Mathematics of Finance (2): Actu. 461

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Theses notes are based on the following references

1. **Robert McDonald: Derivatives markets 3rd edition 2013**

2. **John C. Hull: Options, Futures, and Other Derivatives, Pearson, Prentice Hall 8th Ed. 2012**

Chapter 4

Futures contracts

4.1 Forwards: Alternative derivation of formula

Spot transaction

- Price agreed to.
- Price paid/received.
- Item exchanged.

Prepaid forward contract

- Price agreed to.
- Price paid/received.
- Item exchanged in T–years.

Forward contract

- Price agreed to
- Price paid/received in T–years.
- Item exchanged in T–years.

A forward contract has two risks: **market risk** and **credit risk**. The market risk is related with the **volatility** of the asset price. The credit risk is related with the solvency of each party.

4.1.1 Futures: Definition

A future contract is a standardized agreement in which two counterparts agree to buy/sell an asset for a specified price at a specified period. The buyer in the future contract is said to be in long position (LP) on futures.

The seller in the future contract is said to be in short position (SP) on futures. The main reasons to enter into a future contract are hedging and speculation.

Difference between forwards and futures

Recall forward contracts are privately negotiated and are not standardized. Forward contracts are entirely flexible. Forward contracts are tailor–made contracts.
Futures contracts are standardized instruments and have clearing houses that guarantee the transactions, which drastically lowers the probability of default to almost never.

The specific details concerning settlement and delivery are quite distinct.

Futures contracts are **marked–to–market** daily or weekly, settlement for futures can occur over a range of dates.

A clearing house is an agency or separate corporation of a futures exchange responsible for settling trading accounts, clearing trades, collecting and maintaining margin monies, regulating delivery and reporting trading data. Clearinghouses act as third parties to all futures and options contracts as a buyer to every clearing member seller and a seller to every clearing member buyer. Like forward contracts, futures contracts are contracts for deferred delivery. But, unlike forward contracts, futures contracts are marked to market daily or weekly. Consider "corresponding" forward and futures contracts:

- Same underlying asset.
- Delivery date in two days.
- The contracts are identical except:
- i) Forward contract is settled at maturity.
- ii) Futures contract is settled daily.
- Forward ignore taxes, transaction costs, and the treatment of margins.

4.2 Forward prices & futures prices

Example: Suppose we have for $T = 2$:
Day 0: $G(0, 0, 2) = 20$ SAR
Day 1: $G(0, 1, 2) = 10$ SAR with a 50% probability and $G(0, 1, 2) = 30$ SAR with a 50% probability
Day 2: $G(0, 2, 2) = S_2$ since the futures contract terminates.

Suppose that the interest rate is a constant 10% (effective per day).

If on day 1 $G(0, 1, 2) = 10$ SAR, the P&L of the buyer is $G(0, 1, 2) - G(0, 0, 2) = -10$ SAR. She (He) would borrow this amount at $r = 10\%$ and have to repay 11 SAR **on day 2**.

If on day 1 $G(0, 1, 2) = 30$ SAR, the P&L of the buyer is $= G(0, 1, 2) - G(0, 0, 2) = 10$ SAR. She (He) would invest this amount at $r = 10\%$ and have 11 SAR **on day 2**.

Since there is a 50% chance of paying interest of 1 SAR and a 50% chance of earning interest of 1 SAR, there is no expected benefit from marking to market **on day 1**.

Since futures contract offers no benefit as compared to the forward contract $F(0, 0, T) = G(0, 0, T)$.

Now suppose that the interest rate is not constant. Suppose that $r = 12\%$ on day 1 if $G(0, 1, 2) = 30$ SAR and $r = 8\%$ on day 1 if $G(0, 1, 2) = 10$ SAR.

If on day 1 $G(0,1,2) = 10$ SAR then the P&L of the buyer is $G(0,1,2) - G(0,0,2) = -10$ SAR. She (He) would borrow this amount at $r = 8\%$ and have to repay 10.8 SAR **on day 2**.

If on day 1 $G(0,1,2) = 30$ SAR then the P&L of the buyer is $G(0,1,2) - G(0,0,2) = 10$ SAR. She (He) would invest this amount at $r = 12\%$ and have 11.2 SAR **on day 2**.

Now there is an expected gain from marking to market = $(50\% \times 0.12 - 50\% \times 0.08) = 0.02$ SAR.

Since the futures contract offers a benefit as compared to the forward contract, $G(0,0,T)$ must exceed $F(0,0,T)$.

Now suppose that the interest rate is not constant. Suppose that $r = 8\%$ on day 1 if $G(0,1,2) = 30$ SAR and $r = 12\%$ on day 1 if $G(0,1,2) = 10$ SAR.

If on day 1 $G(0,1,2) = 10$ SAR then the P&L of the buyer is $G(0,1,2) - G(0,0,2) = -10$ SAR. She (He) would borrow this amount at $r = 12\%$ and have to repay 11.2 SAR **on day 2**.

If on day 1 $G(0,1,2) = 30$ SAR then the P&L of the buyer is $G(0,1,2) - G(0,0,2) = 10$ SAR. She (He) would invest this amount at $r = 8\%$ and have 10.8 SAR **on day 2**.

Now there is an expected P&L from marking to market = $(50\% \times 0.08 - 50\% \times 0.12) = -0.02$ SAR.

Since the futures contract produces a loss as compared to the forward contract, $F(0,0,T)$ must exceed $G(0,0,T)$.

With this reasoning situations:

1. $G(0,0,T) = F(0,0,T)$ when interest rates are uncorrelated with the futures price.

2. $G(0,0,T) \geq F(0,0,T)$ when interest rates are positively correlated with the futures price.

3. $G(0,0,T) \leq F(0,0,T)$ when interest rates are negatively correlated with the futures price.

Stock index futures contracts

- Stock index: a weighted average of the prices of a selected number of stocks.

- Underlying: the portfolio of stocks comprising the index.

- Stock index futures contracts are heavily traded

- Examples of stock indices (futures exchanges):

  - S&P/TSX Canada 60 Index (ME)
  - S&P500 Composite Index (CME)
  - NYSE Composite Index (NYFE)

4.3 Where you buy and/or sell futures contracts

Futures are bought and sold in organized futures exchanges. The biggest future exchanges are:

- South African Futures Exchange (SAFEX)
- China Financial Futures Exchange (CFFEX)
- Shanghai Futures Exchange (SHFE)
- International Petroleum Exchange of London
- New York Mercantile Exchange
- London Metal Exchange
- Tokyo Commodity Exchange
- Hong Kong Futures Exchange (HKFE)
- Taiwan Futures Exchange (TAIFEX)
- Turkish Derivatives Exchange (TURDEX)
- Agricultural Futures Exchange of Thailand (AFET)
- Mercado Español de Futuros Financieros (MEFF)
- ICE Futures Europe, formerly London International Financial Futures and Options Exchange (LIFFE)

Futures. Examples of underlying assets on which futures contracts are traded.

