Chapter 3

Weight-Volume Relationships

Omitted Parts:
Sections 3.6 & 3.7
➢ Soil deposits comprise the accumulated solid particles plus the void space between the particles.

➢ The void spaces are partially or completely filled with water or other liquid.

➢ Voids space not occupied by fluid are filled with air or other gas.

➢ Hence soil deposits are referred to as three-phase system, i.e. Solid + Liquid (water) + Gas (air)

S: Solid
W: water
A: Air
Bulk soil as it exists in nature is a more or less random accumulation of soil particles, water, and air as shown above.

Properties such as strength, compressibility, permeability are directly related to the ratio and interaction of these three phases.

Therefore, an understanding of the terminology and definitions relating to soil composition is fundamental to the study of soil mechanics and geotechnical engineering as a whole.
For purpose of study and analysis it is convenient to represent the soil mass by a **PHASE DIAGRAM**, with part of the diagram representing the solid particles, part representing water or liquid, and another part air or other gas.
Phase diagram in terms of mass
Possible Cases

Two phases:

- Dry soil (solid + air)
- Fully saturated soil (solid + water)

Three phases:

- Partially saturated soil (solid + water + air)
The total volume of a given soil sample can be expressed as:

\[ V = V_s + V_v = V_s + V_w + V_a \]

Where

- \( V \) = Total volume
- \( V_s \) = Volume of soil solids
- \( V_v \) = Volume of voids
- \( V_w \) = Volume of water
- \( V_a \) = Volume of air

Assuming that the weight of the air is negligible, we can give the total weight of the sample as

\[ W = W_s + W_w \]

Where

- \( W_s \) = weight of solids
- \( W_w \) = weight of water

In engineering practice we usually measure the total volume, \( V \), the mass of water, \( M_w \), and the mass of dry solid \( M_s \).
Volume Relationships

There are three volumetric ratios that are very useful in geotechnical engineering, and these can be determined directly from the phase diagram.

1. Void Ratio
   \[ e = \frac{V_v}{V_s} \]

2. Porosity
   \[ n = \frac{V_v}{V} = \frac{e}{1 + e} \]

3. Degree of Saturation
   \[ S = \frac{V_w}{V_v} \]

Porosity and degree of saturation are commonly expressed as a percentage.
Given $V_v = V_s$, $V_w = 0.5V_v$

$e = ?$

$n = ? \%$

$S = ? \%$
There are **three** weight ratios that are very useful in geotechnical engineering, and these can be determined directly from the phase diagram.

1. **Moisture content (Water content)**

\[ w = \frac{W_w}{W_s} \]

2. **Unit weight (total, bulk, moist, wet)**

\[ \gamma = \frac{W}{V} \quad \text{unit: kN/m}^3 \]

3. **Specific gravity**

\[ G_s = \frac{W_s}{W_w} = \frac{W_s}{V_s \cdot \gamma_w} \]
1. Unit weight (total, wet, bulk or moist unit weight)

\[ \gamma = \frac{W}{V} \]

2. Solid unit weight

\[ \gamma_s = \frac{W_s}{V_s} \]

3. Unit weight of water

\[ \gamma_w = \frac{W_w}{V_w} \]

\( (\gamma_w = 9.807 \approx 10 \text{ kN/m}^3) \)

4. Dry unit weight

\[ \gamma_d = \frac{W_s}{V} \]

5. Saturated unit weight

\[ \gamma_{sat} = \frac{W_s + W_w}{V} \quad (S = 100\%) \]

6. Submerged unit weight

\[ \gamma' = \gamma - \gamma_w \]
Density vs Unit Weight

Density
\[ \rho = \frac{M}{V} \quad \text{unit: kg/m}^3 \]

Dry density
\[ \rho_d = \frac{M_s}{V} \]
\[ \rho_d = \frac{\rho}{1 + w} \]

Unit Weight
\[ \gamma = \rho \cdot g \]
\[ \gamma \ (\text{kN/m}^3) = \frac{g \cdot \rho (\text{kg/m}^3)}{1000} \]
\[ g = 9.81 \ m/sec^2 \]
\[ \gamma_w = 9.81 \text{ kN/m}^3 = 1000 \text{ kgf/m}^3 \]

\[ G_s = \frac{M_s}{M_w} = \frac{M_s}{V_s \cdot \rho_w} \]
\[ G_s = \frac{W_s}{W_w} = \frac{W_s}{V_s \cdot \gamma_w} \]
Simple Rules

- Remember the basic definitions of e, n, w, s, G_s, γ, ... etc.

