King Saud University
Department of Mathematics

Final Exam in Math151, Semester 1, 1444H.
Calculators are not allowed
(The exam is <u>two-pages</u> long)

Q1. (a) Without using truth tables, show that $(p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee q)$ is logically equivalent to $p \wedge q$. (3pts)

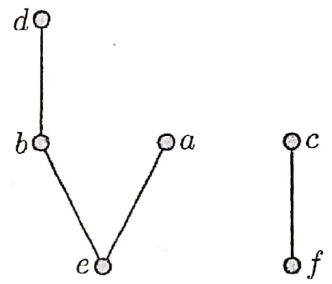
(b) Use induction to show the following for every $n \geq 1$:

$$2 + 8 + 14 + \cdots + (6n - 4) = n(3n - 1). \text{ (4pts)}$$

(c) Prove by contradiction: "For $m \in \mathbb{Z}$, if $m^2 - 3m + 5$ is even, then m is even". (2pts)

Q2. (a) Let R be the relation on $\mathbb{Z}$ defined by mRn if and only if 10 divides $m^4 - n^4$.
(i) Show that R is an equivalence relation. (3pts)
(ii) Show that $(m, -m) \in R$ for every $m \in \mathbb{Z}$. (1pts)
(iii) Is $[3] = [-1]$? (Justify your answer.) (1pts)

(b) Let P be the partial order on $A = \{a, b, c, d, e, f\}$ represented by the following Hasse diagram.



(i) List all ordered pairs of P. (2pts)
(ii) Is P a total order? (Justify your answer.) (1pts)

(c) Let $S = \{(1,1), (1,2), (2,3), (3,2), (3,4), (4,4), (5,5)\}$ be a relation on $B = \{1, 2, 3, 4, 5\}$.
(i) Represent S by a digraph. (1pts)
(ii) Is S antisymmetric? (Justify your answer.) (1pts)

Q3. (a) Let $G = (V, E)$ be an r-regular graph with $|V| = |E|$. Find the value of r. (1pts)
(b) Let H be a graph with degree-sequence $b - 1$, b, $b + 1$, $b + 2$. Find the value of b if H has 9 edges. (2pts)

(c) Let N be the simple graph represented by the following adjacency matrix.

$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

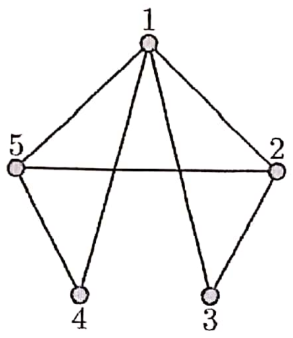
(i) Draw N. (1pts)

(ii) Is N connected? (Justify your answer.) (1pts)

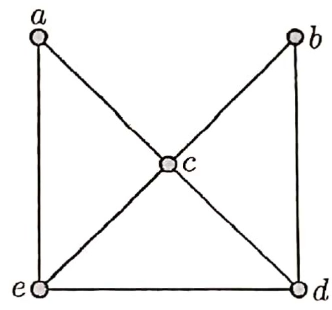
(iii) Determine whether N is bipartite. If so, give a bipartite representation. (2pts)

(d) Determine whether the following graphs I and J are isomorphic. (2pts)

I



J

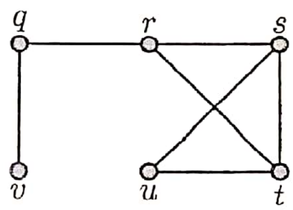


Q4. (a) For the graph L below, find a spanning tree with root r,

(i) using *depth-first* search; (1pts)

(ii) using *breadth-first* search. (1pts)

L



(b) (i) Using alphabetical order, form a binary search tree for the words *Makkah, Madin* *Riyadh, Jeddah, Qassim, Al-Khobar, Dammam.* (2 pts)

(ii) Is the tree in (i) full binary? (Justify your answer.) (1pts)

Q5. (a) Let $f(x, y, z) = (\overline{x} + \overline{y})(x + z)$ be a Boolean function.

