King Saud University
Faculty of Sciences
Department of Mathematics

Midterm Exam	Math 106	October 2024

Question 1 : (3+2+3)

1. If $F(x) = \int_{\cos x}^{\sin x} \sqrt{1-t^2}\,dt$, find $F'(\frac{\pi}{4})$.
2. Compute the integral $\int \frac{1+(\sec x)^2}{(\tan x + x)^5}dx$.
3. Use the trapezoid rule with $n = 4$ to approximate the integral $\int_0^1 \frac{dx}{1+x^2}$.

Question 2 : (3+2+3)

1. Use the substitution $u = \ln|\sec x + \tan x|$ to compute the integral $\int (\sec x)\ln|\sec x + \tan x|dx$.
2. If $y = (x^2+1)^{\ln x + \cos x}$, find $\frac{dy}{dx}$.
3. Evaluate the integral $\int \frac{7^{\tanh^{-1}(2x)}}{1-4x^2}dx$.

Question 3 : (3+3+3)

1. Compute the integral $\int \frac{1+\ln x}{\sqrt{(x\ln x)^2+4}}dx$.
2. Evaluate $\int \frac{dx}{x\sqrt{9-x^{10}}}$.
3. Find $\int \frac{x^4}{\sqrt{3^{x^5}-1}}dx$.

Question 1 : (3+2+3)

1. If $F'(x) = \cos x|\cos x| + \sin x|\sin x|$, then $F'(\frac{\pi}{4}) = 1$. **(2)+(1)**

2.

$$\int \frac{1+(\sec x)^2}{(\tan x + x)^5} dx \overset{u=\tan x+x}{=} \int \frac{du}{u^5} \quad (1)$$

$$= \frac{-1}{4(\tan x+x)^4} + c. \quad (1)$$

3.

k	x_k	$f(x_k)$	m_k	$m_k f(x_k)$
0	0	1	1	1
1	0.25	0.9117	2	1.8234
2	0.5	0.8	2	1.6
3	0.75	0.64	2	1.28
4	1	0.5	1	0.5
				6.2034

(2.5)

$$\int_0^1 \frac{dx}{1+x^2} \approx 0.7754. \quad \textbf{(0.5)}$$

Question 2 : (3+2+3)

1. $u = \ln|\sec x + \tan x|$, $du = \sec x dx$, then

$$\int (\sec x) \ln|\sec x + \tan x| dx = \frac{1}{2}\ln^2|\sec x + \tan x| + c. \quad \textbf{(2.5)} + \textbf{(0.5)}$$

2. If $y = (x^2+1)^{\ln x + \cos x}$, $\ln y = (\ln x + \cos x)\ln(x^2+1)$, **(0.5)**

$$y' = \left((\frac{1}{x} - \sin x)\ln(x^2+1) + \frac{2x}{x^2+1}(\ln x + \cos x) \right) (x^2+1)^{\ln x + \cos x}. \quad \textbf{(1.5)}$$

3. $u = \tanh^{-1}(2x)$, $du = \dfrac{2}{1-4x^2}$

$$\int \frac{7^{\tanh^{-1}(2x)}}{1-4x^2}dx = \frac{1}{2}\int 7^u du = \frac{1}{2\ln 7}7^{\tanh^{-1}(2x)} + c. \quad (2)+(1)$$

Question 3 : (3+3+3)

1. $$\int \frac{1+\ln x}{\sqrt{(x\ln x)^2+4}}dx \overset{u=x\ln x}{=} \int \frac{du}{\sqrt{u^2+4}} = \sinh^{-1}(\frac{u}{2}) + c.$$

$$\int \frac{1+\ln x}{\sqrt{(x\ln x)^2+4}}dx = \sinh^{-1}(\frac{x\ln x}{2}) + c. \quad (2)+(1)$$

2. $$\int \frac{dx}{x\sqrt{9-x^{10}}}dx \overset{u=x^5}{=} \frac{1}{5}\int \frac{du}{u\sqrt{9-u^2}}.$$

$$\int \frac{dx}{x\sqrt{9-x^{10}}}dx = -\frac{1}{15}\text{sech}^{-1}(\frac{|x|^5}{3}) + c. \quad (2)+(1)$$

3. $$\int \frac{x^4}{\sqrt{3^{x^5}-1}}dx \overset{u^2=3^{x^5}}{=} \frac{2}{5\ln 3}\int \frac{du}{u\sqrt{u^2-1}} = \frac{2}{5\ln 3}\sec^{-1}(3^{\frac{x^5}{2}}) + c. \quad (2)+(1)$$