Name: Answer Sheet (Dr Borhen)
ID:
Num of attendance:

Time: 45 min

Quiz 2 - Math 204 - Semester I - 1444 H

Marks: 10

Q1) Determine a homogeneous linear differential equation with constant coefficients having the fundamental set of solutions:

$$y_1 = 5; \quad y_2 = 2x; \quad y_3 = e^{-2x}\cos 3x; \quad y_4 = e^{-2x}\sin 3x; \quad y_5 = x^2.$$

Answer: The function $y = 5 + 2x + x^2 + e^{-2x}(\cos(3x) + \sin(3x))$ is a solution of a linear diff eq which is a special case of the general solution: $y_c = c_1 + c_2 x + c_3 x^2 + e^{-2x}(c_4 \cos(3x) + c_5 \sin$

Then the roots of the characteristic eq of the DE will be

$m = 0, 0, 0, \; -2 \pm 3i$

Hence the characteristic eq is $m^3(m+2-3i)(m+2+3i) = 0$

$$m^3((m+2)^2 + 9) = 0$$

$$m^3(m^2 + 4m + 13) = 0$$

So the differential eq is $\boxed{y^{(5)} + 4y^{(4)} + 13y''' = 0}$.

4

Q2) Find the general solution of the differential equation

$$x^3\, y''' - x^2\, y'' - 2x\, y' - 4\, y = 0; \quad x > 0.$$

Answer: Put $y = x^m$; $y' = m x^{m-1}$; $y'' = m(m-1)x^{m-2}$; $y''' = m(m-1)(m$

$x^3[m(m-1)(m-2)x^{m-3}] - x^2[m(m-1)x^{m-2}] - 2x\, m\, x^{m-1} - 4x^m = 0$; $x >$

$x^m[m(m-1)(m-2) - m(m-1) - 2m - 4] = 0$; $x^m \neq 0$ then

$$(m^3 - 3m^2 + \cancel{2m} - m^2 + m - \cancel{2m} - 4) = 0$$

$$m^3 - 4m^2 + m - 4 = 0$$

$$m^2(m-4) + (m-4) = (m-4)(m^2+1) = 0$$

Hence $m = 4$ or $m = \pm i$.

The general solution is $\boxed{y = c_1 x^4 + c_2 \cos(\ln x) + c_3 \sin(\ln x)}$.

3

Q3) Determine only **the form** of the particular solution y_p of the differential equation

$$y'' + 2y' - 3y = 2 + 3x + 3\,x^2e^x + e^{2x} + x\sin x.$$

Answer: The characteristic eq of $y''+2y'-3y=0$

is $m^2+2m-3=0 \Leftrightarrow (m+3)(m-1)=0$ (*)

whose roots are $m=-3$, $m=1$

3

$f(x) = 2+3x+3x^2e^x+e^{2x}+x\sin x$

We observe that $r=1$ is a root of (*) and $r=2$, $r=0$

and $m=\pm i$ are not roots of (*), then the particular

solution of the DE is:

$$\boxed{y_p = (A_0+A_1x) + x(A_2+A_3x+A_4x^2)e^x + A_5e^{2x} + (A_6+A_7x)\cdots + (A_8+A_9x)\cdots}$$