

Question	1	2	3	4	5	6	7	8	9	10
Answer	<u>C/D</u>	<u>C</u>	<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>B</u>	<u>D</u>	<u>B</u>	<u>D</u>

Choose the correct answer (write it in the table above):

1) $\ln(27)$ is equal to $\ln(3^3) = 3\ln 3 = \ln(9 \times 3) = \ln 9 + \ln 3$.

- (a) $9 \ln 3$ (b) $27 \ln 1$ (c) $\ln 9 + \ln 3$ (d) $3 \ln 3$

2) The solution of the equation $\ln(\ln x) = 0$ is

- (a) 0 (b) 1 (c) e (d) e^2

② $e^{\ln(\ln x)} = e^0 \Leftrightarrow \ln x = 1 \Leftrightarrow e^{\ln x} = e^1 \Leftrightarrow x = e$

3) The domain of the function $y = \ln(6-3x)$ is

- (a) $(-\infty, 2)$ (b) $(-\infty, 2]$ (c) $(2, \infty)$ (d) $[2, \infty)$

③ $f(x) = \ln(6-3x)$
 $D_f = \{x / 6-3x > 0\} = \{x / x < 2\}$

4) $\log_2(20) - \log_2(5)$ is equal to

- (a) 1 (b) 2 (c) 4 (d) 5

$\log_2\left(\frac{20}{5}\right) = \log_2(4) = \log_2(2^2) = 2$

5) The solution of equation $(x-3)e^{-\ln(\frac{1}{x})} = 4$ is

- (a) -1 (b) e (c) 4 (d) 6

⑤ $e^{-\ln(1/x)} = e^{\ln x} = x ; x > 0$
 $(x-3)e^{-\ln(1/x)} = x(x-3) = 4$
So $x^2 - 3x - 4 = 0$
 $(x-4)(x+1) = 0$
 $x = 4$ or $x = -1$
because $x > 0$

6) The solution of the equation $\ln(x^2 - 1) - \ln(2x - 1) + \ln 2 = 0$ is

- (a) $\frac{1-\sqrt{3}}{2}$ (b) e (c) $\frac{1+\sqrt{2}}{2}$ (d) $\frac{1+\sqrt{3}}{2}$

⑥ $D_E = \{x / \text{and } x^2 - 1 > 0 \text{ and } 2x - 1 > 0\} = (1, +\infty)$ ← Domain of equation

Please go on to the next page...

Let $x \in D_E$; $\ln(x^2 - 1) - \ln(2x - 1) = -\ln 2$

$\ln\left(\frac{x^2 - 1}{2x - 1}\right) = \ln(1/2) \Leftrightarrow \frac{x^2 - 1}{2x - 1} = 1/2$

$x_1 = \frac{2 - 2\sqrt{3}}{4} = \frac{1 - \sqrt{3}}{2}$ $2(x^2 - 1) - (2x - 1) = 0 \Leftrightarrow 2x^2 - 2x - 1 = 0$
Discriminant $D = 4 + 8 = 12 = (2\sqrt{3})^2$
 $x_2 = \frac{1 + \sqrt{3}}{2}$

7 Domain of inequality
 $D_{Ineq} = \{x \in \mathbb{R} \mid \frac{x+1}{3x-5} > 0 \text{ and } 3x-5 \neq 0\}$
 $D_{Ineq} = (-\infty, -1) \cup (5/3, +\infty)$

7) The solution of the inequality $\ln(\frac{x+1}{3x-5}) < 0$ is

- (a) $(-\infty, -1)$
- (b) $(-\infty, -1) \cup (3, +\infty)$
- (c) $(3, +\infty)$
- (d) $(-\infty, -1) \cup (\frac{5}{3}, +\infty)$

$\ln(\frac{x+1}{3x-5}) < 0 \Leftrightarrow e^{\ln(\frac{x+1}{3x-5})} < e^0$
 $e^{\ln(\frac{x+1}{3x-5})} = \frac{x+1}{3x-5}$
 So $\frac{x+1}{3x-5} \leq 1$

8) The solution of equation $e^{2x} - e^x - 6 = 0$ is

- (a) 2
- (b) $\ln 2$
- (c) 3
- (d) $\ln 3$

$\Rightarrow \frac{x+1}{3x-5} - 1 \leq 0 \Leftrightarrow \frac{6-2x}{3x-5} \leq 0$

x	5/3	3
Q	-	+

9) The solution of $3e^x - 7e^{-x} - 20 = 0$ is

- (a) $\ln 3$
- (b) $\ln 7$
- (c) $\ln 10$
- (d) $2 \ln 5$

Put $u = e^x > 0$; $u^2 - 4 - 6 = 0$
 $(u-3)(u+2) = 0$
 $u = 3$ or $u = -2$ because $u > 0$
 So $e^x = 3$
 we get $x = \ln 3$

10) The solution of equation $x^{\sqrt{x}} = (\sqrt{x})^x$ is

- (a) 1
- (b) 4
- (c) 1 or e
- (d) 1 or 4

Put $u = e^x > 0$
 $3e^x - 7e^{-x} - 20 = 0$
 $3(e^x)^2 - 7 - 20e^x = 0$
 $3u^2 - 20u - 7 = 0$
 $a=3; b=-20; c=-7$
 discriminant
 $D = 400 + 84 = 484 = (22)^2$
 $u_1 = \frac{20-22}{6} = -1/3$
 $u_2 = \frac{20+22}{6} = 7$
 $e^x = 7$
 so $x = \ln 7$

10 $x^{\sqrt{x}} = (\sqrt{x})^x, x > 0$

$\ln(x^{\sqrt{x}}) = \ln((\sqrt{x})^x)$

$\sqrt{x} \ln x = x \ln(\sqrt{x}) = x \ln(x^{1/2}) = \frac{x}{2} \ln x$

$\sqrt{x} \ln x - \frac{x}{2} \ln x = 0$

$\ln x (\sqrt{x} - x/2) = 0$

So $\ln x = 0$ or $\sqrt{x} = x/2$
 $x = 1$ or $x = \frac{x^2}{4} \Leftrightarrow x(\frac{x}{4} - 1) = 0$
 $x \neq 0$ so $x = 4$