Category	**Description**
Stock index	S&P 500 index, Euro Stoxx 50 index, Nikkei 225, Dow-Jones Industrials, Dax, NASDAQ, Russell 2000, S&P Sectors (healthcare, utilities, technology)
Interest rate	30-year U.S. Treasury bond, 10-year U.S. Treasury notes, Fed funds rate, Euro-Bund, Euro-Bobl, LIBOR, Euribor
Foreign exchange	Euro, Japanese yen, British pound, Swiss franc, Australian dollar, Canadian dollar, Korean won
Commodity	Oil, natural gas, gold, silver, copper, aluminum, corn, wheat, lumber, hogs, cattle, milk

Futures transactions in the USA are regulated by the Commodity Futures Trading Commission (CFTC), an agency of the USA government. The clearinghouse matches the purchases and the sales which take place during the day. By matching trades, the clearinghouse never takes market risk because it always has offsetting positions with different counterparts. By having the clearinghouse as counterparty, an individual entering a future contract does not face the possible credit risk of its counterparty.

4.3.1 Determination of forward and futures prices

Notations:

1. t present time F_0 or $F(0,T)$ the delivery price.
2. T expiration date of forward delivery (maturity).
3. S_t, S_T the underlying prices at time t and T.
4. f_t the value of the forward contract at time t.
5. $F(t,T)$ forward price at time t (which matures at T).
6. r the continuously risk–free interest rate at time t for investment of horizon T.
7. q dividend rate on the underlying

Intuitively the froward price has the form

$$F(t,T) = Function(t, T, r, q, S_t, K)$$

But the Function is unknown !

Arbitrage principle

The case where r is not random
For non-dividends paying stock. If $F(t,T) > S_t e^{r(T-t)}$ then, the arbitrageur choose the following strategy:

- Takes short position on the forward/futures at the price $F(t,T)$ for maturity T. The corresponding payoff is is $F(t,T) - S_T$,
- Borrows the amount S_t with the rate r. The corresponding payoff is given by $S_t e^{r(T-t)}$,
- Takes a short position on the stock by buying one share of the asset at the price S_t in order to compensate the risk of the forward/futures contract. The payoff of this position is S_T when selling the share at tile T.

At maturity date T, the broker gets a net profit equal to $F(t,T) - S_t e^{r(T-t)}$.
Now, if $F(t,T) < S_t e^{r(T-t)}$ then the broker choose the following strategy:

- long a forward/futures at the price $F(t,T)$ maturing at time T. The payoff is $S_T - F(t,T)$,
- Short sell one share of the asset S_t, in order to buy it at expiration date. The payoff is $-S_T$,
- Invest in bank the amount S_t with the rate r. The payoff is $-S_t e^{r(T-t)}$

At expiration date T, the broker ends up a profit equal $S_t e^{r(T-t)} - F(t,T)$.

Therefore the fair price is then $F(t,T) = S_t e^{r(T-t)}$.

Pricing of Forwards and Futures for investment assets

Asset	**Forward / Futures price**	**Contract Value at t**
Non–dividends	$S_0 e^{rT}$	$S_t - \dfrac{F_0}{e^{r(T-t)}}$
Return with Present Value I	$(S_0 - I)\, e^{rT}$	$S_t - I_t - \dfrac{F_0}{e^{r(T-t)}}$
Return with rate q	$S_0 e^{(r-q)T}$	$\dfrac{S_t}{e^{q(T-t)}} - \dfrac{F_0}{e^{r(T-t)}}$

where $F_0 = F(0,T)$ for ease of notations is the delivery price which is given by the second column.

Futures and forward contracts on currency We can assimilate the foreign currencies as assets paying dividends with the rate $q = r_f$ which represents the exchange rate of the currency at time T.

$$F(t,T) = S_t e^{(r-r_f)(T-t)}$$

If this formula is not satisfied then there will be arbitrage opportunities.

Futures on consumable assets By consumable assets we mean asset which can be consumed, such as commodities, raw materials in industrial products (gold, copper, oil, etc.) or products food (wheat, oil, cocoa, sugar, coffee, etc.).

For this kind of asset we must consider the costs of carry or warehousing that includes rent. The arbitrage relationship between the futures price and the current price of the asset is given

$$F(t,T) = S_t e^{(c-y)(T-t)} \begin{cases} c = r \text{ non-dividends} \\ c = r - q \text{ with dividends} \\ c = r + u \text{ cost of carry} \end{cases}$$

where c is *the cost of carry* and y is the convenience yield. This parameter reflects the market's expectations concerning the future availability of the commodity.

Forwards: Alternative derivation of formula

Forward price when the underlying asset provides a known yield q: $F_p(0,t,T) = S_t e^{-q(T-t)}$:

$F_p(0,t,T)$ equals the investment required in the asset at time t (today) that will yield one unit of the asset in T–years when physical delivery occurs.

$e^{-q(T-t)}$ units of the asset will grow to $e^{-q(T-t)} \times e^{q(T-t)} = 1$–unit of the asset in T–years, assuming that the income provided by the asset is reinvested in the asset.

$e^{-q(T-t)}$ units of the asset cost $S_t e^{-q(T-t)}$ today (at time t).

$$F(0,t,T) = F_p(0,t,T)e^{r(T-t)} = S_t e^{(r-q)(T-t)}$$

A forward contract allows the long position to delay payment for T–years and requires the short position to delay receipt. The long position can earn interest on the cash that would otherwise have been paid.

The short position foregoes this interest. The forward price (which is arrived at by multiplying the prepaid forward price, equal to $S_t e^{-q(T-t)}$ by $e^{r(T-t)}$ compensates the short position for the delay.

4.4 Futures and hedging

An airline company may want to hedge its bets against an unexpected increase in jet fuel prices. Its traders will therefore seek to enter into a futures contract to lock in a purchase price closer to today's prices for jet fuel.

They may buy a futures contract agreeing to buy 1 million gallons of JP-8 fuel, taking delivery 90 days in the future, at a price of 3 dollars per gallon.

Someone else naturally wants to ensure they have a steady market for fuel.

They also want to protect themselves against an unexpected decline in fuel prices, so they will gladly enter into either a futures contract.

In this example, both parties are hedgers, rather than speculators.

They are turning to the futures market as a way to manage their exposure to risk, rather than make money off of the deal directly.

4.4.1 Futures: Arbitrage trade

There are also people who seek to make money off of changes in the price of the contract itself, when bought or sold to other investors.

Naturally, if the price of fuel rises, the contract itself becomes more valuable, and the owner of that contract could, if it chose, sell that contract for someone else who is willing to pay more for it.

