Programme 11

SERIES

PART 1

1

Series

A series, $u_1, u_2, u_3 \ldots$ is a sequence of terms each of which is formed according to some definite pattern.

e.g. $1, 3, 5, 7, \ldots$ is a series (the next term would be 9)
$2, 6, 18, 54, \ldots$ is a series (the next term would be 3×54, i.e. 162)
$1^2, -2^2, 3^2, -4^2, \ldots$ is a series (the next term would be 5^2)

but $1, -5, 37, 6, \ldots$ is *not* a series since the terms are not formed to a regular pattern and one cannot assess the next term.

A *finite* series contains only a finite number of terms.
An *infinite* series is unending.
So which of the following constitutes a finite series:

(i) All the natural numbers, i.e. $1, 2, 3, \ldots$ etc.
(ii) The page numbers of a book.
(iii) The telephone numbers in a telephone directory.

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The page numbers of a book

Correct. Since they are in regular sequence and terminate at the last page. (The natural numbers form an infinite series, since they never come to an end: the telephone numbers are finite in number, but do not form a regular sequence, so they do not form a series at all.)

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We shall indicate the terms of a series as follows:

u_1 will represent the first term, u_2 the second term, u_3 the third term, etc., so that u_r will represent the r^{th} term, and u_{r+1} the $(r+1)^{\text{th}}$ term, etc.

Also the sum of the first 5 terms will be indicated by S_5.

So the sum of the first n terms will be stated as ....

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S_n

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You will already be familiar with two special kinds of series which have many applications. These are (i) *arithmetic series* and (ii) *geometric series*. Just by way of revision, however, we will first review the important results relating to these two series.

1. *Arithmetic series* (or arithmetic progression) denoted by A.P.

An example of an A.P. is the series

$$2, 5, 8, 11, 14, \dots \dots$$

You will note that each term can be written from the previous term by simply adding on a constant value 3. This regular increment is called the *common difference* and is found by selecting any term and subtracting from it the previous term

$$\text{e.g. } 11 - 8 = 3; \quad 5 - 2 = 3; \text{ etc.}$$

Move on to the next frame.

4

The *general arithmetic series* can therefore be written:

$$a, \ a + d, \ a + 2d, \ a + 3d, \ a + 4d, \dots \dots \dots \quad \text{(i)}$$

where a = first term and d = common difference.

You will remember that

(i) the n^{th} term $= a + (n - 1)d$ .... (ii)

(ii) the sum of the first n terms is given by

$$S_n = \frac{n}{2}(2a + \overline{n-1}\, d) \dots \dots \dots \quad \text{(iii)}$$

Make a note of these three items in your record book.

By way of warming up, find the sum of the first 20 terms of the series:

$$10, 6, 2, -2, -6, \dots \text{ etc.}$$

Then turn to frame 5.

5

$$\boxed{S_{20} = -560}$$

Since, for the series 10, 6, 2, −2, −6, . . . etc.

$$a = 10 \text{ and } d = 2 - 6 = -4$$

$$S_n = \frac{n}{2}(2a + \overline{n-1}\,d)$$

$$\therefore S_{20} = \frac{20}{2}(20 + 19[-4])$$

$$= 10(20 - 76) = 10(-56) = \underline{-560}$$

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Here is another example:

If the 7th term of an A.P. is 22 and the 12th term is 37, find the series.

$$\left.\begin{array}{ll} \text{We know } 7^{th} \text{ term} = 22 & \therefore a + 6d = 22 \\ \text{and } 12^{th} \text{ term} = 37 & \therefore a + 11d = 37 \end{array}\right\} 5d = 15 \quad \therefore d = 3, \quad \therefore a = 4$$

So the series is $\underline{4, 7, 10, 13, 16, \ldots \text{ etc.}}$

Here is one for you to do:

The 6th term of an A.P. is −5 and the 10th term is −21. Find the sum of the first 30 terms.

6

$$\boxed{S_{30} = -1290}$$

since:

$$\left.\begin{array}{ll} 6^{th} \text{ term} = -5 & \therefore a + 5d = -5 \\ 10^{th} \text{ term} = -21 & \therefore a + 9d = -21 \end{array}\right\} 4d = -16 \quad \therefore d = -4, \quad a = 15$$

$$\therefore a = 15,\ d = -4,\ n = 30,\ S_n = \frac{n}{2}(2a + \overline{n-1}\,d)$$

$$\therefore S_{30} = \frac{30}{2}(30 + 29[-4])$$

$$= 15(30 - 116) = 15(-86) = \underline{-1290}$$

Arithmetic mean

We are sometimes required to find the arith. mean of two numbers, P and Q. This means that we have to insert a number A between P and Q, so that P, A and Q form an A.P.

$$A - P = d \quad \text{and} \quad Q - A = d$$

$$\therefore A - P = Q - A \quad 2A = P + Q \quad \therefore \underline{A = \frac{P + Q}{2}}$$

The arithmetic mean of two numbers, then, is simply their average. Therefore, the arithmetic mean of 23 and 58 is ....

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The arithmetic mean of 23 and 58 is $\boxed{40{\cdot}5}$

If we are required to insert 3 arithmetic means between two given numbers, P and Q, it means that we have to supply three numbers, A, B, C between P and Q, so that P, A, B, C, Q form an A.P.

Example. Insert 3 arithmetic means between 8 and 18.

Let the means be denoted by A, B, C.

Then 8, A, B, C, 18 form an A.P.

First term, $a = 8$. fifth term $= a + 4d = 18$

$$\left.\begin{aligned} \therefore \quad a &= 8 \\ a + 4d &= 18 \end{aligned}\right\} 4d = 10 \quad \therefore \; d = 2{\cdot}5$$

$$\left.\begin{aligned} A &= 8 + 2{\cdot}5 = 10{\cdot}5 \\ B &= 8 + 5 \;\; = 13 \\ C &= 8 + 7{\cdot}5 = 15{\cdot}5 \end{aligned}\right\} \quad \text{Required arith. means are } \underline{10{\cdot}5,\ 13,\ 15{\cdot}5}$$

Now, you find five arithmetic means between 12 and 21·6.

Then turn to frame 8.

8

Required arith. means: $\boxed{13{\cdot}6,\ 15{\cdot}2,\ 16{\cdot}8,\ 18{\cdot}4,\ 20}$

Here is the working:

Let the 5 arith. means be A, B, C, D, E.

Then 12, A, B, C, D, E, 21·6 form an A.P.

$$\therefore \; a = 12; \quad a + 6d = 21{\cdot}6$$

$$\therefore \; 6d = 9{\cdot}6 \quad \therefore \; d = 1{\cdot}6$$

Then		
	$A = 12 + 1{\cdot}6 = 13{\cdot}6$	$A = 13{\cdot}6$
	$B = 12 + 3{\cdot}2 = 15{\cdot}2$	$B = 15{\cdot}2$
	$C = 12 + 4{\cdot}8 = 16{\cdot}8$	$C = 16{\cdot}8$
	$D = 12 + 6{\cdot}4 = 18{\cdot}4$	$D = 18{\cdot}4$
	$E = 12 + 8{\cdot}0 = 20{\cdot}0$	$E = 20.$

So that is that! Once you have done one, the others are just like it.

Now we will see how much you remember about Geometric Series.

So, on to frame 9.