Answer

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Name:.............................
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Time: 40 Min **Quiz 1 - Math 107- Semester II- 1443 H** Marks: 10

Q.1) Determine the values of β for which the system has no solutions, exactly one solution, or infinitely many solutions $\begin{cases} x+y+\beta z &= 1 \\ -x-2y &= -1 \\ \beta x+8z &= 2 \end{cases}$. [4 Marks]

Answer:

(1) $\begin{vmatrix} 1 & 1 & \beta \\ -1 & -2 & 0 \\ \beta & 0 & 8 \end{vmatrix} = (-16+0+0)-(-2\beta^2+0-8) = 2\beta^2-8 = 2(\beta^2-4) = 2(\beta-2)(\beta+2)$

(1) • if $\beta \in \mathbb{R}\setminus\{\pm 2\}$ the system has unique solution

(1) • if $\boxed{\beta=-2}$, then the augmented matrix is $\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ -1 & -2 & 0 & -1 \\ -2 & 0 & 8 & 2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & 2 & 4 & 4 \end{array}\right)$

$\to \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & \boxed{4} \end{array}\right)$ so the system no solution.

(1) • if $\boxed{\beta=2}$, then the augmented matrix is $\left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ -1 & -2 & 0 & -1 \\ 2 & 0 & 8 & 2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & -2 & 4 & 0 \end{array}\right)$

$\to \left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ so infinitely solution.

Q.2) Solve the system by inverting the coefficient matrix

$$(*) \begin{cases} x_1-x_2-x_3 &= 0 \\ 2x_1+x_2+x_3 &= 3 \\ x_1+2x_2+x_3 &= 0 \end{cases}.$$

Answer: (*) Can be written as $AX=B$ with $A=\begin{pmatrix} 1 & -1 & -1 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$; $X\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$; $B\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$

$(A|I) = \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 3 & 3 & -2 & 1 & 0 \\ 0 & 3 & 2 & -1 & 0 & 1 \end{array}\right)$

$\to \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2/3 & 1/3 & 0 \\ 0 & 0 & -1 & 1 & -1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/3 & -2/3 & 1 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right)$

$\to \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1/3 & -2/3 & 1 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & 1/3 & 0 \\ 0 & 1 & 0 & 1/3 & -2/3 & 1 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right)$

(3) So $A^{-1} = \begin{pmatrix} 1/3 & 1/3 & 0 \\ 1/3 & -2/3 & 1 \\ -1 & 1 & -1 \end{pmatrix}$

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We deduce that $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A^{-1}B = \begin{pmatrix} 1/3 & 1/3 & 0 \\ 1/3 & -2/3 & 1 \\ -1 & 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

(1) $x_1 = 1$; $x_2 = -2$ and $x_3 = 3$

Q.3) Find the values of α for which A is invertible, where

$$A = \begin{pmatrix} 1 & -1 & 1 \\ \frac{1}{2} & \alpha & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \alpha \end{pmatrix}.$$

Answer: A is invertible iff $|A| \neq 0$

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$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 1/2 & \alpha & 1/2 \\ 1/2 & 1/2 & \alpha \end{vmatrix} = \left(\alpha^2 - 1/4 + 1/4\right) - \left(\frac{\alpha}{2} + \frac{1}{4} - \frac{\alpha}{2}\right)$$

$$= \alpha^2 - \frac{1}{4} = \left(\alpha - \frac{1}{2}\right)\left(\alpha + 1/2\right)$$

A is invertible iff $\alpha \in \mathbb{R} \setminus \{\pm 1/2\}$