KING SAUD UNIVERSITY

Math Department **June 2026**

Time: 180mn

Final exam Math106

Question 1(3+2+3)

a) Find the number(s) c that satisfies the conclusion of the mean value theorem for the function $f(x) = 2 + 3\sqrt{x}$ on $[1, 4]$.

b) Evaluate the integral $\int \frac{dx}{xlnx-x}$

c) Use Riemann sums to compute $\int_0^2 4 - x^2 dx$

Question 2(3+3+3)

a) Find $\lim_{x \to +\infty} (1 + e^x)^{\frac{1}{x}}$

b) Find the indefinite integral $\int \sqrt{x}(lnx)^2 dx$

c) Compute $\int (tanx)^5 (secx)^8 dx$

Question 3(3+3+2)

a) Evaluate $\int \frac{1}{x^2\sqrt{4+x^2}} dx$

b) Evaluate the integral $\int \frac{3x-1}{x^2+4x+8} dx$

c) Compute $\int (sin5x)cosxdx$

Question 4(3+3+3)

a) Does the integral $\int_0^{\pi/2} \frac{cosx}{\sqrt{sinx}} dx$ converge? Find its value if it does.

b) Sketch the region bounded by $x = y^2,\ x - y = 2$, and find its area.

c) Find the volume of the solid obtained by revolving the region bounded by

$y = (x-1)^2,\ y = 0\ , x = 1, x = 2$ about the y-axis.

Question5(3+3)

a) Find the area of the surface obtained by revolving the curve $y = 2\sqrt{x+1}$,$0 \le x \le 3$ about the x-axis.

b) Sketch the region inside $r = 2 + 2cos\theta$ and outside $r = 2 - 2cos\theta$ and find its area.

Grading scheme final 106-June 2026

Q1

a)

$\int_1^4 2 + 3\sqrt{x}dx = 20 = 3(2 + 3\sqrt{c})$ **(2)**

So $c = \frac{196}{81}$. **(1)**

b) $\int \frac{dx}{xlnx-x} = \int \frac{du}{u-1} = ln|lnx - 1| + C$. **(1) +(1)**

c) $R_n = \sum_1^n f(u_i)\Delta x_i = \sum_1^n \left(4 - \frac{4i^2}{n^2}\right)\frac{2}{n} = 8 - \frac{8n(n+1)(2n+1)}{6n^3}$. **(2)**

thus $\int_0^2 4 - x^2 dx = 16/3$. **(1)**

Q2

a) If $y = (1 + e^x)^{\frac{1}{x}}$

$\lim_{x\to\infty} \frac{ln(1+e^x)}{x} = \lim_{x\to+\infty} \frac{e^x}{1+e^x} = 1$ **(2.5)**

Thus $\lim_{x\to+\infty} (1 + e^x)^{1/x} = e$. **(0.5)**

b) $\int \sqrt{x}(lnx)^2 dx = \frac{2}{3}x^{3/2}(lnx)^2 - \frac{4}{3}\int \sqrt{x}lnxdx$

$= \frac{2}{3}x^{3/2}(lnx)^2 - \frac{8}{9}x^{\frac{3}{2}}lnx + \frac{16}{27}x^{\frac{3}{2}} + C$

(1.5) +(1.5)

c) $\int (tanx)^5(secx)^8 dx = \int (u^2 - 1)^2 u^7 du$

$= \frac{(secx)^{12}}{12} - \frac{(secx)^{10}}{5} + \frac{(secx)^8}{8} + C$

(1.5) + (1.5)

Q3

a) $\int \frac{dx}{x^2\sqrt{4+x^2}} = \int \frac{cos\theta d\theta}{4(sin\theta)^2} = \frac{-1}{4sin\theta} + C = -\frac{\sqrt{4+x^2}}{4x} + C$

(2) + (1)

b) $\int \frac{3x-1}{x^2+4x+8} dx = \int \frac{3u-7}{u^2+4} du = \frac{3}{2} ln(x^2+4x+8) - \frac{7}{2}\tan^{-1}\frac{(x+2)}{2} + C$

(1.5) +(1.5)

c)

$\int (sin5x)cosxdx$

$$= \frac{1}{2}\int (sin6x + sin4x)dx = \frac{-1}{12}cos6x - \frac{1}{8}cos4x + C$$

(1) + (1)

Q4

a) $\int_c^{\pi/2} \frac{cosx}{\sqrt{sinx}} dx = \int_{sinc}^{1} \frac{du}{\sqrt{u}} = 2 - 2\sqrt{sinc}.$

So the integral converges and its value is 2.

(2) +(1)

b) Sketch **(1)**

$y + 2 = y^2 \Leftrightarrow y = -1\ or\ y = 2$ **(0.5)**

$A = \int_{-1}^{2} y + 2 - y^2 dy = \frac{9}{2}$ **(1.5)**

c) $\int_1^2 2\pi x(x-1)^2 dx = 2\pi \int_0^1 (u+1)u^2 du = \frac{7\pi}{6}$

(2)+(1)

Q5

a)

$$S = \int_0^3 4\pi\sqrt{x+1}\sqrt{1+\frac{1}{x+1}}dx = 4\pi\int_0^3 \sqrt{x+2}dx$$

$$= \frac{8\pi}{3}(5\sqrt{5} - 2\sqrt{2})$$

(2) +(1)

b) Graph **(1)**

$$A = \int_0^{\pi/2} (2 + 2cos\theta)^2 - (2 - 2cos\theta)^2 d\theta$$

$$= 16\int_0^{\pi/2} cos\theta d\theta = 16$$

(1.5) +(0.5)