جامعة الملك سعود
King Saud University

Miterm 1 – Math 111- semester 1 – 1446H
Time: 1h 30 min

1) Use Rieman sum to evaluate $\int_0^2 (x^2+3)dx$ (3 marks)

2) Find $\acute{F}(x)$ if $F(x)=\int_{tan(^x/_2)}^{3^{5x}} \sqrt{t^2+1}\,dt$ (2 marks)

Find $\acute{y}$ of the following: (2+2)

3) $y = [cos^{-1}(3x)]\ log\,|csc(x) - cot(x)|$

4) $y = (cot\,x)^{csc\,x} + 3^{x^2}$

Q2: Evaluate the following integrals (2 marks for each integral)

1) $\int \left(x^{\frac{2}{5}}\sqrt[5]{sin\,x^3}\right)^5 dx$

2) $\int \frac{-1}{\sqrt{e^{6x}-4}} dx$

3) $\int_0^1 e^{2ln(x)}\,3^{3x^3} dx$

4) $\int \frac{x+1}{\sqrt{1-x^2}} dx$

5) $\int \frac{sec(ln\,x^2)}{x} dx$ $\frac{2x}{x^2}$ $\frac{1}{x}$

6) $\int \frac{tan\left(\sqrt[3]{x}\right)}{x^{\frac{3}{2}}} dx$

7) $\int \frac{(1+sec^{-1}x)^5}{\sqrt{x^4-x^2}} dx$

8) $\int (cos\,x)\,(sin^2 x)^{-1} dx$

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① Use Rieman Sum to evaluate

$$\int_0^2 (x^2+3)\,dx$$

Ans: $a=0$; $b=2$; $f(x)=x^2+3$

- $\Delta x = \frac{b-a}{n} = 2/n$
- $x_k = a + k\Delta x = \frac{2k}{n}$; $0 \le k \le n$
- $f(x_k) = \left(\frac{2k}{n}\right)^2 + 3 = \frac{4k^2}{n^2} + 3$
- $\sum_{k=0}^{n} f(x_k) = \frac{4}{n^2}\left(\sum_{k=0}^{n} k^2\right) + 3\left(\sum_{k=0}^{n} 1\right)$

$$= \frac{4}{n^2}\,\frac{n(n+1)(2n+1)}{6} + 3(n+1)$$

②

$$\int_0^2 (x^2+3)\,dx = \lim_{n\to+\infty} \frac{2}{n}\left(\sum_{k=0}^{n} f(x_k)\right)$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[\frac{4(n+1)(2n+1)}{6n} + 3(n+1)\right]$$

$$= \frac{16}{6} + 6 = \frac{8}{3} + 6 = \frac{26}{3}$$

Check $\left[\frac{x^3}{3} + 3x\right]_0^2 = \frac{8}{3} + 6 = \frac{26}{3}$

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② $F(x) = \int_{\tan(\frac{x}{2})}^{3^{5x}} \sqrt{t^2+1}\, dt$

$= \int_{g(x)}^{h(x)} f(t)\, dt$

where $g(x) = \tan(x/2)$; $h(x) = 3^{5x}$; $f(t) = \sqrt{t^2+1}$

$F'(x) = f(h(x))\, h'(x) - f(g(x))\, g'(x)$

$= f(3^{5x})(5\ln 3)\, 3^{5x} - f(\tan(x/2))\, \frac{1}{2}\sec^2(x/2)$

$= \sqrt{(3^{5x})^2+1}\, (5\ln 3)\, 3^{5x} - \frac{1}{2}\sqrt{\tan^2(x/2)+1}\, \sec^2(x/2)$.

