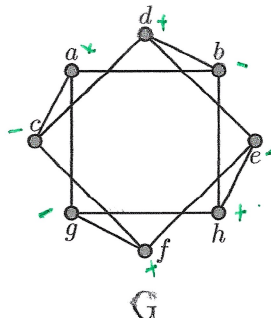


Final Exam in Math 151
Semester 1, 1443 H.

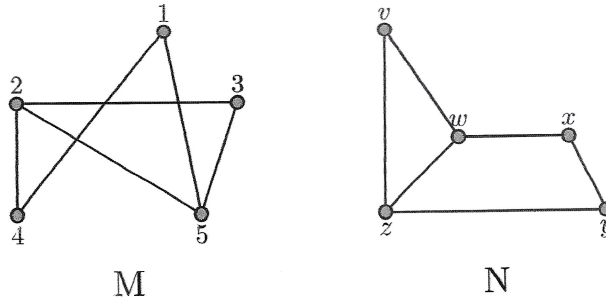
Calculators are not allowed
The Examination contains 2 pages

- Q1.** (a) Without using truth tables show that $(p \wedge q) \vee (\neg p \wedge q) \vee (p \wedge \neg q) \equiv p \vee q$. (2pts)
(b) Suppose a , b and c are integers. Prove that if a divides $(b - 1)$ and a divides $(c - 1)$; then a divides $(bc - 1)$. (3pts)
(c) Use induction to show that $1 + 5 + 9 + \dots + (4n - 3) = n(2n - 1)$ for all $n \geq 1$. (4pts)
- Q2.** (a) Let R be the relation on $\mathbb{Q}$ (the set of rational numbers) defined by xRy if and only if $x - y$ is an integer.
(i) Show that R is an equivalence relation. (3pts)
(ii) Find the equivalence class $[0]$. (1pts)
(iii) Show that $[7/16] \neq [-1/8]$. (1pts)
(b) Let $P = \{(a, a), (a, c), (b, b), (b, d), (c, c), (d, d)\}$ be a relation on the set $A = \{a, b, c, d\}$.
(i) Represent P by a digraph. (1pts)
(ii) Show that P is a partial order. (3pts)
(iii) Is P a total order? (Justify your answer.) (1pts)
(iv) Represent P by a Hasse diagram. (1pts)
- Q3.** (a) Find the number of vertices n of a complete graph K_n which has 36 edges. (2pts)
(b) Let G be the graph below.

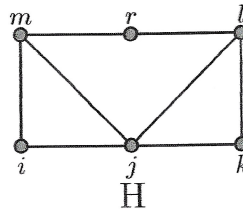


- (i) Determine whether G is bipartite. (Justify your answer)(2pts)
(ii) Find a simple path from a to h of length 4. (1pts)
(iii) Find a cycle from a to a of length 6. (1pts)

- (c) Determine whether the following graphs M and N are isomorphic. (Justify your answer)(2pts)



- Q4.** (a) For the graph H below, find a spanning tree with root r ,



- (i) using *depth-first* search; (1 pts)
(ii) using *breadth-first* search. (1 pts)
- (b) Form a binary search tree for the words *english*, *chinese*, *russian*, *german*, *arabic*, *spanish*, *french*. (using alphabetical order). (2 pts)

- Q5.** (a) Prove the following Boolean identity. (2pts)

$$(x + y)(x + \bar{y})(\bar{x} + y) = xy.$$

- (b) Find the complete sum-of-products expansion (CSP) for the function $f(x, y, z) = (x + \bar{y}z)(\bar{x} + z)$. (2pts)
- (c) Find the complete product-of-sums expansion (CPS) for the function $g(x, y, z) = \bar{y} + x\bar{z}$. (2pts)
- (d) Use a K-map to simplify the following sum-of-products expansion (i.e., write in MSP form).(2pts)

$$xyz + x\bar{y}\bar{z} + x\bar{y}z + \bar{x}y\bar{z} + \bar{x}\bar{y}z.$$

Answer final examination
Math 151.
Semester 1 1443.

