King Saud University

Math department

April 2026

Midterm2 106(90mn)

Question1(3+3+3)

a) Find $\lim_{x \to +\infty} (1 + \frac{1}{x})^{x^2}$

b) Compute the integral $\int x^2 e^{4x} dx$

c) Evaluate the integral $\int (sin2x)^5 (cos2x)^{10} dx$

Question2 (2+3+3)

a) Compute $\int (cos4x)(cos3x)dx$

b) Evaluate $\int \frac{dx}{x^2\sqrt{x^2-1}}$

c) Find $\int \frac{3x^2-7x+5}{(x-1)(x-2)^2} dx$

Question3(3+2+3)

a) Compute the integral $\int \frac{dx}{x^{1/3}+x^{2/3}}$

b) Find $\int \frac{dx}{\sqrt{x^2+2x+10}}$

c) Does the integral $\int_0^{+\infty} x^2 e^{-2x^3} dx$ converge? Find its value if it does.

Grading scheme midterm2 April2026

Question1

a) If $y = (1 + \frac{1}{x})^{x^2}$

$$\lim_{x \to +\infty} lny = \lim_{x \to +\infty} \frac{ln(1 + \frac{1}{x})}{1/x^2} = \lim_{x \to +\infty} \frac{x}{2(1 + \frac{1}{x})} = +\infty$$

Thus $\lim_{x \to +\infty} y = +\infty$

(2.5) +(0.5)

b) $\int x^2 e^{4x} dx = \frac{1}{4} x^2 e^{4x} - \frac{1}{2} \int x e^{4x} dx$

$$= \frac{1}{4} x^2 e^{4x} - \frac{1}{8} x e^{4x} + \frac{1}{32} e^{4x} + C$$

(1.5) + (1.5)

c)

$$\int (sin2x)^5 (cos2x)^{10} dx = \frac{-1}{2} \int (1 - u^2)^2 u^{10} du$$

$$= -\frac{1}{22} (cos2x)^{11} - \frac{1}{30} (cos2x)^{15} + \frac{1}{13} (cos2x)^{13} + C$$

(1.5) + (1.5)

Question2

a) $\int cos4x . cos3x dx = \frac{1}{2} \int (cos7x + cosx) dx$

$$\frac{sin7x}{14} + \frac{sinx}{2} + C$$

(1) + (1)

b) $\int \frac{dx}{x^2\sqrt{x^2-1}} = \int cos d\theta = sin\theta + C$

$= \frac{\sqrt{x^2-1}}{x} + C$

(2) +(1)

c) $\frac{3x^2-7x+5}{(x-1)(x-2)^2} = \frac{1}{x-1} + \frac{2}{x-2} + \frac{3}{(x-2)^2}$

$$\int \frac{3x^2-7x+5}{(x-1)(x-2)^2}dx = \ln|x-1| + 2\ln|x-2| - \frac{3}{x-2} + C$$

(1.5) +(1.5)

Question3

a) $\int \frac{dx}{x^{1/3}+x^{2/3}} = 3\int (1 - \frac{1}{1+u})du$

$= 3(x^{1/3} - \ln(1 + x^{1/3}) + C$

(2) + (1)

b) $\int \frac{dx}{\sqrt{x^2+2x+10}} = \int \frac{du}{\sqrt{u^2+9}} = \sinh^{-1}\left(\frac{x+1}{3}\right) + C$

(1.5) +(0.5)

c) $\int_0^c x^2e^{-2x^3}dx = \frac{1}{6}(1 - e^{-2c^3})$

Thus $\int_0^{+\infty} x^2e^{-2x^3}dx$ converges and its value is 1/6

(2) +(1)