(i) Find the complete sum-of-products expansion (CSP) of f. (2pts)

(ii) Find the complete product-of-sums expansion (CPS) of f. (2pts)

(b) Let $g(x, y, z) = xyz + xy\overline{z} + x\overline{y}z + \overline{x}y\overline{z} + \overline{x}\,\overline{y}\,\overline{z}$ be a Boolean function.

(i) Build the K-map of g. (1pts)

(ii) Simplify g (i.e., write in MSP form). (2pts)

Answer Final math 151

1444 (I).

9

P1) a) $(p\vee q)\wedge(p\vee \neg q)\wedge(\neg p\vee q)\equiv[p\vee(q\wedge\neg q)]\wedge(\neg p\vee q)$

$\equiv[p\vee F)\wedge(\neg p\vee q)$

3

$\equiv p\wedge(\neg p\vee q)$

$\equiv(p\wedge\neg p)\vee(p\wedge q)$

$\equiv F\vee(p\wedge q)=p\wedge q.$

b)

B.S: L.H.S $=2$; R.H.S $=1(3-1)=2$

$\Rightarrow P(1)$ is true.

I.S: Let $k\geq 1$; we prove that $P(k)\longrightarrow P(k+1)$.

we assume that $P(k)$ is true and we prove that $P(k+1)$ is true.

$P(k): 2+8+14+\cdots+(6k-4)=k(3k-1).$

$P(k+1): 2+8+14+\cdots+(6k+2)=(k+1)(3k+2).=3k^2+5k+2.$

4

$2+8+14+\cdots+(6k-4)+(6k+2)=k(3k-1)+6k+2.$

$=3k^2-k+6k+2.$

$=3k^2+5k+2.$

$\Rightarrow P(k+1)$ is true $\Rightarrow \forall n\geq 1,\ 2+8+14+\cdots+(6n-4)=n(3n-1).$

c) Let $m\in\mathbb{Z}$; we have m^2-3m+5 is even.

we assume that m is odd $\Rightarrow m=2k+1;\ k\in\mathbb{Z}.$

$\Rightarrow m^2-3m+5=(2k+1)^2-3(2k+1)+5$

2

$=4k^2+2k+1-6k-3+5$

$=4k^2-4k+3=2(2k^2-2k+1)+1$ is odd

contradiction

$\Rightarrow m$ is even.

Q_2) a)

(i) $\bullet\, m \in \mathbb{Z};\ m^4 - m^4 = 0 \Rightarrow 10 | 0 \Rightarrow 10 | m^4 - m^4.$

$\Rightarrow m R m$

$\Rightarrow R$ is reflexive (1).

1

$\bullet\, m, n \in \mathbb{Z};\ m R n \Rightarrow 10 | m^4 - n^4 \Rightarrow 10 | n^4 - m^4$

$\Rightarrow n R m \Rightarrow R$ is symmetric (2).

1

$\bullet\, m, n, p \in \mathbb{Z};\ m R n;\ n R p \Rightarrow 10 | m^4 - n^4;\ 10 | n^4 - p^4$

$\Rightarrow m^4 - n^4 = 10 q_1,\ n^4 - p^4 = 10 q_2$

$\Rightarrow m^4 - n^4 + n^4 - p^4 = 10 q_1 + 10 q_2$

$\Rightarrow m^4 - p^4 = 10(q_1 + q_2) \Rightarrow 10 | m^4 - p^4$

$\Rightarrow m R p.$

1

$\Rightarrow R$ is transitive. (3).

(1), (2), (3) $\Rightarrow R$ is an equivalence relation.

