MID TERM EXAMINATION, SEMESTER I, 1443
DEPARTMENT OF MATHEMATICS, COLLEGE OF SCIENCE
KING SAUD UNIVERSITY
MATH - 107 FULL MARK: 30 TIME: 2 HOURS

[N. B.: Marks: Q1. [5]; Q2. [5]; Q3.[2+2+2=6]; Q4. [2+3+3=8] Q5. [2+4=6]]

**Q1**. Solve the system of linear equations by Gaussian elimination:

$$\begin{aligned} 2x_1 + 3x_2 + x_3 &= 5 \\ x_1 + x_2 + x_3 &= 2 \\ 4x_1 + 3x_2 - 3x_3 &= 11 \end{aligned}$$

**Q2.** Let

$$A = \begin{bmatrix} 1 & -1 & \alpha \\ -1 & 2 & -\alpha \\ \alpha & 1 & 1 \end{bmatrix}$$

Find the values of $\alpha$ for which the matrix $A$ is invertible.

**Q3**. (i) Find the angle between the vectors $\mathbf{u} = 10\mathbf{i} + 9\mathbf{j}$ and $\mathbf{v} = -4\mathbf{i} + 2\mathbf{j}$.

(ii) Let $\mathbf{a} = \langle 1, 0, 0 \rangle$ and $\mathbf{b} = \langle -6, 2, 1 \rangle$. Determine the component of $\mathbf{b}$ along $\mathbf{a}$.

(iii) Determine whether $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j}$ are perpendicular to each other.

**Q4**. (a) Find the work done that is exerted by a constant force $\mathbf{F} = 3\mathbf{i} + \mathbf{j} - 5\mathbf{k}$ to move a particle from a point $P(-1, 1, 2)$ to another point $Q(2, 4, 3)$.

(b) Find the equation of the plane determined by the points $P(4, -3, 1)$, $Q(6, -4, 7)$ and $R(1, 2, 2)$.

(c) Sketch the graph of the equation $4x^2 - 9y^2 + z^2 = 36$ in an $xyz$-coordinate system, and identify the surface.

**Q5**. Let $C$ be the curve determined by $\mathbf{r}(t) = (1 + t^3)\mathbf{i} + \sqrt{2t - 1}\,\mathbf{j} + t^3\mathbf{k}$.

(a) Find the domain of $\mathbf{r}(t)$.

(b) Find parametric equations for the tangent line to $C$ at the point $(2, 1, 1)$.

MT (M-107) Solutions

P1

Q1

$$\left(\begin{array}{ccc|c} 2 & 3 & 1 & 5 \\ 1 & 1 & 1 & 2 \\ 4 & 3 & -3 & 11 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right)$$ whence

(3+2)

$x_3 = -\frac{1}{2}$, $x_2 = 1 + x_3 = 1 - \frac{1}{2} = \frac{1}{2}$, $x_1 = 2 - x_2 - x_3 = 2 - \frac{1}{2} + \frac{1}{2} = 2$

So, $x_1 = 2$, $x_2 = \frac{1}{2}$, $x_3 = -\frac{1}{2}$.

Q2.

$$\det(A) = \begin{vmatrix} 1 & -1 & \alpha \\ -1 & 2 & -\alpha \\ \alpha & 1 & 1 \end{vmatrix} = 1 - \alpha^2$$

(3+2)

A is invertible $\iff \det(A) \neq 0 \iff 1 - \alpha^2 = 0 \iff \alpha \neq \pm 1$.

Q3 (i) $\cos\theta = \dfrac{\underline{u} \cdot \underline{v}}{\|\underline{u}\| \|\underline{v}\|} = \dfrac{-22}{\sqrt{(181)}\sqrt{20}}$ (2)

(ii) $\text{comp}_{\underline{a}}\, \underline{b} = \dfrac{\underline{a} \cdot \underline{b}}{\|\underline{a}\|} = \dfrac{-6}{1} = -6$ (2)

(iii) $\underline{a} \cdot \underline{b} = \langle 2, -3, -5 \rangle \cdot \langle 3, 2, 0 \rangle = 6 - 6 = 0$

$\Rightarrow$ $\underline{a}$ and $\underline{b}$ are perpendicular to each other. (2)

Q4 (a) $\vec{F} = \langle 3, 1, -5 \rangle$, $\vec{PQ} = \langle 3, 3, 1 \rangle$ (2)

$W = \vec{F} \cdot \vec{PQ} = 7$ units.