It may make sense for another airline to pay 10 cents per gallon for a contract to save 20 cents. And so there is a lively and relatively liquid market for these contracts, and they are bought and sold daily on exchanges.

Example: The S&P 500 Futures Contract

Specifications for the S&P500 index futures contract	
Underlying	S&P 500 index
Where traded	Chicago Mercantile Exchange
Size	$250 \times$ S&P 500 index
Months	March, June, September, December
Trading ends	Business day prior to determination of settlement price
Settlement	Cash-settled, based up on opening price of S&P500 on third Friday of expiration month

The S&P 500 futures contract has the S&P 500 stock index as the underlying asset. Futures on individual stocks have recently begun trading in the United States. The notional value, or size, of the contract is the dollar value of the assets underlying one contract. In this case it is by definition $250\$ \times 1300 = 325,000.12$ The S&P 500 is an example of a cash-settled contract: Instead of settling by actual delivery of the underlying stocks, the contract calls for a cash payment that equals the profit or loss as if the contract were settled by delivery of the underlying asset.

On the expiration day, the S&P 500 futures contract is marked-to-market against the actual cash index. This final settlement against the cash index guarantees that the futures price equals the index value at contract expiration. It is easy to see why the S&P 500 is cash-settled. A physical settlement process would call for delivery of 500 shares (or some large subset thereof) in the precise percentage they make up the S&P 500 index. This basket of stocks would be expensive to buy and sell. Cash settlement is an inexpensive alternative.

4.4.2 Margins and Marking to Market

Let us explore the logistics of holding a futures position. Suppose the futures price is 1100 and you wish to acquire a 2.2 million US $ position in the S&P500 index. The notional value of one contract is $250 \times 1100 = 275000$: this represents the amount you are agreeing to pay at expiration per futures contract. To go long 2.2 million USA $ of the index, you would enter into $2.2 million/0.275 million = 8$ long futures contracts. The notional value of eight contracts is $8 \times 250 \times 1100 = 2000 \times 1100 = 2.2$ million $.

The margin on the S&P500 contract has generally been less than the 10% we assume in this example.

See Excel sheets for practice
Example: some common futures

1. Crude oil futures trade in units of 1000 U.S. barrels (42,000 gallons). The underlying is a US barrel. The notional amount is 1000 barrels. The current price is $70 per barrel. Hence, the current value of a future contract on crude oil is $70000.

2. S&P500 future contracts trade on 250 units of the index. They are cash settled. At expiration time, instead of a sale, one of the future counterpart receive a payment according with S&P500 spot price at expiration. The current price of S&P500 is 1500. The current value of a future contract on S&P500 is $(250)(1500) = \$375000$.

Suppose that two parties agree in a future contact for crude oil for delivery in 18 months. The contract is worth $70000. Usually future positions are settled into the margin account either every day or every week. By every day we mean every day which the market is open. Let us suppose that a clearinghouse settles accounts daily.

Suppose that the annual continuously compounded interest rate is r. Every day, the profit or loss is calculated on the investor's futures position. If there exists a loss, the investor's broker transfers that amount from the investor's margin account to the clearinghouse. If a profit, the clearinghouse transfers that amount to investor's broker who then deposits it into the investor's margin account. The profit for a long position in a future contract is

$$M_{t-(1/365)} \times (\exp(r/365) - 1) + N(S_t - S_{t-(1/365)}).$$

where $M_{t-(1/365)}$ is the yesterday's balance in the margin account, N is the notional amount, S_t is the current price, $S_{t-(1/365)}$ is the yesterday price. Hence, after the settlement, the balance in the investor's (buyer) margin account is

$$M_t = M_{t-(1/365)} \times \exp(r/365) + N(S_t - S_{t-(1/365)}).$$

The profit for a short position in a future contract is

$$M_{t-(1/365)} \times (1 - \exp(r/365)) + N(S_{t-(1/365)} - S_t).$$

Marking–to–market is to calculate the value of a future contract according with the current value of the asset.

On July 5, 2007, ABC enters a long future contract for 1,000 U.S. barrels of oil at \$71.6 per barrel. The margin account is 50% of the market value of the futures' underlier. The annual continuously compounded rate of return is 6%.
(i) On July 6, 2007, the price of oil is \$70.3. What is the balance in ABC's margin account after settlement?
(ii) On July 7, 2007, the price of oil is \$72.1.
What is the balance in ABC's margin account after settlement?
Solution: (i) The initial balance in ABC's margin account is $0.50 \times 1000 \times 71.6 = 35800$.
The balance in ABC's margin account on July 6, 2007, after settlement, is

$$M_{t-(1/365)} \exp(r/365) + N(S_t - S_{t-(1/365)})$$

$$= (35800) \exp(0.06/365) + (1000)(70.3 - 71.6) = 35105.89.$$

Since the price of the oil decreases, the value of having 1000 barrels in 18 months decreases.
Solution: (ii) The balance in ABC's margin account on July 6, 2007, after settlement, is

$$M_{t-(1/365)} \exp(r/365) + N(S_t - S_{t-(1/365)})$$

$$= (35105.89) \exp(0.06/365) + (1000)(72.1 - 70.3)$$

$$= 35711.56.$$

Notice that this balance is different from

$$(35800) \exp(0.06(2/365)) + (1000)(72.1 - 71.6) = 36311.77.$$

In the first day, ABC's account balance was smaller. So, ABC lost interest because the drop on price on July 6, 2007.

If the balance in the margin account falls the clearinghouse has less protection against default. Investors are required to keep the margin account to a minimum level. This level is a fraction of the initial margin. The maintenance margin is the fraction of the initial margin which participants are asked to hold in their accounts. If the balance in the margin account falls below this level, an investor's broker will require the investor to deposit funds sufficient to restore the balance to the initial margin level. Such a demand is called a margin call. If an investor fail to the deposit, the investor's broker will immediately liquidate some or all of the investor's positions. A company enters into a short futures contract to sell 100000 pounds of frozen orange juice for \$1.4 cents per pound. The initial margin is 30% and the maintenance margin is 20%. The annual effective rate of interest is 4.5%. The account is settled every week. What is the minimum next week price which would lead to a margin call?

Solution: The initial balance in the margin account is

$$(0.30) \times (100000) \times (1.4) = 42000.$$

The minimum balance in the margin account is

$$(0.20) \times (100000) \times (1.4) = 28000.$$

After settlement next week balance is

$$42000(1.045)^{1/52} + 100000(1.4 - S_{1/52}).$$

A margin call happens if

$$28000 \quad > \quad 42000(1.045)^{1/52} + 100000(1.4 - S_{1/52}),$$

or

$$S_{1/52} \quad > \quad 1.4 - \frac{28000 - 42000(1.045)^{1/52}}{100000} = 1.540355672.$$

Advantages of futures versus forwards

The two main advantages of futures versus forwards are liquidity and counter–party risk. It is much easier to cancel before expiration a future contract than a forward contract. Since the trade is made against a clearinghouse, a participant does face credit risk. At the same time, the margin and the marking to market reduces the default risk.