(ii) $m^4 - (-m)^4 = m^4 - m^4 = 0;\ 10 | 0 \Rightarrow 10 | m^4 - (-m)^4$

$\Rightarrow m R (-m).$

1

(iii)

$3^4 - (-1)^4 = 81 - 1 = 80;\ 10 | 80 \Rightarrow 3 R (-1)$

$\Rightarrow [3] = [-1]$

1

10

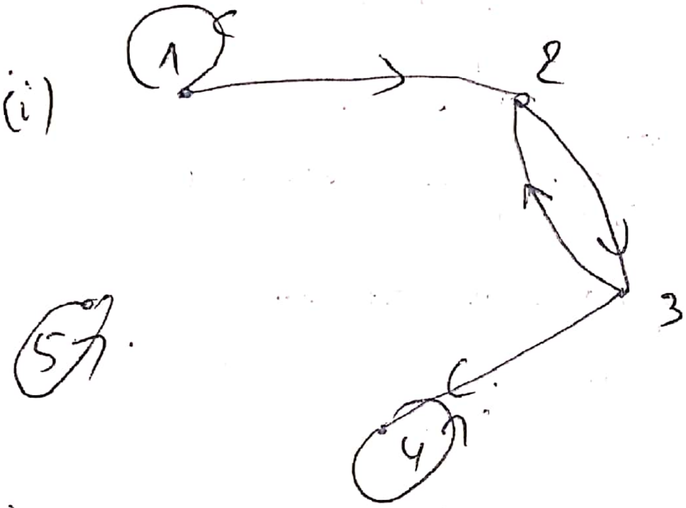
b)

(i) $P = \{(e,e),(e,a),(e,b),(e,d),(f,f),(f,c),(a,a),(b,b),(b,d),(c,c),(d,d)$ 2

(ii) P is not a total order;

$(a,c) \notin P$; $(c,a) \notin P \Rightarrow a, c$ are incomparable 1

c) (i)



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(ii) S is not antisymmetric.

$(2,3) \in S$; $(3,2) \in S$ 1

P3) → 9

a) $|V| = n = |E|$; $|E| = \frac{n \cdot r}{2} = n$ 1

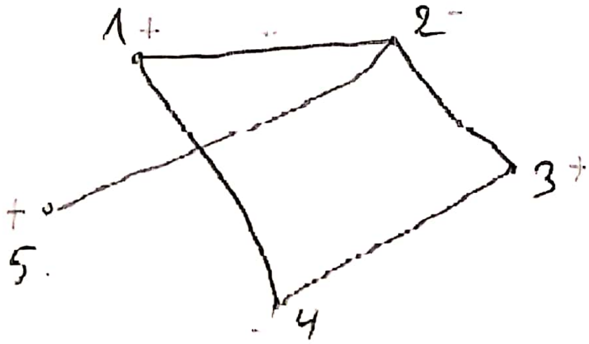
$\Rightarrow nr = 2n \Rightarrow r = 2$

b)

$b-1+b+b+1+b+2 = 18$

$\Rightarrow 4b+2 = 18 \Rightarrow 4b = 16 \Rightarrow b = 4$ 2

c) (i)



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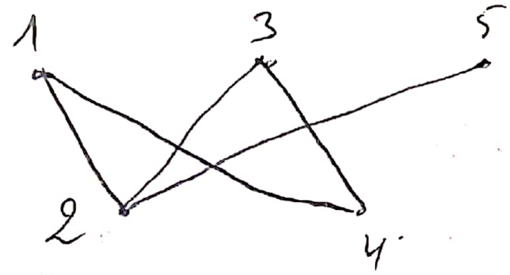
... there is a path between every two vertices. (1)

(iii) N is bipartite.



(2)

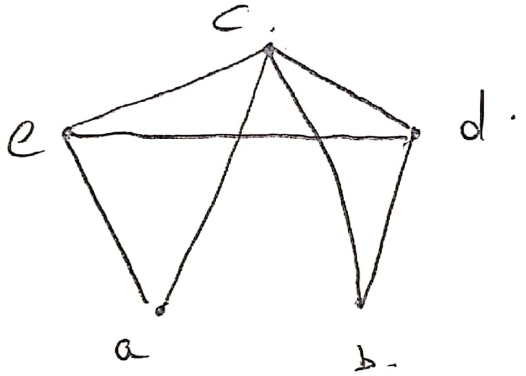
d)

I and J have both 5 vertices; 7 edges, 2 vertices of degree 3; 2 vertices of degree 2 and on vertex of degree 4.

