Assignment 4(Solution)

Questions:

1. Find the current in a long straight wire that would produce a magnetic field twice the strength of the Earth's at a distance of 5.0 cm from the wire where B due to the wire is taken to be1.0×10^{-4} T.

Strategy

The Earth's field is about 5.0×10^{-5} T, and so here B due to the wire is taken to be 1.0×10^{-4} T. The equation $B = \frac{\mu_0 I}{2\pi r}$ can be used to find I, since all other quantities are known.

Solution

Solving for I and entering known values gives

$$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(5\times10^{-2}m)(1\times10^{-4}T)}{4\pi\times10^{-7}T.m/A} I = 25A$$

Discussion

So, we find a moderately large current produces a significant magnetic field at a distance of 5.0 cm from a long straight wire.

2. What is the field inside a 2.00 m long solenoid that has 2000 loops aand carries a 1600A current?

Strategy

To find the field strength inside a solenoid, we use $B = \mu_0 nI$. First, we note the number of loops per unit length is

$$n = \frac{N}{l} = \frac{2000}{2m} = 1000m^{-1}$$

Solution

Substituting known values gives

$$B = \mu_0 nI = \left(4\pi\times10^{-7}T.\frac{m}{A}\right)(1000m^{-1})(1600\text{A}) = 2.01 \text{ T}$$

Discussion

This is a large field strength that could be established over a large-diameter solenoid, such as in medical uses of magnetic resonance imaging (MRI). The very large current is an indication that the fields of this strength are not easily achieved, however. Such a large current through 1000 loops squeezed into a meter's length would produce significant heating. Higher currents can be achieved by using superconducting wires, although this is expensive. There is an upper limit to the current, since the superconducting state is disrupted by very large magnetic fields.

3. A strong electromagnet produces a uniform field of 1.60 T over a crosssectional area of 0.200 m^2 . We place a coil having 200 turns and a total resistance of 20.0 Ω around the electromagnet. We then smoothly decrease the current in the electromagnet until it reaches zero in 20.0 ms. What is the current induced in the coil?

Strategy
A strong magnetic field turned off in a short time (20.0 ms) will produce a large induced voltage (emf), maybe on the order of 1 kV . With only 20.0 Ω of resistance in the coil, the induced current produced by this emf will probably be larger than 10 A but less than 1000 A.

Solution
According to Faraday's law, if the magnetic field is reduced uniformly, then a constant emf will be produced. The definition of resistance can be applied to find the induced current from the emf.
Noting unit conversions from F = qv × B and U = qV , the induced voltage is

$$e = N\frac{d\Phi}{dt} = N\frac{d(B.A)}{dt} = N\frac{(B_i A cos\theta)}{dt} = \frac{200(1.6T)(0.2m^2)(cos0^o)}{20\times10^{-3}s} = 3200V$$

Discussion
This is a large current, as we expected. The positive sign is indicative that the induced electric field is in the positive direction around the loop (as defined by the area vector for the loop).

4. A helicopter has blades of length 3.0 m, about a central hub. If the vertical component of Earth's magnetic field is $5.0x10^{-5}$ T, what is the induced voltage between the blade tip and the central Hub where the time is about 500 ms?

Strategy
During each revolution, one of the rotor blades sweeps out a horizontal circler area of radius r, A= πr^2 and N=1.

Solution
The nuber of magnetic filed lines cut per revolution is $d\Phi_B = \mathrm{B.A}$, The induced voltage is

$$e = N\frac{d\Phi}{dt} = N\frac{d(B.A)}{dt} = N\frac{(B_i A cos\theta)}{dt} = N\frac{(B.\pi r^2 cos\theta)}{dt}$$

$$= \frac{5\times10^{-5}\times\pi\times(3m)^2(cos0^o)}{0.5s} = 28\times10^{-3}V = 28mV$$