Difference between forward and futures contracts

	Forward	**Futures**
Standardized contract (delivery date, quality, quantity)	No	Yes
Traded in primary market standardized exchanges	O-T-C	Yes
Credit risk (default risk)	Yes	No
Settlement	Maturity	Daily or Weekly
Clearinghouse	No	Yes
Margin requirement	No	Yes
Transaction cost	High	Low
Regulations	No	Yes

4.4.3 Using forward and futures for hedging

Consider a company **ABC** based in USA which imports goods from Germany

ABC	Outlays	Portfolio	Maturity	forward price
ABC	10 Million €	long forward	3 months	1.1503

The forward price agreed to pay \$11.503 Million. If at the end of 3 months the exchange spot rate is 1.1400, without hedging the outlay will be \$11.400 Million which is less than 11.503 Million \$. But if exchange spot rate is 1.17, the 10 Million € worth \$11.7 Million the company ABC will regret no hedging.

Oil : Hedging a short position on the stock.

Assume that an oil producer sign a contract with physical settlement of **1 Million** barrels of oil to deliver on July 25, 2016 with the (**spot price**). This corresponds to short the stock.

Remark that each decrease of the price by one **cent** leads to a **loss** of 10000\$.
Question how to mange or reduce this risk

Oil : Hedging a short position on the stock.

Answer : Go the Futures market (NYMEX) and short a futures contract on 1 Million barrels for July 25, 2016.

In the standards of oil each contract contains 1000 barrels.

Strategy	Spot at time 0	futures price	Spot at time T	Payoff
1000 SP	41\$	39\$	$\begin{cases} 44\$ \\ 36\$ \end{cases}$	$\begin{cases} -5\$ \\ +3\$ \end{cases}$
SP symb	S_0	F_0	S_T	$F_0 - S_T$

The payoff of the global position is $= 1000 \times 1000(F_0 - S_T) + 1\text{Million} \times S_T = 1\text{Million} \times F_0$

Copper: Hedging a long position on the stock.

A company needs copper for his industry process and should buy 100,000 pounds of copper on June 15, 2016. This company signs a contract with physical delivery to buy copper from a producer but with the **spot price**. Remark that each increase of the price by one **cent** leads to a **loss** of 10000\$.

Question how to mange or reduce this risk ?
Copper: Hedging a long position on the stock.

Answer : Go to the Futures market (COMEX) and long 4 futures contract on copper for June 15, 2016.

For copper each contract contains 25000 pounds.

Strategy	Spot at 0	futures price	Spot at T	Payoff
4 LP	168 cent	150 cent	$\begin{cases} 160 \text{ cent} \\ 140 \text{ cent} \end{cases}$	$\begin{cases} 10 \text{ cent} \\ -10 \text{ cent} \end{cases}$
LP symb	S_0	F_0	S_T	$S_T - F_0$

The payoff of the global position is $= 4 \times 25,000(S_T - F_0) - 100,000S_T = 100,000F_0$

Using futures or forward for arbitrage trade

Consider once again the futures contract where the underlying asset is ABC. Moreover assume that:
1. In the spot market the asset ABC is selling for 1000 SAR
2. Asset XYZ pays the holder (with certainty) 120 SAR per year in four quarterly payments of 30 SAR, and the next quarterly payment is exactly 3 months from now.
3. The futures contract requires delivery 3 months from now.
4. The current 3-month interest rate at which funds can be loaned or borrowed is 2% per year.

Suppose that an investor or a broker suggests to buy a futures contract for 1008 SAR with cash delivery in 3 months.

Consider the following strategy:

1. Short futures: Sell the futures contract at 1010 SAR.
2. Borrow 1000 for 3 months at 2% per year.
3. Purchase an unit of the asset ABC at the market for $S_0 = 1000$.

At maturity (in 3 months):

1. Sell one unit of the asset for $S_{3months}$.
2. Settle the futures contract for $1010 - S_{3months}$. This to positions leads to a cash of 1010 SAR.
3. Repay the loan which sum-up to $1000 \times (1 + 2/4 \times 0,005) = 1005$.

This strategy ends up with a profit = 5 SAR without risk and without initial investment.

Chapter 5

Options

5.1 Introduction and motivations

We have seen that forward and futures contract are binding contract, a forward/futures contract obligates the buyer (the holder of the long position) to pay the forward price at expiration, even if the value of the underlying asset at expiration is less than the forward price. Because losses are possible with a forward/futures contract, it is natural to wonder: Could there be a contract where the buyer has the right to walk away from the deal?

The answer is yes; a **call option** is a contract where the buyer has the right to buy, but not the obligation to buy.

5.1.1 Learning objectives

- Understand how call and put options are used and how they are priced
- Examine the instruments traded on the options market
- Understand how options can be used for either risk management or for speculative purposes

5.2 Call options

5.2.1 Definitions and examples

Definition. A financial option contract gives its owner the right (but not the obligation) to purchase or sell an asset at a fixed price at some future date.

An option is a financial instrument like a stock or bond, an option is a derivative security. It is also a non–binding contract with strictly defined terms and properties.

Here is an example illustrating how a call option works at expiration.

Example 1. *Suppose that the call buyer agrees to pay $1020 for the ABC index in 6 months but is not obligated to do so. (The buyer has purchased a call option.)If in 6 months the ABC price is $1100, the buyer will pay $1020 and receive the index. This leads to a payoff of $80 per unit of the index. If the ABC price is $900, the buyer walks away, hence his payoff is 0.*

Remark 2. *From the seller's point of view the buyer is in control of the option, deciding when to buy the index by paying $1020. Thus, the rights of the option buyer are obligations for the option seller.*

Example 3. *If in 6 months the ABC price is $1100, the seller will receive $1020 and give up an index worth more, for a loss of $80 per unit of the index. If the ABC price is less than $1020, the buyer will not buy, so the seller has no obligation. Thus, at expiration, the seller will have a payoff that is zero (if the ABC price is less than $1020) or negative (if the ABC price is greater than $1020).*

5.3 Different type of options

Two distinct kinds of option contracts exist: **call options and put options**. A **call option** gives the owner the right to **buy** the asset;a **put option** gives the owner the right to **sell** the asset. Because an option is a contract between two parties, for every owner of a financial option, there is also an option writer, the person who takes the other side of the contract.