- Draw a phase diagram

- Assume either V_s = 1.0 or V=1.0, if NOT given.

- Often use s*e = w* G_s

\[ G_s = \frac{W_s}{V_s \gamma W} \]
\[ \gamma = \frac{G_s (1 + w)}{1 + e} \gamma_w \]

\[ \gamma_d = \frac{G_s}{1 + e} \gamma_w \]

\[ G_s = \frac{W_s}{V_s \gamma_w} \]

\[ \gamma = \frac{G_s + S \cdot e}{1 + e} \gamma_w \]

\[ \gamma_{sat} = \frac{G_s + e}{1 + e} \gamma_w \]

\[ S \cdot e = w \cdot G_s \]
**Exercise**

Prove that

\[
\gamma = \frac{G_s (1 + w)}{1 + e} \gamma_w
\]

\[
\gamma_d = \frac{G_s}{1 + e} \gamma_w
\]

\[
S \ast e = w \ast G_s
\]

\[
\gamma = \frac{G_s + S \ast e}{1 + e} \gamma_w
\]

\[
\gamma_{sat} = \frac{G_s + e}{1 + e} \gamma_w
\]

\[
n = \frac{e}{1 + e}
\]

\[
\gamma = \frac{W}{V} = \frac{W_w + W_s}{V_s + V_v} = \frac{\gamma_w V_w + \gamma_s V_s}{V_s + V_v} = \frac{\gamma_w V_w + \gamma_w G_s V_s}{V_s + V_v}
\]
# Weight-Volume Relationships

## Table 3.1 Various Forms of Relationships for \( \gamma, \gamma_d, \) and \( \gamma_{sat} \)

<table>
<thead>
<tr>
<th>Moist unit weight ( (\gamma) )</th>
<th>Dry unit weight ( (\gamma_d) )</th>
<th>Saturated unit weight ( (\gamma_{sat}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Given</strong></td>
<td><strong>Relationship</strong></td>
<td><strong>Given</strong></td>
</tr>
<tr>
<td>( w, G_s, e )</td>
<td>( \frac{(1+w)G_s\gamma_w}{1+e} )</td>
<td>( \gamma )</td>
</tr>
<tr>
<td>( S, G_s, e )</td>
<td>( \frac{(G_s + Se)\gamma_w}{1+e} )</td>
<td>( G_s, e )</td>
</tr>
<tr>
<td>( w, G_s, S )</td>
<td>( \frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}} )</td>
<td>( G_s, n )</td>
</tr>
<tr>
<td>( w, G_s, n )</td>
<td>( G_s\gamma_w(1-n)(1+w) )</td>
<td>( e, w_{sat} )</td>
</tr>
<tr>
<td>( S, G_s, n )</td>
<td>( G_s\gamma_w(1-n) + nS\gamma_w )</td>
<td>( e, w_{sat} )</td>
</tr>
<tr>
<td>( \gamma_{sat}, e )</td>
<td>( \frac{\gamma_{sat} - e\gamma_w}{1+e} )</td>
<td>( \gamma_d, n )</td>
</tr>
<tr>
<td>( \gamma_{sat}, n )</td>
<td>( \gamma_{sat} - n\gamma_w )</td>
<td>( \gamma_d, S )</td>
</tr>
<tr>
<td>( \gamma_{sat}, G_s )</td>
<td>( \frac{(\gamma_{sat} - \gamma_w)G_s}{(G_s - 1)} )</td>
<td></td>
</tr>
</tbody>
</table>
Example 3.2

For a moist soil sample, the following are given.