I and J are isomorphic.

x	1	5	2	3	4
$f(x)$	c	e	d	b	a

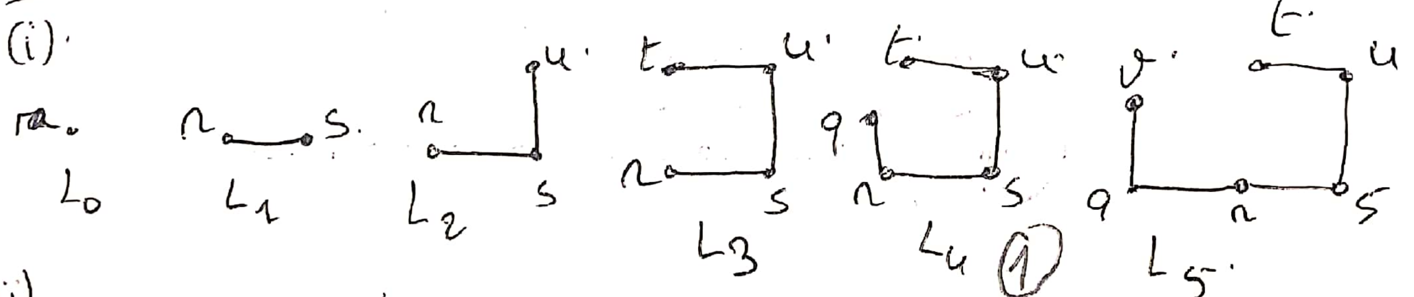
(2)



Q4) (5)

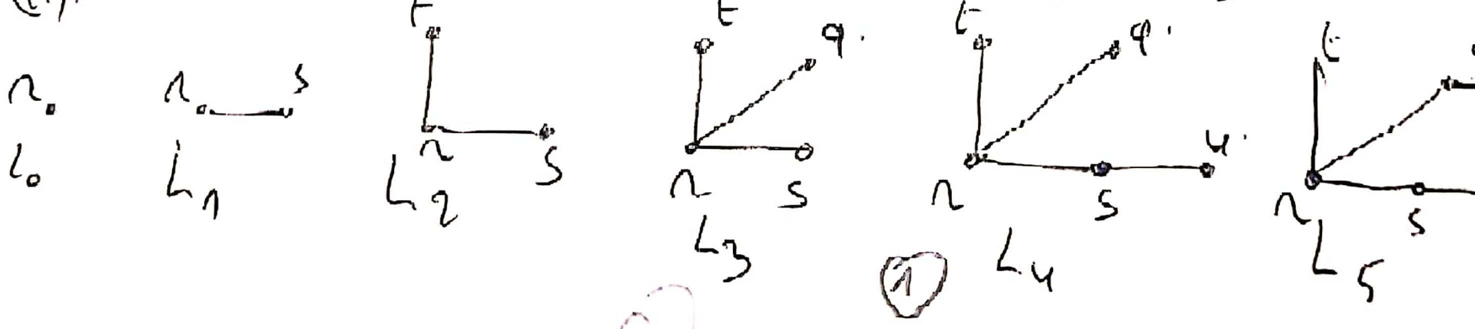
a)

(i)



(1)