Initial payment is necessary Because the buyer can decide whether to buy, the seller cannot make money at expiration. This situation suggests that the seller must get something to enter into the contract in the first place. At the time the buyer and seller agree to the contract, the buyer must pay the seller an initial price, the **premium**. This initial payment compensates the seller for being at a disadvantage at expiration .Contrast this with a forward/futures contract, for which the initial premium is **zero**

Example 4. *The most commonly encountered option contracts are options on shares of stock. A stock option gives the holder the option to buy or sell a share of stock on or before a given date for a given price. For example, a call option on 3M Corporation stock might give the holder the right to purchase a share of 3M for $75 per share at any time up to, for example, July 19, 2016.Similarly, a put option on 3M stock might give the holder the right to sell a share of 3M stock for $50 per share at any time up to, say, June 16, 2016.*

5.4 Option Terminology

Here are some key terms used to describe options:

1. **Strike price:** The strike price, or exercise price, of a call option is what the buyer pays for the asset. In the example above, the strike price was $1020. The strike price can be set at any value.

2. **Exercise:** The exercise of a call option is the act of paying the strike price to receive the asset. In the abode example, the buyer decided after 6 months whether to exercise the option–that is, whether to pay $1020 (the strike price) to receive the ABC index.

3. **Expiration:** The expiration of the option is the date by which the option must either be exercised or it becomes worthless. The option in previous example had an expiration of 6 months.

4. The (**owner**) buyer of a call option is called the **option call holder**. The holder of a call option is said to have a **long call position**.

5. The seller of a call option is called the **option call writer**.

6. The writer of a call is said to have a **short call position**. Assets used in call options are in *commodities, currency exchange, stock shares and stock indices.*

7. A call option needs to specify the **type** and **quality** of the **underlying**. The asset used in the call option is called the **underlier** or **underlying asset**.

8. **Notional**: The amount of the underlying asset to which the call option applies is called the **notional** amount.

9. **Exercise style:** The exercise style of the option governs the time at which exercise can occur. In the above example, exercise could occur only at expiration. Such an option is said to be a **European–style option**.

10. If the buyer has the right to exercise at any time during the life of the option, it is an **American–style option**.

11. If the buyer can only exercise during specified periods, but not for the entire life of the option, the option is a **Bermudan–style option**.

The terms "European" and "American", have nothing to do with geography. European, American, and Bermudan options are bought and sold worldwide.

An European call option gives the owner of the call the right, but not the obligation, to buy the underlying asset on the expiration date by paying the strike price. The option described in two examples above is a 6–month European–style ABC call with a strike price of \$1020.The buyer of the call can also be described as having a **long position** in the call.

5.5 Payoff and Profit for options

5.5.1 Payoff and Profit for a purchased call option

We can graph call options as we did forward contracts. The buyer is not obligated to buy the index, and hence will only exercise the option if the payoff is greater than zero. The algebraic expression for the payoff to a purchased call is therefore

$$\text{Purchased call payoff} = \max(\text{spot price at expiration} - \text{strike price}; 0) \tag{5.1}$$

The expression $\max(a, b)$ means take the greater of the two values a and b.

Example 5. *Consider a call option on the ABC index with 6 months to expiration and a strike price of \$1000.Suppose the index in 6 months is \$1100. Clearly it is worthwhile to pay the \$1000 strike price to acquire the index worth \$1100.Using the equation (5.1), the call payoff is*

$$\max(\$1100 - \$1000, 0) = \$100$$

If the index is \$900 at expiration, it is not worthwhile paying the \$1000 strike price to buy the index worth \$900. The payoff is then

$$\max(\$900 - \$1000, 0) = \$0$$

See the link for more information:

http://www.cboe.com/delayedquote/quotetable.aspx?ticker=SPX

Notice that the payoff does not take account of the initial cost of acquiring the position. For a purchased option, the premium is paid at the time the option is acquired. In computing profit at expiration, suppose we defer the premium payment; then by the time of expiration we accrue 6 months' interest on the premium. The option profit is computed as

$$\begin{aligned} \text{Purchased call profit} &= \max(\text{spot price at expiration} - \text{strike price}, 0) \\ &\quad - \text{future value of option premium} \end{aligned} \tag{5.2}$$

Example illustrating the computation of the profit Use the same option as before, and suppose that the risk–free rate is 2% over 6 months. Assume that the index spot price is \$1000 and that the premium for this call is \$93.81.Hence, the future value of the call premium is

$$\$93.81 \times 1.02 = \$95.68.$$

If the ABC index price at expiration is \$1100, the owner will exercise the option. Using equation (5.2), the call profit is

$$\max(\$1100 - \$1000, 0) - \$95.68 = \$4.32.$$

If the index is \$900 at expiration, the owner does not exercise the option. It is not worthwhile paying the \$1000 strike price to buy the index worth \$900.The Profit is then

$$\max(\$900 - \$1000, 0) - \$95.68 = -\$95.68$$

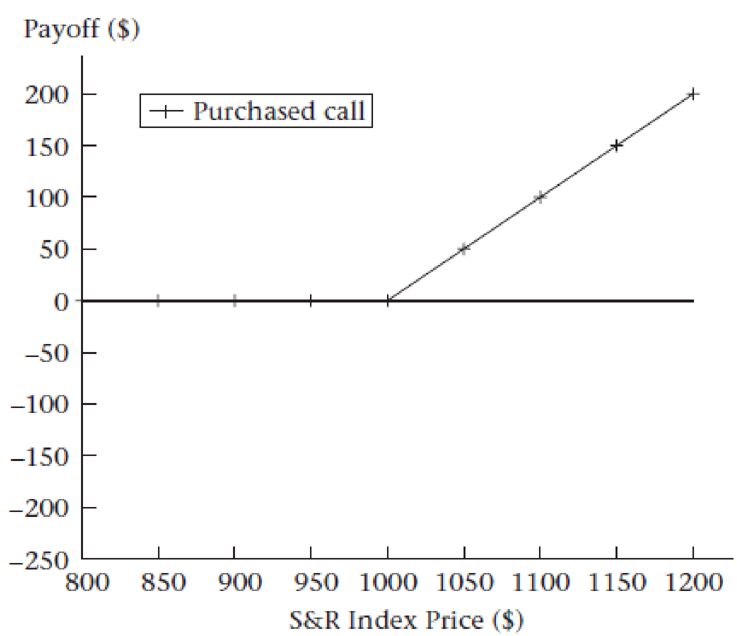
reflecting the loss of the premium.

We graph the call payoff by computing, for any index price at expiration, the payoff on the option position as a function of the price. We graph the call profit by subtracting from this the future value of the option premium.

