Application of Integration
(Arc Length and Surface of Revolution)

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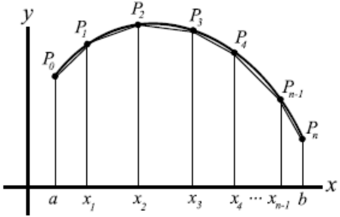

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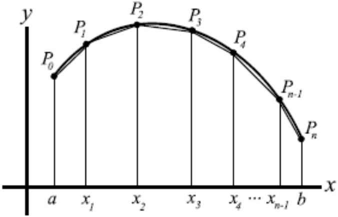

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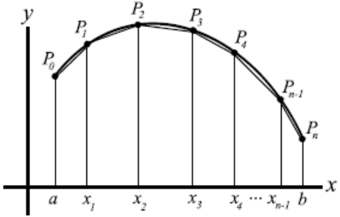

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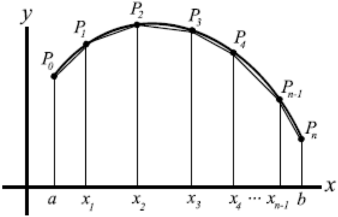

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$$\boxed{L_a^b = \int_a^b \sqrt{1 + [f'(x)]^2}dx} \quad or \quad \boxed{L_c^d = \int_c^d \sqrt{1 + [g'(y)]^2}dy}$$

Examples (Swokowsoki,335)

[1] If $f(x) = 3x^{2/3} - 10$, *find the arc length of the graph of* f *from the point* $A(8, 2)$ *to* $B(27, 17)$. *(answer apprx. 2.4)*
[2] Set up an integral for finding the arc length of the graph of the equation $y^3 - y - x = 0$ *from* $A(O, -1)$ *to* $B(6, 2)$.

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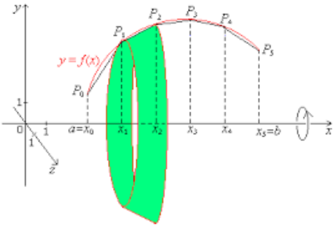

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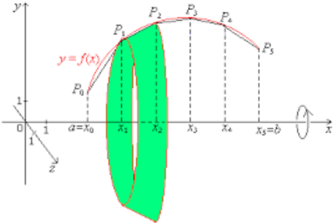

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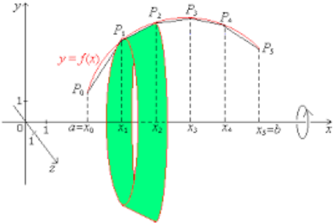

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$$S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}dx$$

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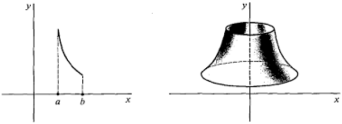

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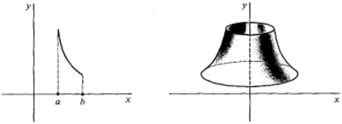

If the graph of g is revolved about the y-axis from (a, c) to (b, d), a surface of revolution is generated and the area is given by the formula:

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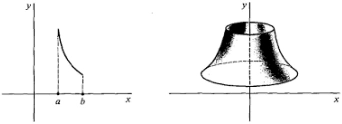

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Example (2, Swokowsoki, exercises 342)

The graph of the equation from A *to* B *is revolved about the* x*-axis. Find the area of the resultingsurface.*

a $4x = y^2; A(0, 0), B(1, 2)$.

b $y = x^3; A(1, 1), B(2, 8)$.

c $y = 2x + 1; A(0, 2), B(3, 4)$.

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Example (3, Swokowsoki, exercises 342)

The graph of the equation from A *to* B *is revolved about the* y*-axis. Find the area of the resulting surface.*

a $y = 2\sqrt[3]{x}; A(1,2), B(8,4)$.

b $x = 4\sqrt{y}; A(4,1), B(12,9)$.