③ $y = \cos^{-1}(3x) \log|\csc x - \cot x|$

$y' = \frac{-3}{\sqrt{1-(3x)^2}} \log|\csc x - \cot x| + \frac{\cos^{-1}(3x)}{\ln 10} \frac{\csc^2 x - \csc x \cot x}{\csc x - \cot x}$

④ $y = (\cot x)^{\csc x} + 3^{x^2}$

$= y_1 + y_2$

$y' = y_1' + y_2'$

$y_1 = (\cot x)^{\csc x}$

$\ln y_1 = \csc x \cdot \ln(\cot x)$

$\frac{y_1'}{y_1} = -\csc x \cot x \ln(\cot x) + \csc x \frac{-\csc^2 x}{\cot x}$

$y_1' = \left[-\csc x \cot x \ln|\cot x| - \frac{\csc^3 x}{\cot x}\right] (\cot x)^{\csc x}$

$y_2' = (\ln 3)(2x)\, 3^{x^2}$

So $y' = y_1' + y_2'$

① $I = \int \left(x^{2/5} \sqrt[5]{\sin(x^3)}\right)^5 dx$

$= \int x^2 \sin(x^3)\, dx$

$\overset{u=x^3}{=} \frac{1}{3}\int \sin u\, du$

$= -1/3 \cos u + C$

③ $K = \int_0^1 e$

④ $L = \int \frac{x+1}{\sqrt{1-}}$

⑤ $M = \int \frac{Arc}{}$

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$$\int \frac{u'(x)}{2\sqrt{u(x)}}\,dx = \sqrt{u(x)}+c$$

$$\left[-\frac{\csc^3 x}{\cot x}\right](\cot x)^{\csc x}$$

$$I=\int \left(x^{2/5}\sqrt[5]{\sin(x^3)}\right)^5 dx = -\frac{1}{3}\cos(x^3)+c$$

(2) $$J=-\int \frac{dx}{\sqrt{e^{6x}-4}} = -\int \frac{dx}{\sqrt{(e^{3x})^2-2^2}}$$

$u=e^{3x}$

$du=3e^{3x}dx$

$du=3u\,dx$

$$=-\frac{1}{3}\int \frac{du}{u\sqrt{u^2-2^2}} = -\frac{1}{6}\sec^{-1}\left(\frac{u}{2}\right)+c$$

$$=-\frac{1}{6}\sec^{-1}\left(\frac{e^{3x}}{2}\right)+c$$

(3) $$K=\int_0^1 e^{2\ln x}\, 3^{3x^3}dx = \int_0^1 x^2\, 3^{3x^3} dx = \frac{1}{9}\int_0^3 3^u du = \frac{1}{9\ln 3}\left[3^u\right]_0^3 = \frac{26}{9\ln 3}$$

$u=3x^3$

$du=9x^2dx$

(4) $$L=\int \frac{x+1}{\sqrt{1-x^2}}dx = -\int \frac{-2x}{2\sqrt{1-x^2}}dx + \int \frac{dx}{\sqrt{1-x^2}} = -\sqrt{1-x^2}+\sin^{-1}x+c$$

(5) $$M=\int \frac{\sec(\ln(x^2))}{x}dx = \frac{1}{2}\int \sec u\,du = \frac{1}{2}\ln|\sec u+\tan u|+c$$

$$=\frac{1}{2}\ln|\sec(2\ln x)+\tan(2\ln x)|+c$$

$u=\ln(x^2)=2\ln x$

$du=\frac{2}{x}dx$

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⑥ $$N = \int \frac{\tan(\sqrt[3]{x})}{x^{2/3}}\,dx = 3\int \tan u \, du$$

$$= 3\ln|\sec u| + c$$

$$= 3\ln|\sec(x^{1/3})| + c$$

$$u = x^{1/3}$$

$$du = \frac{1}{3}x^{-2/3}\,dx = \frac{1}{3}\,\frac{dx}{x^{2/3}}$$

⑦ $$P = \int \frac{(1+\sec^{-1}x)^5}{\sqrt{x^4-x^2}}\,dx = \int \frac{(1+\sec^{-1}x)^5}{x\sqrt{x^2-1}}\,dx$$

$$= \int u^5\,du$$

$$= u^6/6 + c$$

$$= \frac{1}{6}(1+\sec^{-1}x)^6 + c$$

$$u = 1+\sec^{-1}x$$

$$du = \frac{dx}{x\sqrt{x^2-1}}$$

⑧ $$Q = \int \frac{\cos x}{\sin^2 x}\,dx$$

$$= \int \underbrace{\frac{\cos x}{\sin x}} \cdot \underbrace{\frac{1}{\sin x}}\,dx$$

$$= \int \cot x \, \csc x \, dx$$

$$= -\csc x + c$$

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