Question 1: (9 marks)

a) ②

$$(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q)$$

$$\equiv (Q \wedge (P \vee \neg P)) \vee (P \wedge \neg Q)$$

$$\equiv (Q \wedge T) \vee (P \wedge \neg Q)$$

$$\equiv (Q \vee P) \wedge (Q \vee \neg Q) \equiv (Q \vee P) \wedge T \equiv Q \vee P.$$

b) ③

$$a|b-1 \Rightarrow b-1 = q_1 a \Rightarrow b = q_1 a + 1.$$

$$d|c-1 \Rightarrow c-1 = q_2 a \Rightarrow c = q_2 a + 1.$$

$$bc = (q_1 a + 1)(q_2 a + 1).$$

$$= q_1 q_2 a^2 + q_1 a + q_2 a + 1.$$

$$\Rightarrow bc - 1 = q_1 q_2 a^2 + q_1 a + q_2 a$$

$$= a(q_1 q_2 a + q_1 + q_2)$$

$$\Rightarrow a|bc-1.$$

c) ④

$$P(n): 1+5+9+\dots+(4n-3) = n(2n-1) \quad \forall n \geq 1.$$

B.S. $P(1): 1 = 1(2-1) = 1$ True.

I.S. Let $h \geq 1$; we assume that $P(h)$ is true

$$1+5+9+\dots+(4h-3) = h(2h-1) \quad \text{I.H.}$$

and we prove that $P(h+1)$ is true.

$$1+5+9+\dots+(4(h+1)-3) = (h+1)(2(h+1)-1) \\ = (h+1)(2h+1).$$

$$1+5+9+\dots+(4h+1)$$

$$= 1+5+9+\dots+(4h-3) + (4h+1)$$

$$= h(2h-1) + 4h+1.$$

$$= 2h^2 - h + 4h + 1$$

$$= 2h^2 + 3h + 1 = \text{L.H.S.}$$

$$\text{R.H.S.}: (h+1)(2h+1) = 2h^2 + 2h + h + 1 \\ = 2h^2 + 3h + 1 = \text{L.H.S.}$$

So $P(h+1)$ is true

$\Rightarrow P(1)$ true

$P(h) \rightarrow P(h+1) \quad \forall h \geq 1 \quad \Rightarrow \forall n \geq 1$
 $P(n)$ is true.

Question 2: (12 marks)

a)

(i) ③ Let $x \in \mathbb{Q}; x - x = 0 \in \mathbb{Z}$

$\Rightarrow x R x \Rightarrow R$ is reflexive ①.

Let $x, y \in \mathbb{Q}$ such that $x R y$

$$\Rightarrow x - y \in \mathbb{Z} \Rightarrow y - x = -(x - y) \in \mathbb{Z}$$

$$\Rightarrow y R x$$

$\Rightarrow R$ is symmetric ②.

Let $x, y, z \in \mathbb{Q}$ such that $x R y$ and $y R z$

$$\Rightarrow x - y \in \mathbb{Z} \text{ and } y - z \in \mathbb{Z}$$

$$\Rightarrow (x - y) + (y - z) \in \mathbb{Z}$$

$$\Rightarrow x - z \in \mathbb{Z} \Rightarrow x R z$$

$\Rightarrow R$ is transitive ③

①, ② and ③ $\Rightarrow R$ is an equivalence relation on $\mathbb{Z}$.

(ii) ①

$$[0] = \{x \in \mathbb{Q}; x R 0\}.$$

$$= \{x \in \mathbb{Q}; x - 0 \in \mathbb{Z}\}$$

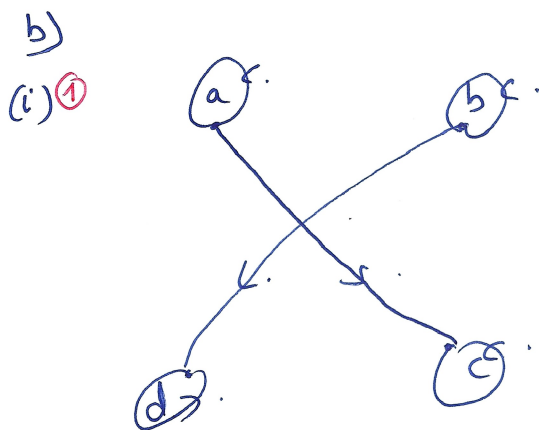
$$= \{x \in \mathbb{Q}; x \in \mathbb{Z}\} = \mathbb{Z}.$$