Total volume: \( V = 1.2 \) m\(^3\)
Total mass: \( M = 2350 \) kg
Moisture content: \( w = 8.6\% \)
Specific gravity of soil solids: \( G_s = 2.71 \)

Determine the following.

a. Moist density
b. Dry density
c. Void ratio
d. Porosity
e. Degree of saturation
f. Volume of water in the soil sample

Solution

Part a
From Eq. (3.13),
\[
\rho = \frac{M}{V} = \frac{2350}{1.2} = 1958.3 \text{ kg/m}^3
\]

Part b
From Eq. (3.14),
\[
\rho_d = \frac{M_s}{V} = \frac{M}{(1 + w)V} = \frac{2350}{(1 + \frac{8.6}{100})(1.2)} = 1803.3 \text{ kg/m}^3
\]

Part c
From Eq. (3.23),
\[
\rho_d = \frac{G_s \rho_w}{1 + e}
\]
\[
e = \frac{G_s \rho_w}{\rho_d} - 1 = \frac{(2.71)(1000)}{1803.3} - 1 = 0.503
\]

Part d
From Eq. (3.7),
\[
n = \frac{e}{1 + e} = \frac{0.503}{1 + 0.503} = 0.335
\]

Part e
From Eq. (3.19),
\[
S = \frac{w G_s}{e} = \frac{(8.6)(2.71)}{0.503} = 0.463 = 46.3\%
\]

Part f
The volume of water is
\[
\frac{M_w}{\rho_w} = \frac{M - M_s}{\rho_w} = \frac{M - \frac{M_s}{(1 + w)}}{\rho_w} = \frac{2350 - \left( \frac{2350}{1 + \frac{8.6}{100}} \right)}{1000} = 0.186 \text{ m}^3
\]
**Alternate Solution**

Refer to Figure 3.7.

Part a

\[ \rho = \frac{M}{V} = \frac{2350}{1.2} = 1958.3 \text{ kg/m}^3 \]

Part b

\[ M_s = \frac{M}{1 + w} = \frac{2350}{1 + \frac{8.6}{100}} = 2163.9 \text{ kg} \]

\[ \rho_d = \frac{M_s}{V} = \frac{M}{(1 + w)V} = \frac{2350}{(1 + \frac{8.6}{100})(1.2)} = 1803.3 \text{ kg/m}^3 \]

Part c

The volume of solids: \[ \frac{M_s}{G_s \rho_w} = \frac{2163.9}{(2.71)(1000)} = 0.798 \text{ m}^3 \]

The volume of voids: \[ V_v = V - V_s = 1.2 - 0.798 = 0.402 \text{ m}^3 \]

Void ratio: \[ e = \frac{V_v}{V_s} = \frac{0.402}{0.798} = 0.503 \]

Part d

Porosity: \[ n = \frac{V_v}{V} = \frac{0.402}{1.2} = 0.335 \]

**Figure 3.7**

Part e

\[ S = \frac{V_w}{V_v} \]

Volume of water: \[ V_w = \frac{M_w}{\rho_w} = \frac{186.1}{1000} = 0.186 \text{ m}^3 \]

Hence,

\[ S = \frac{0.186}{0.402} = 0.463 = 46.3\% \]

Part f

From Part e,

\[ V_w = 0.186 \text{ m}^3 \]
The moist unit weight of a soil is 19.2 kN/m³. Given that $G_s$ 2.69 and $w = 9.8\%$, determine

a. Dry unit weight
b. Void ratio
c. Porosity
d. Degree of saturation

\[
\gamma = \frac{(S_e + G_s)}{1 + e} \gamma_w
\]

Think instead of

\[
S_e = wG_s
\]

\[
\gamma_d = \frac{\gamma}{1 + w} = \frac{19.2}{1 + \frac{9.8}{100}} = 17.5 \text{ kN/m}^3
\]

\[
\gamma_d = 17.5 = \frac{G_s \gamma_w}{1 + e} = \frac{(2.69)(9.81)}{1 + e}; \quad e = 0.51
\]

\[
n = \frac{e}{1 + e} = \frac{0.51}{1 + 0.51} = 0.338
\]

\[
S = \frac{wG_s}{e} = \frac{(0.098)(2.69)}{0.51} \times 100 = 51.7\%
\]
A sample of soil has a total volume of 0.0282 m³, a saturation rate of 56% and a water content of 18.5%. If the specific gravity of the soil is 2.529, determine the values of the wet and dry densities and void ratio of the soil.