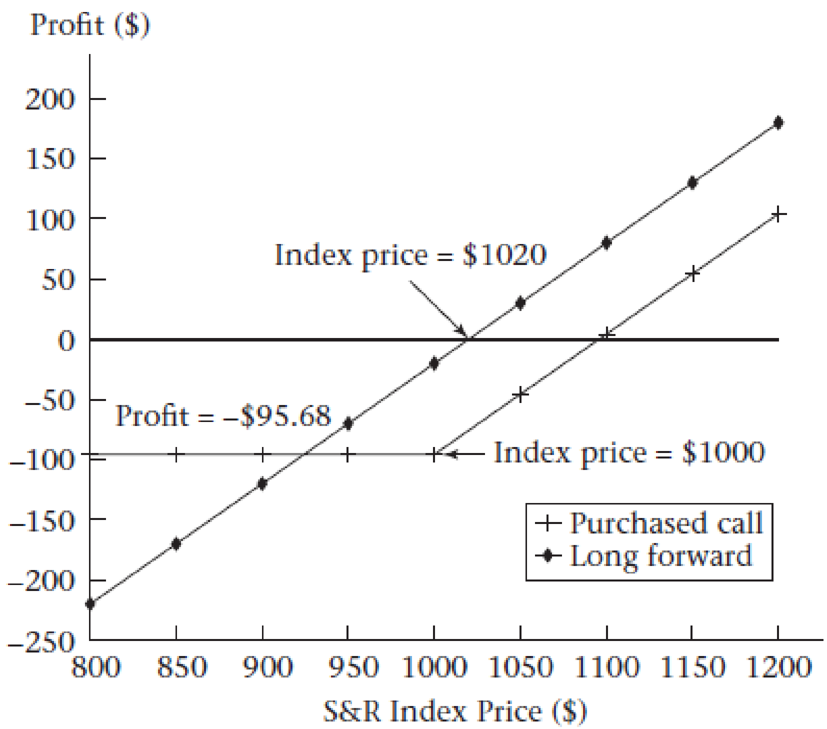
Payoff and profit after 6 months from a purchased \$1000–strike ABC call option with a future value of premium of \$95.68. The option premium is assumed to be \$93.81 and the effective interest rate is 2% over 6 months. The payoff is computed using equation (5.1), and the profit using equation (5.2).

S&R Index in 6 Months	Call Payoff	Future Value of Premium	Call Profit
800	$0	–$95.68	–$95.68
850	0	–95.68	–95.68
900	0	–95.68	–95.68
950	0	–95.68	–95.68
1000	0	–95.68	–95.68
1050	50	–95.68	–45.68
1100	100	–95.68	4.32
1150	150	–95.68	54.32
1200	200	–95.68	104.32

The payoff at expiration of a purchased ABC call with a $1000 strike price.



Profit at expiration for purchase of a 6-month ABC index call with a strike price of $1000 versus profit on a long ABC index forward position.



5.5.2 Payoff and Profit for a Written Call Option

Now, let us look at the option from the point of view of the seller.
The seller is said to be the **option writer**, or to have a **short position** in a call option.

The option writer is the counterparty to the option buyer.

The writer receives the premium for the option and then has an obligation to sell the underlying security in exchange for the strike price if the option buyer exercises the option.

The payoff and profit to a written call are just the opposite of those for a purchased call:

$$\text{Written call payoff} = -\max(\text{spot price at expiration} \; - \; \text{strike price}; 0)$$

and

$$\begin{aligned}\text{Written call profit} \quad &= \quad -\max(\text{spot price at expiration} - \text{strike price}, 0) \\ &\quad\; +\text{future value of option premium}\end{aligned} \qquad (5.3)$$

The following example illustrates the option writer's payoff and profit. Just as a call buyer is long in the call, the call seller has a short position in the call.

Example 6. *Consider a \$1000–strike call option on the ABC index with 6 months to expiration. At the time the option is written, the option seller receives the premium of \$93.81. Suppose the index in 6 months is \$1100. It is worthwhile for the option buyer to pay the \$1000 strike price to acquire the index worth \$1100.*
*Thus, the option writer will have to sell the index, worth \$1100, for the strike price of \$1000. Using equation (**??**), the written call payoff is*

$$-\max(\$1100 - \$1000, 0) = -\$100$$

The premium has earned 2% interest for 6 months and is now worth \$95.68. Profit for the written call is

$$-\$100 + \$95.68 = -\$4.32$$

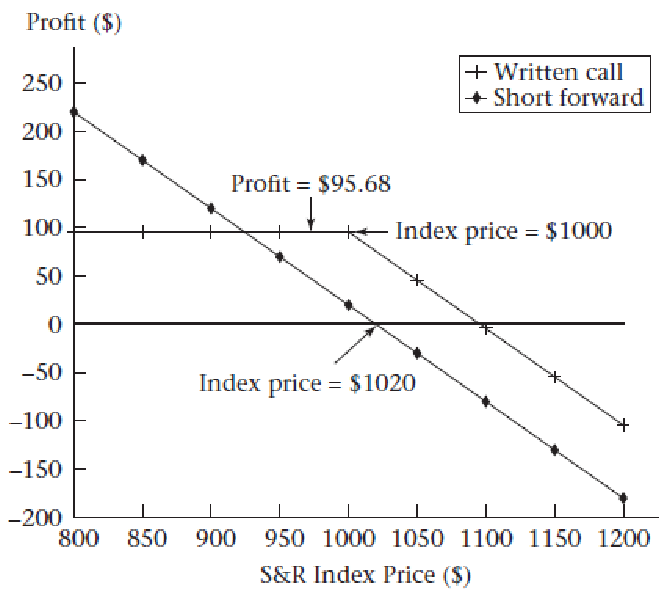
If the index is \$900 at expiration, it is not worthwhile for the option buyer to pay the \$1000 strike price to buy the index worth \$900. The payoff is then

$$-\max(\$900 - \$1000, 0) = \$0.$$

The option writer keeps the premium, for a profit after 6 months of

$$\$95.68.$$

Profit for the writer of a 6-month ABC call with a strike of \$1000 versus profit for a short ABC forward.



5.6 Put options

5.6.1 Definitions and examples

We introduced a call option by comparing it to a forward contract in which the buyer need not buy the underlying asset if it is worth less than the agreed-to purchase price. Perhaps you wondered if there could also be a contract in which the seller could walk away if it is not in his or her interest to sell. The answer is yes.

Definition. A put option is a contract where the seller has the right to sell, but not the obligation. Here is an example to illustrate how a put option works.

Example 7. *Suppose that the seller agrees to sell the ABC index for \$1020 in 6 months but is not obligated to do so. (The seller has purchased a put option.) If in 6 months the ABC* *price is \$1100, the seller will not sell for \$1020 and will walk away. If the ABC price is \$900, the seller will sell for \$1020 and will earn \$120 at that time.*

Remark 8. *A put must have a premium for the same reason a call has a premium. The buyer of the put controls exercise; hence the seller of the put will never have a positive payoff at expiration. A premium paid by the put buyer at the time the option is purchased compensates the put seller for this no-win position.*

5.6.2 Payoff and Profit for a Purchased Put Option

We now see how to compute payoff and profit for a purchased put option. The put option gives the put buyer the right to sell the underlying asset for the strike price. The buyer does this only if the asset is less valuable than the strike price. Thus, the payoff on the put option is

$$\text{Put option payoff} = \max\left(\text{strike price} \ - \ \text{spot price at expiration}, 0\right) \tag{5.4}$$

The put buyer has a long position in the put.

