King Saud University
Department of Mathematics

Second Midterm Exam in Math 151, S1-1445H.
Calculators are not allowed

Q1. (a) Let R be the relation on $\mathbb{Z}$ defined by: $aRb \Leftrightarrow a+b > 12$. Determine whether R is reflexive, symmetric, antisymmetric or transitive. [4]

(b) Let E be the relation on $\mathbb{Z}$ defined by: $mEn \Leftrightarrow 6 \mid (2m-2n)$.

(i) Show that E is an equivalence relation. [3]

(ii) Find $[0]$. [1]

(c) Let $P = \{(1,1),(2,1),(2,2),(2,4),(3,1),(3,3),(3,4),(4,1),(4,4)\}$ be a relation on $A = \{1,2,3,4\}$.

(i) Represent P with a digraph. [1]

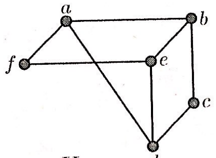
(ii) Show that P is a partial order. [3]

(iii) Is P a total order? (justify your answer.) [1]

(iv) Represent P with a Hasse diagram. [2]

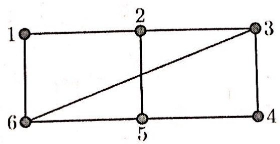
Q2. (a) If a graph $G = (V,E)$ has 5 edges and degree-sequence $1,1,x,x,x^2$, then find the value of x. [2]

(b) Determine if the graph H below is bipartite. If so, find a bipartite representation. [2]



H

(c) Determine whether the graph H in **(b)** is isomorphic to the graph L below. [2]



L

(d) Find the value (values) of n for which K_{2n} is isomorphic to $K_{n,n}$. [2]

(e) Find the number of edges of the complement of $K_{4,6}$. [2]

Answer Sheet Midterm2 - Math151 - Sem1 - 2023

Q1 (a)

- R is not reflexive because $1 \not R 1$ 1
- R is symmetric because if aRb then $a+b>12$ 1
  so $b+a>12 \Leftrightarrow bRa$
- R is not antisymmetric because $7R6$ & $6R7$ but $6\neq 7$ 1
- R is not transitive because $1R12$ and $12R3$ 1
  but $1 \not R 3$.

(b)

(i)
- E is reflexive on $\mathbb{Z}$ because $6|0 \Leftrightarrow 0=6.0$
  1 So $6 \mid 2m-2m \Leftrightarrow mEm$.
- E is symmetric on $\mathbb{Z}$ because if mEn then
  1 $6 \mid 2m-2n \Leftrightarrow 2m-2n=6c$ with $c\in\mathbb{Z}$
  so $2n-2m=-6c=6(-c) \Leftrightarrow 6 \mid 2n-2m \Leftrightarrow nEm$
- E is transitive on $\mathbb{Z}$ because if mEn and nEP
  1 then $6 \mid 2m-2n \Leftrightarrow 2m-2n=6c_1 \ ^{(1)}$ with $c_1\in\mathbb{Z}$
  and $6 \mid 2n-2p \Leftrightarrow 2n-2p=6c_2 \ ^{(2)}$ with $c_2\in\mathbb{Z}$
  $(1)+(2) \Rightarrow 2m-2p=6(c_1+c_2) \Leftrightarrow 6 \mid 2m-2p \Leftrightarrow mEp$

As E reflexive, symmetric and transitive on $\mathbb{Z}$ then E is an equivalence relation on $\mathbb{Z}$.

(ii) $[0]=\{m\in\mathbb{Z} / mE0\}=\{m\in\mathbb{Z} / 6|2m\}$

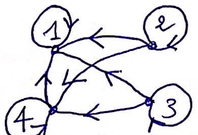
1 $[0]=\{m\in\mathbb{Z} / 2m=6k \text{ with } k\in\mathbb{Z}\}$

$[0]=\{m=3k / k\in\mathbb{Z}\}=\{0; \pm3; \pm6; \pm9; \cdots\}$

(c)

(i)

1

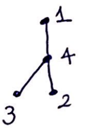


(ii)
- 1 P is reflexive because $I_A=\{(1,1),(2,2);(3,3),(4,4)\}\subset P$
- 1 P is antisymmetric because if aPb then $b \not P a$
- 1 P is transitive because $PoP\subseteq P$

(iii) P is not total order because 2 & 3 are not comparable
$(2,3)$ & $(3,2)\notin P$

1

(iv)



(2)

Q2)

(a) As $\sum_{v \in V(G)} \deg(v) = 2|E(G)|$ then

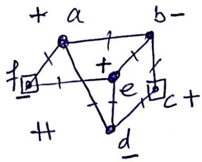
$$1+1+x+x+x^2 = 10$$

(2)

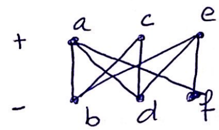
$$x^2+2x-8=0 \quad ; x \neq -4$$

$$(x-2)(x+4)=0 \quad \text{So } \underline{x=2}$$

(b)



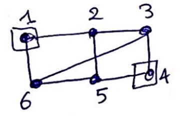
$\simeq$



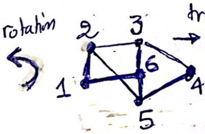
(2)

H is bipartite because it does not contain oddcycles.

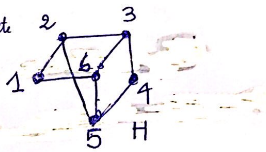
(c)



rotation



translate



(2) $L \simeq H$ because there exists an isomorphism f:

$x \in V(H)$	a	b	c	d	e	f
$f(x) \in V(L)$	2	3	4	5	6	1

(d) K_{2n} has $(2n)$ vertices and $\frac{2n(2n-1)}{2}$ edges and $K_{n,n}$ has $(2n)$ vertices and n^2 edges

As $K_{2n} \simeq K_{n,n}$ then $|E(K_{2n})| = |E(K_{n,n})|$

(2)

$$\frac{\cancel{2}n(2n-1)}{\cancel{2}} = n^2 \iff 2n^2-n=n^2$$

$$\iff n^2-n=0$$

$$\iff n(n-1)=0$$

$K_2 \simeq K_{1,1}$ So $\boxed{n=1}$

(e) $|E(\overline{K_{4,6}})| = ?$

(2)

As $K_{4,6} \cup \overline{K_{4,6}} = K_{10}$ so $|E(K_{4,6})| + |E(\overline{K_{4,6}})| = |E(K_{10})|$

$$24 + |E(\overline{K_{4,6}})| = \frac{10 \times 9}{2} = 45$$

We deduce that $|E(\overline{K_{4,6}})| = 21$.