King Saud University
Faculty of Sciences
Department of Mathematics

Midterm Exam	Math 106	October 2026

Question 1 : (2+2+3)

1. If $F(x) = \int_{3x}^{x^4} \sqrt{1+t^3}dt$, find $F'(x)$.
2. Compute the integral $\int \frac{dx}{x \ln \sqrt{x}}$.
3. Use Simpson formula, with $n = 4$, to approximate the integral $\int_0^{\pi} \frac{dx}{\sqrt{1+(\sin x)^2}}$.

Question 2 : (3+3+3)

1. Find the numbers c in the mean value theorem for the function $f(x) = 1 + \cos(2x)$ on the interval $[0, \pi]$.
2. If $y = (2+\cos x)^{x^2+3x}$, find $\frac{dy}{dx}$.
3. Evaluate the integral $\int \frac{\cosh x}{\sqrt{25-(\sinh x)^2}}dx$.

Question 3 : (3+3+3)

1. Compute the integral $\int \frac{dx}{(\sin x)(\cos x)\sqrt{\tan^2 x - 1}}$.
2. Compute the integral $\int \frac{x\cos x + \sin x}{\sqrt{9+(x\sin x)^2}}dx$.
3. Find $\int \frac{dx}{x\sqrt{36-x^8}}$.

Solution of the first examination Math 106 Semester I
October 2025

Question 1 : (2+2+3)

1. $F'(x) = 4x^3\sqrt{1+x^{12}} - 3\sqrt{1+27x^3}$. $((1+1)$

2. $\int \frac{2dx}{x\ln x} \overset{t=\ln x}{=} \int \frac{2dt}{t} = 2\ln|\ln x| + c..$ $(1+1)$

3.

k	x_k	$f(x_k)$	m_k	$m_k f(x_k)$
0	0	1	1	1
1	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{\sqrt{3}}$	4	≈ 3.2659
2	$\frac{\pi}{2}$	$\frac{1}{\sqrt{2}}$	2	≈ 0.7071
3	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{\sqrt{3}}$	4	≈ 3.2659
4	π	1	1	1
				≈ 11.239

(2.5)

$$\int_0^{\pi} \frac{dx}{\sqrt{1+(\sin x)^2}} \approx 2.94.89. \quad \textbf{(0.5)}$$

Question 2 : (3+3+3)

1. $\frac{1}{\pi}\int_0^{\pi}(1+\cos(2x))dx = 1$, then $\cos(2c) = 0$ and $c = \frac{\pi}{4}$ or $c = \frac{3\pi}{4}$. $(2)+(1)$

2. If $y = y = (2+\cos x)^{x^2+3x}$, $\ln y = (x^2+3x)\ln(2+\cos x)$, (1)

$$y' = y\left((2x+3)\ln(2+\cos x) - \frac{(x^2+3x)\sin x}{2+\cos x}\right). \quad (2)$$

3.

$$\int \frac{\cosh x}{\sqrt{25-(\sinh x)^2}}dx \overset{t=\sinh x}{=} \int \frac{dt}{\sqrt{25-t^2}} \quad (1)$$

$$= \sin^{-1}(\frac{1}{5}\sinh x)+c. \quad (2)$$

Question 3 : (3+3+3)

1.

$$\int \frac{dx}{(\sin x)(\cos x)\sqrt{\tan^2 x-1}} \overset{\tan x=t}{=} \int \frac{dt}{t\sqrt{t^2-1}} \quad (1.5)$$

$$= \sec^{-1}(\tan x)+c. \quad (1.5)$$

2.

$$\int \frac{x\cos x+\sin x}{\sqrt{9+(x\sin x)^2}}dx \overset{t=x\sin x}{=} \int \frac{dt}{\sqrt{9+t^2}} \quad (1.5)$$

$$= \sinh^{-1}(\frac{x\sin x}{3})+c. \quad (1.5)$$

3.

$$\int \frac{dx}{x\sqrt{36-x^8}} \overset{t=x^4}{=} \frac{1}{4}\int \frac{dt}{t\sqrt{36-t^2}} \quad (1.5)$$

$$= -\frac{1}{24}\text{sech}^{-1}(\frac{x^4}{6})+c. \quad (1.5)$$