Example 9. *Consider a put option on the ABC index with 6 months to expiration and a strike price of \$1000. Suppose the index in 6 months is \$1100. It is not worthwhile to sell the index worth \$1100 for the \$1000 strike price. Using equation (5.4), the put payoff is*

$$\max(\$1000 - \$1100, 0) = \$0$$

If the index were 900 at expiration, it is worthwhile selling the index for \$1000. The payoff is then

$$\max(\$1000 - \$900, 0) = \$100$$

As with the call, the payoff does not take account of the initial cost of acquiring the position. At the time the option is acquired, the put buyer pays the option premium to the put seller; we need to account for this in computing profit. If we borrow the premium amount, we must pay 6 months' interest. The option profit is computed as

$$\begin{aligned} \text{Purchased put profit} \ &= \ \max(\text{strike price} \ - \ \text{spot price at expiration}; 0) \\ &\quad -\text{future value of option premium} \end{aligned} \tag{5.5}$$

The following example illustrates the computation of profit on the put.

Example 10. *Use the same option as in Example above and suppose that the risk-free rate is 2% over 6 months. Assume that the premium for this put is \$74.20. The future value of the put premium is* $\$74.20 \times 1.02 = \$75.68$. *If the ABC index price at expiration is \$1100, the put buyer will not exercise the option. Using equation (5.5), profit is*

$$\max(\$1000 - \$1100; 0) - \$75.68 = -\$75.68$$

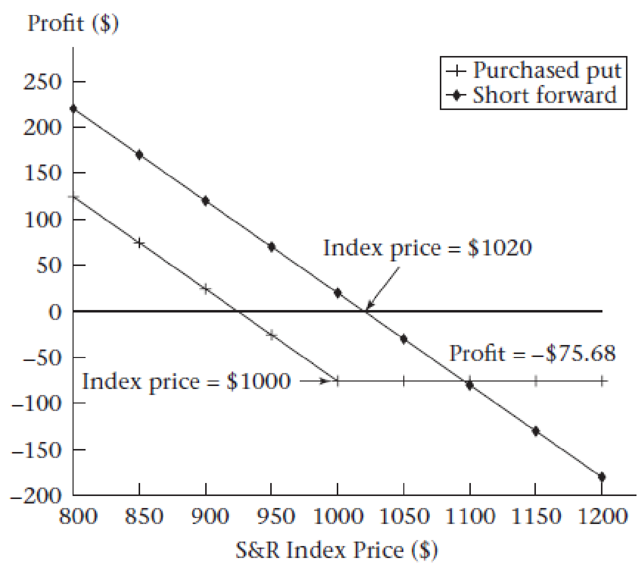
reflecting the loss of the future value of the premium. If the index is \$900 at expiration, the put buyer exercises the put, selling the index for \$1000. Profit is then

$$\max(\$1000 - \$900; 0) - \$75.68 = \$24.32$$

Profit after 6 months from a purchased \$1000–strike ABC put option with a future value of premium of \$75.68.

S&R Index in 6 Months	Put Payoff	Future Value of Premium	Put Profit
\$800	\$200	–\$75.68	\$124.32
850	150	–75.68	74.32
900	100	–75.68	24.32
950	50	–75.68	–25.68
1000	0	–75.68	–75.68
1050	0	–75.68	–75.68
1100	0	–75.68	–75.68
1150	0	–75.68	–75.68
1200	0	–75.68	–75.68

Profit on a purchased ABC index put with a strike price of \$1000 versus a short ABC index forward.



5.6.3 Payoff and Profit for a Written Put Option

Now we examine the put from the perspective of the put writer. The put writer is the counterparty to the buyer. Thus, when the contract is written, the put writer receives the premium. At expiration, if the put buyer elects to sell the underlying asset, the put writer must buy it.

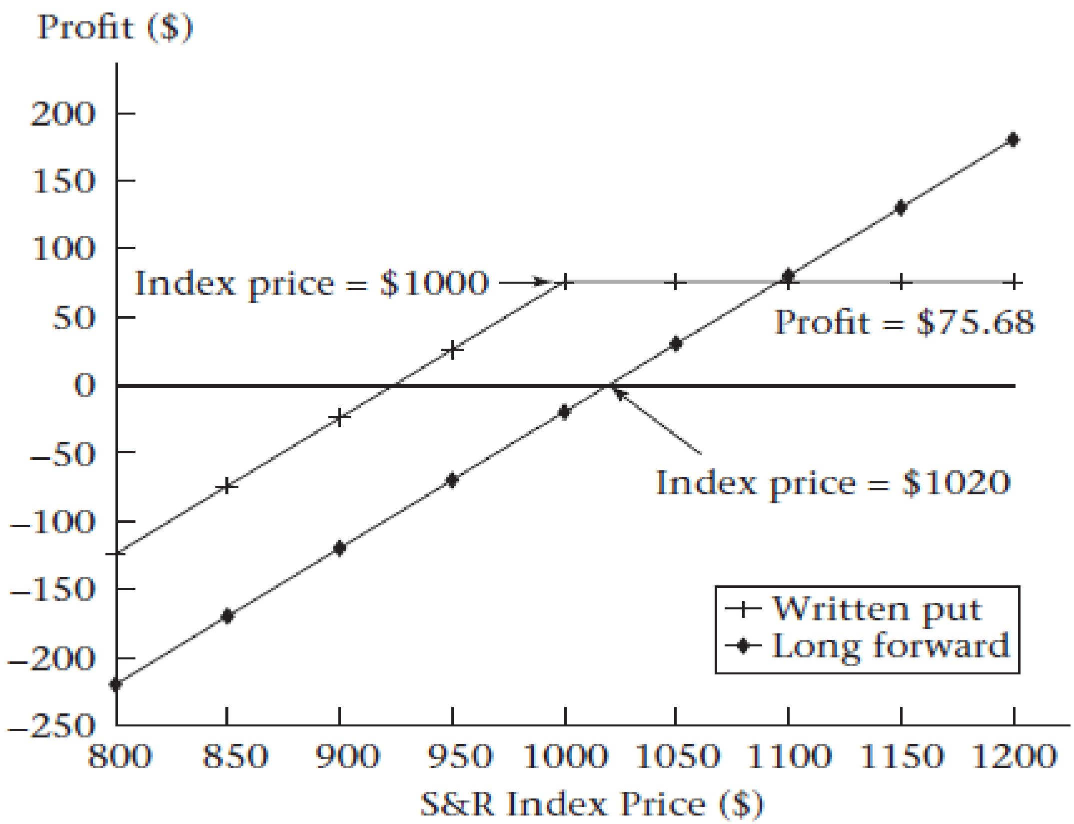
The payoff and profit for a written put are the opposite of those for the purchased put:

$$\text{Written put payoff} = -\max(\text{strike price} - \text{spot price at expiration}; 0) \qquad (5.6)$$

$$\text{Written put profit} = -\max(\text{strike price} - \text{spot price at expiration}; 0) + \text{future value of option premium} \quad (5.7)$$

The put seller has a short position in the put.