(iii) ①

$$\frac{7}{16} - (-\frac{1}{8}) = \frac{7}{16} + \frac{1}{8}$$

$$= \frac{7+2}{16} = \frac{9}{16} \notin \mathbb{Z}.$$

$$\Rightarrow \frac{7}{16} R (-\frac{1}{8}) \Rightarrow [\frac{7}{16}] \neq [-\frac{1}{8}].$$



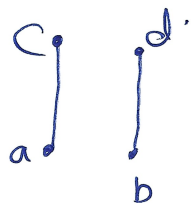
(ii) ③

- $(a,a), (b,b), (c,c), (d,d) \in P$
 $\Rightarrow P$ is reflexive ①
- $(a,c) \in P; (c,a) \notin P$
 $(b,d) \in P; (d,b) \notin P$ } $\Rightarrow P$ is anti-sym ②
- P is transitive ③

①, ② and ③ $\Rightarrow P$ is a partial order.

(iii) ① P is not a total order because a, b are not comparable

(iv) ①



Question 3: (8 marks)

a) ②

$$|E(K_n)| = \frac{n(n-1)}{2} \Rightarrow \frac{n(n-1)}{2} = 36.$$

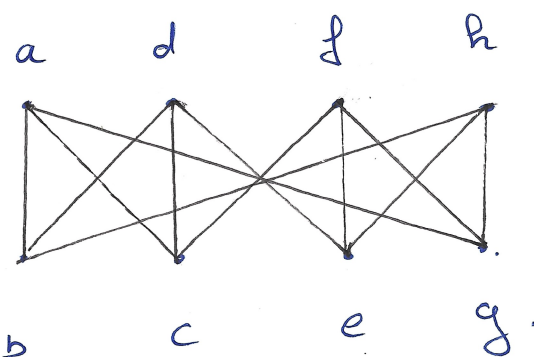
$$\Rightarrow n(n-1) = 72.$$

$$\Rightarrow n = 9.$$

K_n contain 9 vertices.

b)

(i) ② ps



(ii) ①

$a, g, f, e, h.$

(iii) ①

$a, b, d, e, f, c, a.$

c) ②

$$S_1$$

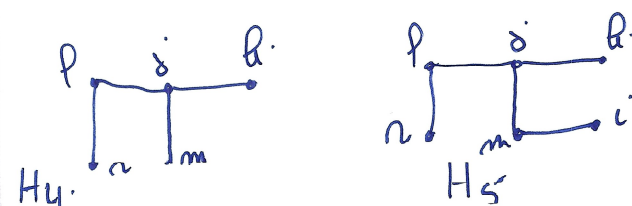
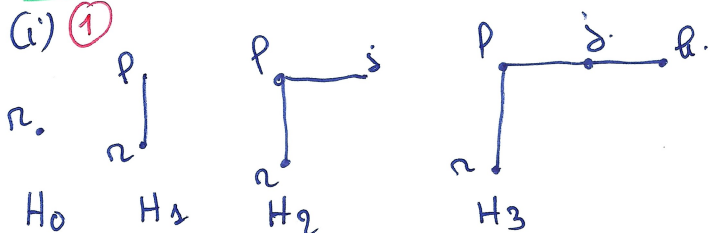
a	2	5	3	4	1
f(a)	w	z	v	x	y