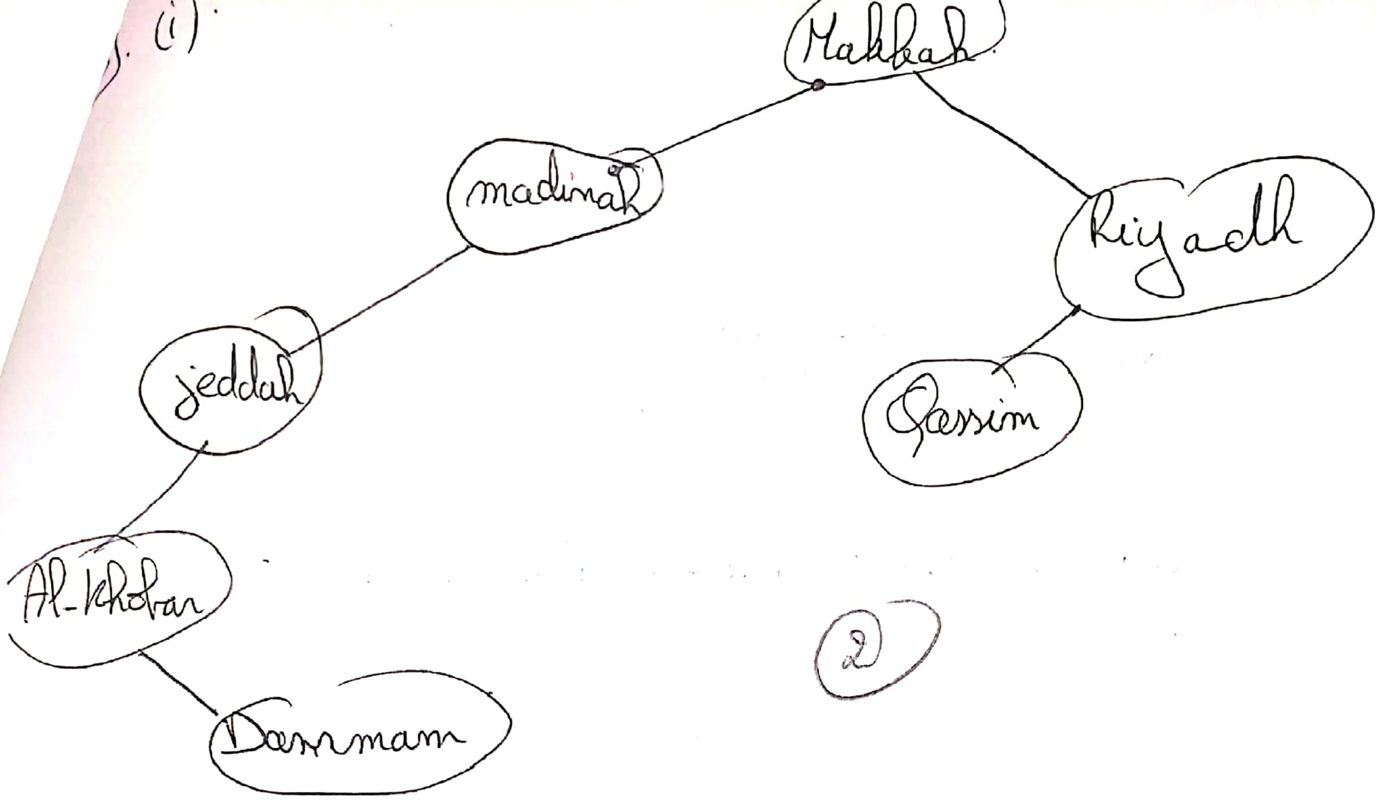
(ii)



(1)

(4)

(i)



(2)

(ii) Not full binary, because not all internal vertices has 2 children. (1)

Q5 (7)

a)

(i) $f(x,y,z) = (\bar{x}+\bar{y})(x+z) = \bar{x}z + \bar{y}x + \bar{y}z$

$= \bar{x}yz + \bar{x}\bar{y}z + x\bar{y}z + x\bar{y}\bar{z} + \cancel{x\bar{y}z} + \cancel{\bar{x}\bar{y}z}$ (2)

$= \bar{x}yz + \bar{x}\bar{y}z + x\bar{y}z + x\bar{y}\bar{z}$

$= CSP(f)$

(ii) $\bar{f}(x,y,z) = xy + \bar{x}\bar{z} = xyz + xy\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z} = CSP(\bar{f})$

$CPS(f) = (\bar{x}+\bar{y}+\bar{z})(\bar{x}+\bar{y}+z)(x+\bar{y}+z)(x+y+z)$.

b) (i)

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
x	1	1		1
$\bar{x}$		1	1	

(2) (1)

(ii) $MSP(g) = xy + xz + \bar{x}\bar{z}$ (2)

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