Name: Answer ID: Section: **Quiz1- First Semester 1443H – Math107** **Duration: 30 min**	جامعة الملك سعود King Saud University

Question 1 [Marks: 4]:

Solve the following linear system by using Gaussian elimination:

$$\begin{aligned} x \quad +y \quad +2z &= 7 \\ x \qquad\quad +z &= 2 \\ 2x - y + 3z &= 5. \end{aligned}$$

Answer :

The system can be written as $AX=B$ with $A=\begin{pmatrix}1&1&2\\1&0&1\\2&-1&3\end{pmatrix}$, $X\begin{pmatrix}x\\y\\z\end{pmatrix}$

and $B=\begin{pmatrix}7\\2\\5\end{pmatrix}$

$-2 \quad -2 \quad -4 \quad -14$

The augmented matrix is $\left(\begin{array}{ccc|c}1&1&2&7\\1&0&1&2\\2&-1&3&5\end{array}\right) \rightarrow \left(\begin{array}{ccc|c}1&1&2&7\\0&-1&-1&-5\\0&-3&-1&-9\end{array}\right)$

$0 \quad 3 \quad 3 \quad 15$

$\rightarrow \left(\begin{array}{ccc|c}1&1&2&7\\0&1&1&5\\0&0&2&6\end{array}\right) \rightarrow \left(\begin{array}{ccc|c}1&1&2&7\\0&1&1&5\\0&0&1&3\end{array}\right)$

So $z=3$ and $y+z=5$ it means $y=2$

Also $x+y+2z=7$ so $x=7-2-6=-1$

The unique solution is $\{(-1,2,3)\}$.

Question 2 [Marks: 2]:

Let $\mathbf{A}=\begin{bmatrix}0&0&4\\0&-2&0\\3&0&0\end{bmatrix}$. Find the inverse of **A**.

Answer:

$|A| = (0+0+0)-(-24+0+0) = 24 \neq 0$ so A has an inverse.

$[A\,|\,I] = \left[\begin{array}{ccc|ccc}0&0&4&1&0&0\\0&-2&0&0&1&0\\3&0&0&0&0&1\end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc}3&0&0&0&0&1\\0&-2&0&0&1&0\\0&0&4&1&0&0\end{array}\right]$

$= \left[\begin{array}{ccc|ccc}1&0&0&0&0&1/3\\0&1&0&0&-1/2&0\\0&0&1&1/4&0&0\end{array}\right] = [I\,|\,A^{-1}]$

So $A^{-1} = \begin{pmatrix}0&0&1/3\\0&-1/2&0\\1/4&0&0\end{pmatrix}$

Question 3 [Marks: 4]:

Determine the value of β so that the following linear system has unique solution:

$$\begin{aligned} x + y + z &= 3 \\ 2x - y - z &= 1 \\ 3x + \beta y + \beta^2 z &= 7. \end{aligned}$$

Answer:

The system can be written as $AX = B$ with

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 3 & \beta & \beta^2 \end{pmatrix} ; \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} ; \quad B = \begin{pmatrix} 3 \\ 1 \\ 7 \end{pmatrix}$$

The system has unique solution if and only if $|A| \neq 0$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 3 & \beta & \beta^2 \end{vmatrix} = \left[(-\beta^2 - 3 + 2\beta) - (-3 - \beta + 2\beta^2) \right]$$

$$= -3\beta^2 + 3\beta = 3\beta(1-\beta)$$

$|A| \neq 0$ iff $(\beta \neq 0 \text{ or } \beta = 1)$