Application of Integration (Area)

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1. Area

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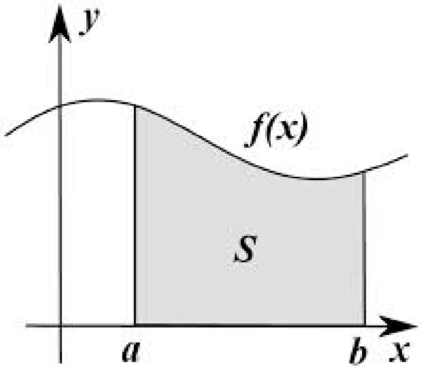

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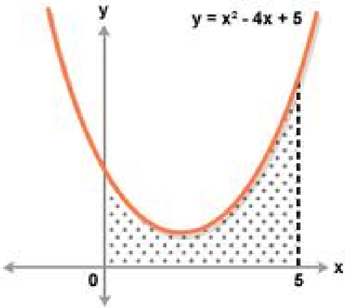

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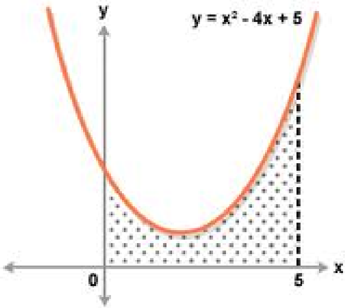

$$A = \int_0^5 (x^2 - 4x + 5)dx = \frac{50}{3}.$$

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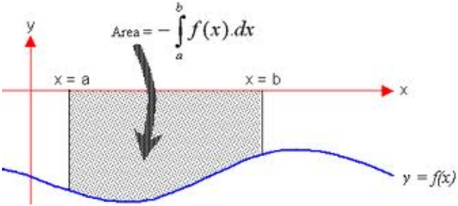

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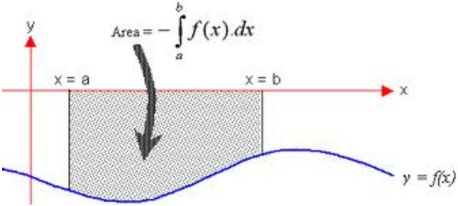

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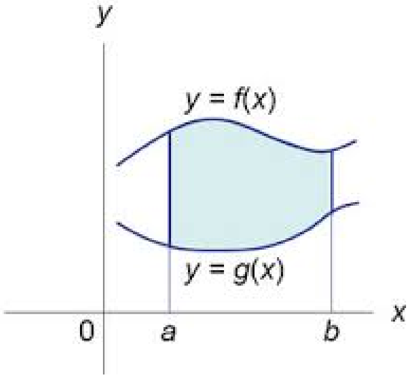
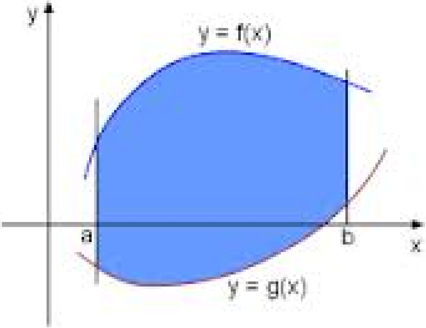
$$A = \int_a^b [f(x) - g(x)]\, dx.$$



Exercise: Express the area between the graphs in the following diagram as a definite integral:



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(2) Find the area of the region bounded by the graphes $y - x = 6, y = x^3$ and $2y + x = 0$.

Exercise: Find the shaded area for the following diagrams:

[1]

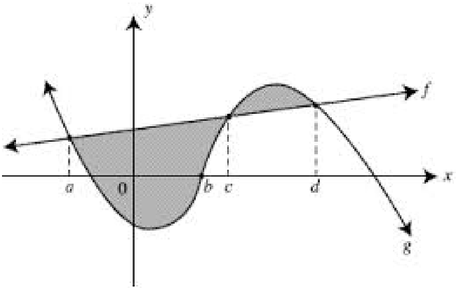
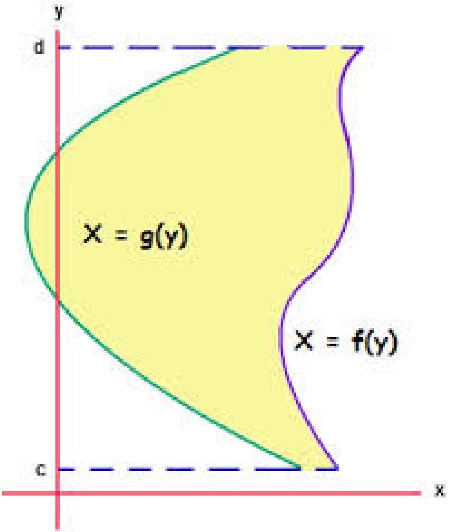


Figure 12.3-8

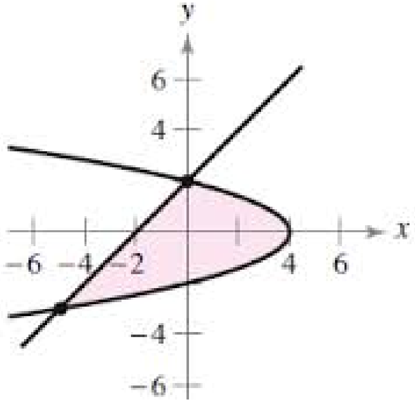
[2]



[3]

$$x = 4 - y^2$$

$$x = y - 2$$



[4]