$$S_2$$

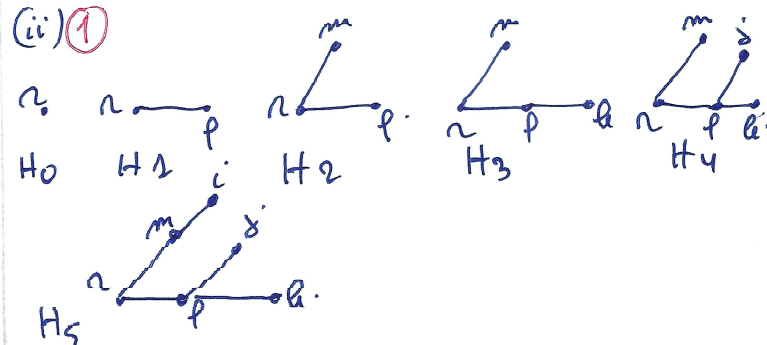
a	2	5	3	4	1
f(a)	z	w	v	y	x

Question 4: (4 marks)

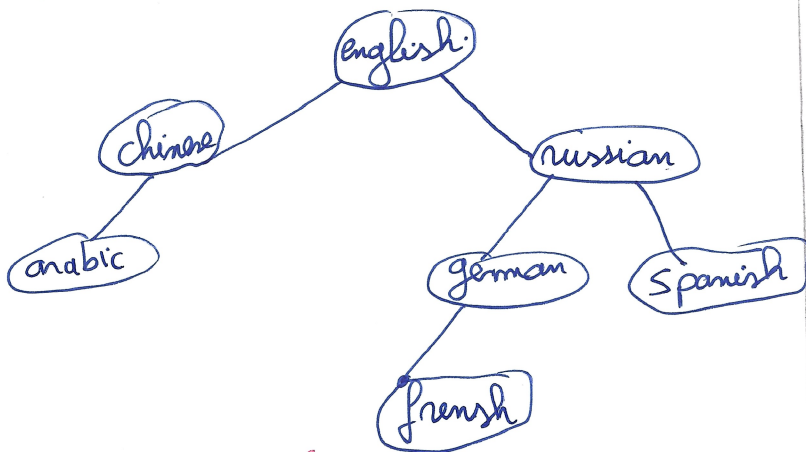
(i) ①



(ii) ①



b) ②



Question 5: (8 marks)

a) ② $(x+y)(x+y)(\bar{x}+y)$
 $= (x + x\bar{y} + xy + y\bar{y})(\bar{x}+y)$
 $= (x + x\bar{y} + xy)(\bar{x}+y)$
 $= x\bar{x} + x\bar{y} + x\bar{y}\bar{x} + x\bar{y}y + xy\bar{x} + xy y$
 $= x\bar{y} + xy = xy$

b) ②

$$f(x, y, z) = (x + \bar{y}z)(\bar{x} + z)$$

$$= x\bar{x} + xz + \bar{x}\bar{y}z + \bar{y}zz$$

$$= xz + \bar{x}\bar{y}z + \bar{y}z$$

$$= xyz + x\bar{y}z + \bar{x}\bar{y}z + x\bar{y}z + \bar{x}\bar{y}z$$

$$CSP(f) = xyz + x\bar{y}z + \bar{x}\bar{y}z + x\bar{y}z$$

c) ②

$$g(x, y, z) = \bar{y} + x\bar{z}$$

$$\Rightarrow \bar{g}(x, y, z) = y \cdot (\bar{x} + z)$$

$$= y\bar{x} + yz$$

$$= \bar{x}yz + \bar{x}y\bar{z} + xyz + \bar{x}y\bar{z}$$

$$CSP(\bar{g}) = \bar{x}yz + \bar{x}y\bar{z} + xyz$$

$$\Rightarrow CPS(g) = (x + \bar{y} + \bar{z})(x + \bar{y} + z)(\bar{x} + \bar{y} + \bar{z})$$

d)

	yz	$y\bar{z}$	$\bar{y}z$	$\bar{y}\bar{z}$
x	1	0	1	1
$\bar{x}$	0	1	0	1

$$MSP(h) = xz + x\bar{y} + \bar{y}z + \bar{x}y\bar{z}$$