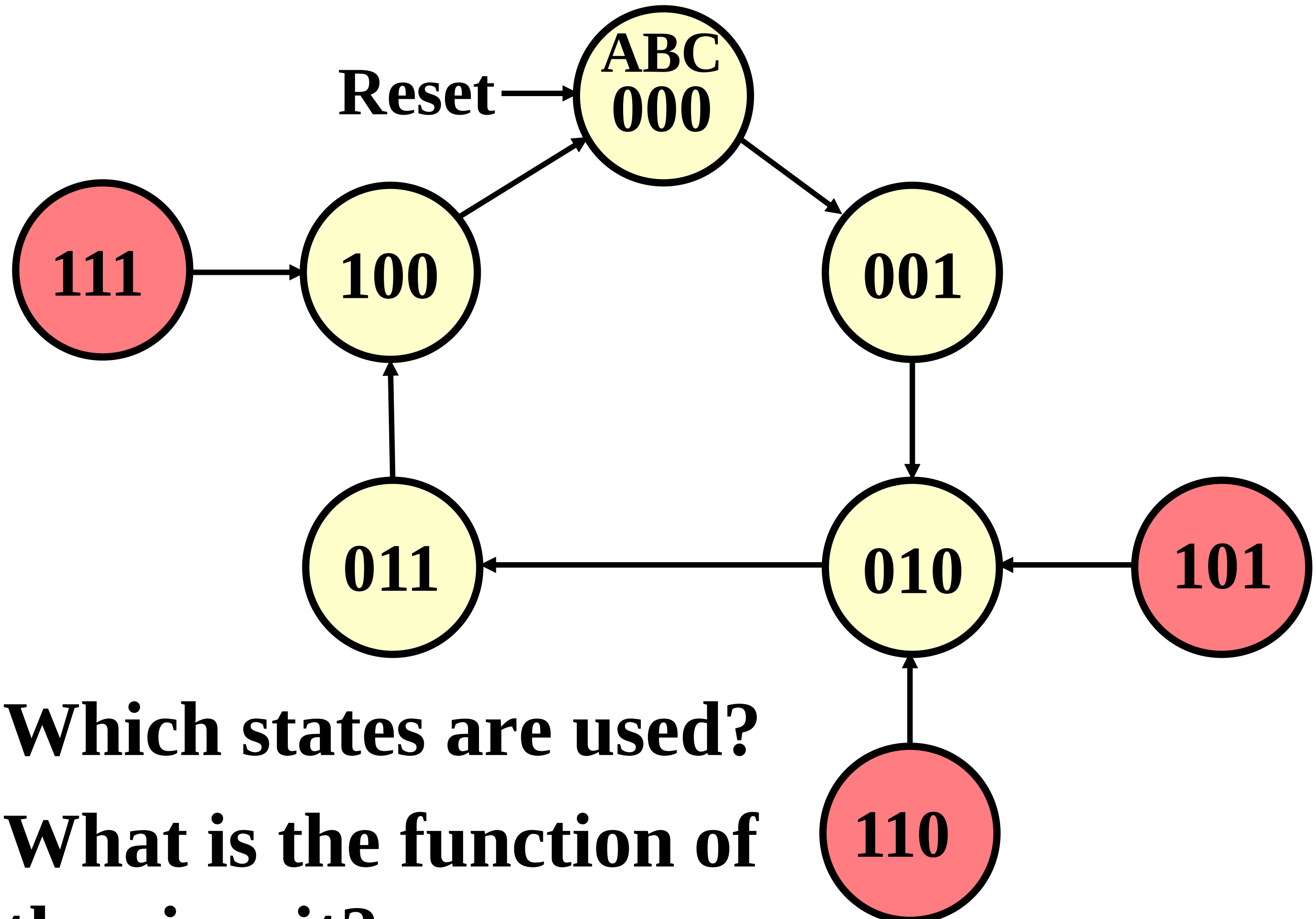
- Variables
  - Inputs: None
  - Outputs: Z
  - State Variables: A, B, C
- Initialization: Reset to (0,0,0)
- Equations
  - A(t+1) = BC
  - B(t+1) = B'C+ BC'
  - C(t+1) = A' C '
  - Z = A

Example 2: State Table

$X' = X(t+1)$

A B C	A'B'C'	Z
0 0 0	0 0 1	0
0 0 1	0 1 0	0
0 1 0	0 1 1	0
0 1 1	1 0 0	0
1 0 0	0 0 0	1
1 0 1	0 1 0	1
1 1 0	0 1 0	1
1 1 1	1 0 0	1

Example 2: State Diagram



- **Which states are used?**
- **What is the function of the circuit?**

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