3.3

[1] Find:

(a) $i^i = e^{i\log i}$

$\log i = \ln|i| + i\arg(i)$

$= \ln(1) + i(\pi/2 + 2k\pi)$

$= i(\pi/2 + 2k\pi)$

$\therefore i^i = e^{i[i(\pi/2+2k\pi)]}$

$= e^{-(\pi/2 + 2k\pi)}$

(d) $(1+i)^{1-i} = e^{(1-i)\log(1+i)}$

$= e^{(1-i)[\ln\sqrt{2} + i(\pi/4 + 2k\pi)]}$

$= (1+i)\, e^{-i/2 \log 2 + \pi/4 + 2k\pi}$

[2] $z^0 = 1$

L.H.S $z^0 = e^{0\log z} = e^0 = 1 = $ R.H.S

[3] (b) $i^{2i} = e^{2i(\operatorname{Log} i)}$

$= e^{2i(i\pi/2)} = e^{-\pi}$ = ~~[illegible]~~

[6] (a) $z^{-\alpha} = e^{-\alpha\log z} = \dfrac{1}{e^{\alpha\log z}} = \dfrac{1}{z^\alpha}$

(b) $z^\alpha z^\beta = e^{\alpha\log z} \cdot e^{\beta\log z} = e^{\alpha\log z + \beta\log z} = e^{(\alpha+\beta)\log z} = z^{\alpha+\beta}$

(c) $\dfrac{z^\alpha}{z^\beta} = \dfrac{e^{\alpha\log z}}{e^{\beta\log z}} = e^{\alpha\log z - \beta\log z} = e^{(\alpha-\beta)\log z} = z^{\alpha-\beta}$

[5] take $z_1 = i$, $z_2 = -1+i$, $\alpha = i$

[11] $\sin z = \cos z$

$\Leftrightarrow \dfrac{e^{iz} - e^{-iz}}{\cancel{2}i} = \dfrac{e^{iz} + e^{-iz}}{\cancel{2}}$

$\Rightarrow e^{iz} - e^{-iz} = ie^{iz} + ie^{-iz}$

$\Rightarrow e^{iz} - ie^{iz} = e^{-iz} + ie^{-iz}$

$\Rightarrow (1-i)e^{iz} = (1+i)e^{-iz}$

$\Rightarrow \dfrac{(1-i)}{(1+i)} = \dfrac{e^{-iz}}{e^{iz}} \Rightarrow \dfrac{(1+i)}{(1-i)} = \dfrac{e^{iz}}{e^{-iz}}$

$\Rightarrow \cancel{e^{-2iz} = \dfrac{(1-i)}{(1+i)}}$

$\Rightarrow e^{2iz} = \dfrac{(1+i)}{(1-i)} \times \left[\dfrac{1+i}{1+i}\right] = \dfrac{(1+i)^2}{\cancel{\pm}2} = i$

$\Rightarrow e^{2iz} = i$

$\Rightarrow \log e^{2iz} = \log(i)$

$\Rightarrow 2\cancel{i}z = \cancel{i}(\pi/2 + 2k\pi), \; k \in \mathbb{Z}$

$\Rightarrow \boxed{z = \pi/4 + k\pi}, \; k \in \mathbb{Z}$