Given:

\[ V = 0.0282 \text{ m}^3 \]
\[ S = 56\% \]
\[ w = 18.5\% \]
\[ G_s = 2.529 \]

Required:

\[ e \]
\[ \gamma \]
\[ \gamma_d \]

From \[ wG_s = Se \]
\[ e = 0.835 \]

From \[ \rho = \frac{(Se + G_s)}{1+e} \rho_w \]
\[ \rho = 1633\text{ Kg} / \text{ m}^3 \]

From \[ \rho_d = \frac{G_s}{1+e} \rho_w \]
\[ \rho_d = 1378\text{ kg} / \text{ m}^3 \]

\[ \rho = \frac{M}{V} = \frac{46.043\text{ kg}}{0.0282\text{ m}^3} = 1633 \text{ kg/m}^3 \]

\[ \rho = \frac{M_s}{V} = \frac{38.855\text{ kg}}{0.0282\text{ m}^3} = 1378 \text{ kg/m}^3 \]

\[ e = \frac{V_v}{V_s} = \frac{0.012836\text{ m}^3}{0.015364\text{ m}^3} = 0.835 \]
EXAMPLE

A saturated soil has a moisture content of 25.7% and a void ratio of 0.668. Determine the density and specific gravity of solids.

\[
S = 1 \\
w = 25.7 \\
e = 0.668
\]

\[
S_e = \omega G_s
\]
The total volume of a soil specimen is 80 cm$^3$ and it weighs 144 g. The dry weight of the specimen is 128 g, and the density of the solids is 2.68 Mg/m$^3$. Find the

a) Water content  

b) Wet density  

c) Dry unit weight  

d) Void ratio  

e) Porosity  

f) Degree of saturation  

g) The mass of water to be added to a cubic meter of soil to reach 80% saturation.
In its natural condition, a soil sample has a mass of 2290 g and a volume of $1.15 \times 10^{-3} \text{ m}^3$. After being completely dried in an oven, the mass of the sample is 2035 g. The value of $G_s$ for the soil solids is 2.68.

Determine the bulk density, moist unit weight, water content, void ratio, porosity, and degree of saturation.
The relative density is the parameter that compares the volume reduction achieved from compaction to the maximum possible volume reduction.

The relative density Dr, also called density index, is commonly used to indicate the IN SITU denseness or looseness of granular soils.
Relative Density

$D_r$ can be expressed either in terms of void ratios or dry densities.

\[
D_r = \frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}}
\]

where $D_r$ = relative density, usually given as a percentage

$e = \text{in situ}$ void ratio of the soil

$e_{\text{max}} = \text{void ratio of the soil in the loosest state}$

$e_{\text{min}} = \text{void ratio of the soil in the densest state}$
• The relative density of a natural soil very strongly affects its engineering behavior.

• The range of values of $D_r$ may vary from a minimum of zero for very LOOSE soil to a maximum of 100% for a very DENSE soil.

• Because of the irregular size and shape of granular particles, it is not possible to obtain a ZERO volume of voids. (Do you remember well-graded vs. poorly-graded!!)

• ASTM test designations D-4253 and D-4254 (2007) provide procedure for determining maximum and minimum dry unit weights of granular soils.
Granular soils are qualitatively described according to their relative densities as shown below:

<table>
<thead>
<tr>
<th>Relative Density (%)</th>
<th>Description of soil deposit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-15</td>
<td>Very loose</td>
</tr>
<tr>
<td>15-50</td>
<td>Loose</td>
</tr>
<tr>
<td>50-70</td>
<td>Medium</td>
</tr>
<tr>
<td>70-85</td>
<td>Dense</td>
</tr>
<tr>
<td>85-100</td>
<td>Very dense</td>
</tr>
</tbody>
</table>

The use of relative density has been restricted to granular soils because of the difficulty of determining $e_{\text{max}}$ in clayey soils. Liquidity Index in fine-grained soils is of similar use as $D_r$ in granular soils.
THE END