(b) $P(4, -3, 1)$, $\vec{PQ} = \langle 2, -1, 6 \rangle$, $\vec{PR} = \langle -3, 5, 1 \rangle$

$$\vec{a} = \begin{vmatrix} i & j & k \\ 2 & -1 & 6 \\ -3 & 5 & 1 \end{vmatrix} = \langle -31, -20, 7 \rangle$$

(2+1)

The reqd. eq. of the Plane is

$$-31(x-4) - 20(y+3) + 7(z-1) = 0$$

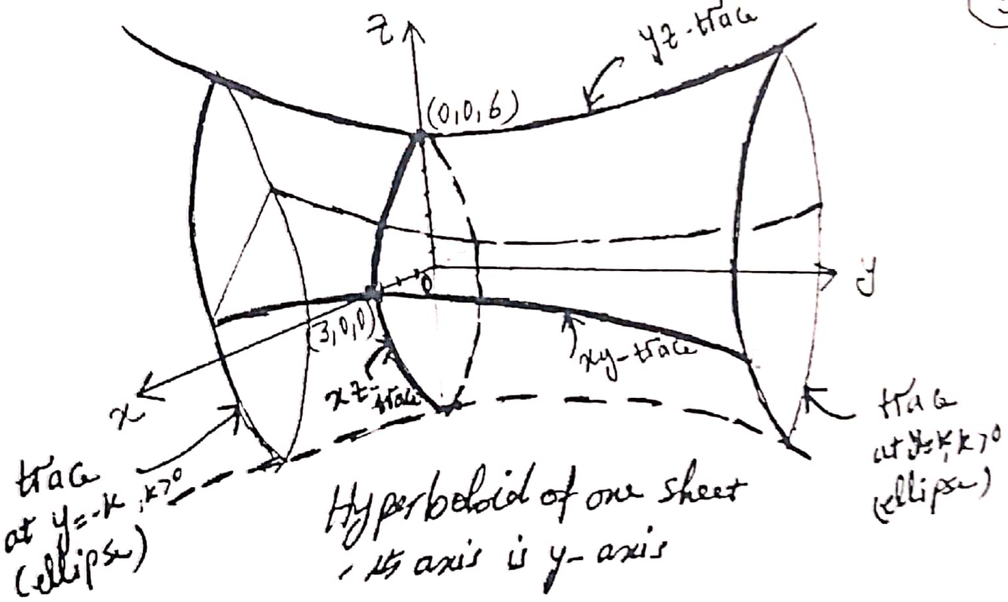
or, $31x + 20y - 7z = 57$.

(Next Page Pl.)

Q4 (c)

| Trace | Equation of trace | Description |
|---|---|---|
| on xy-plane (z=0) | $\frac{x^2}{9}-\frac{y^2}{4}=1$ | Hyperbola |
| on yz-plane (x=0) | $\frac{z^2}{36}-\frac{y^2}{4}=1$ | Hyperbola |
| on xz-plane (y=0) | $\frac{x^2}{9}+\frac{z^2}{36}=1$ | Ellipse |

3




Q5 (a) $D_{\underline{r}} = [\frac{1}{2}, \infty)$ 2+4

(b) $\underline{r}'(t) = 3t^2\,\underline{i} + \frac{1}{\sqrt{2t-1}}\,\underline{j} + 3t^2\,\underline{k}$

Since $t=1$, $\underline{r}'(1) = \langle 3, 1, 3\rangle$,

So the reqd. parametric eqs are:

$$x = 2+3t$$
$$y = 1+t$$
$$z = 1+3t$$