Written ABC index put option with a strike of $1000 versus a long ABC index forward contract.



5.6.4 The "Moneyness" of an Option

Options are often described by their degree of moneyness. This term describes whether the option payoff would be positive if the option were exercised immediately. (The term is used to describe both American and European options even though European options cannot be exercised until expiration.)

An **in-the-money option** is one which would have a positive payoff (but not necessarily positive profit) if exercised immediately.

A call with a strike price less than the asset price and a put with a strike price greater than the asset price are both **in-the-money**.

An **out-of-the-money option** is one that would have a negative payoff if exercised immediately.

A call with a strike price greater than the asset price and a put with a strike price less than the asset price are both **out-of-the-money**.

AMZN 77.03 +1.40

Jul 08 2009 @ 15:26 ET Bid 77.02 Ask 77.03 Size 1 x 3 Vol 6548487

Calls	Last Sale	Net	Bid	Ask	Vol	Open Int	Puts	Last Sale	Net	Bid	Ask	Vol	Open Int
09 Jul 70.00 (ZQN GN-E)	7.65	1.60	7.20	7.30	221	2637	09 Jul 70.00 (ZQN SN-E)	0.36	−0.18	0.36	0.38	684	11031
09 Jul 75.00 (ZQN GO-E)	3.35	0.86	3.20	3.30	943	6883	09 Jul 75.00 (ZQN SO-E)	1.30	−0.66	1.38	1.40	2394	15545
09 Jul 80.00 (QZN GP-E)	0.94	0.24	0.93	0.96	2456	9877	09 Jul 80.00 (QZN SP-E)	4.15	−1.05	4.00	4.10	700	10718
09 Jul 85.00 (QZN GQ-E)	0.22	0.07	0.19	0.21	497	26679	09 Jul 85.00 (QZN SQ-E)	8.25	−1.25	8.25	8.35	112	7215
09 Aug 70.00 (ZQN HN-E)	9.75	1.04	9.60	9.70	51	326	09 Aug 70.00 (ZQN TN-E)	2.77	−0.39	2.75	2.79	225	1979
09 Aug 75.00 (ZQN HO-E)	6.50	0.70	6.40	6.50	65	1108	09 Aug 75.00 (ZQN TO-E)	4.60	−0.55	4.55	4.60	2322	6832
09 Aug 80.00 (QZN HP-E)	4.00	0.50	3.90	4.00	172	2462	09 Aug 80.00 (QZN TP-E)	6.95	−0.95	7.05	7.15	145	2335
09 Aug 85.00 (QZN HQ-E)	2.15	0.15	2.22	2.26	833	5399	09 Aug 85.00 (QZN TQ-E)	10.15	−1.00	10.30	10.40	43	4599

Source: Chicago Board Options Exchange at www.cboe.com

An **at-the-money option** is one for which the strike price is approximately equal to the asset price.

Let S_0 be the price of the underlying asset at time 0 and K the strike price of an option.

The purchased call option is **in–the–money** if $S_0 > K$.

The purchased call option is **out–the–money** if $S_0 < K$.

The purchased call option is **at–the–money** if $S_0 = K$.

Example: Option Quotes for Amazon.com Stock

It is the afternoon of July 8, 2009, and you have decided to purchase 10 August call contracts on

Amazon stock with an exercise price of \$80. Because you are buying, you must pay the ask price.

How much money will this purchase cost you? Is this option in-the-money or out-of-the-money?

Solution

From the table above, the ask price of this option is \$4.00. You are purchasing 10 contracts and each contract is on 100 shares, so the transaction will cost $4.00 \times 10 \times 100 = \$4,000$ (ignoring any brokerage fees). Because this is a call option and the exercise price is above the current stock price (\$77.03), the option is currently out-of-the-money.

Concept check

1. What is the difference between an American option and a European option?
2. Does the holder of an option have to exercise it?
3. Why does an investor who writes (shorts) an option have an obligation?

5.6.5 Positions Long with Respect to the Index

The following positions are long in the sense that there are circumstances in which they represent either a right or an obligation to buy the underlying asset:

Long forward: An obligation to buy at a fixed price.

Purchased call: The right to buy at a fixed price if it is advantageous to do so.

Written put: An obligation of the put writer to buy the underlying asset at a fixed price if it is advantageous to the option buyer to sell at that price.

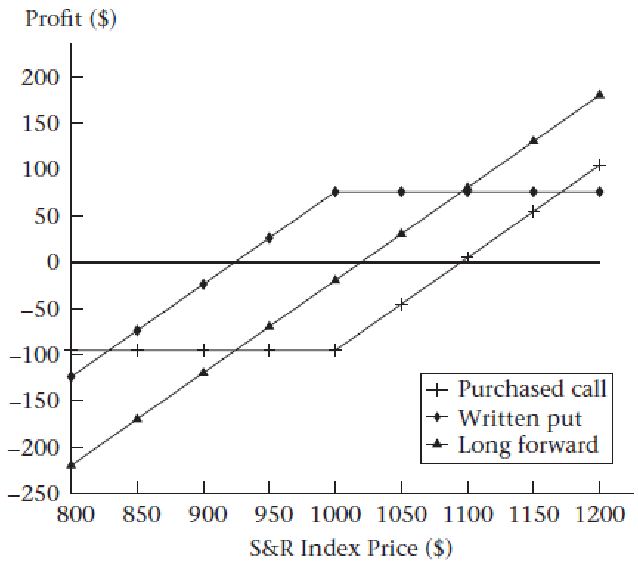
(Recall that the option buyer decides whether or not to exercise.)

Maximum possible profit and loss at maturity for long and short forwards and purchased and written calls and puts. FV(premium) denotes the future value of the option premium.

Position	**Maximum Loss**	**Maximum Gain**
Long forward	-Forward price	Unlimited
Short forward	Unlimited	Forward price
Long call	-FV(premium)	Unlimited
Short call	Unlimited	FV(premium)
Long put	-FV(premium)	Strike price-FV(premium)
Short put	FV(premium)-Strike price	FV(premium)

Remark 11. *The terms "**long**" and "**short**" can be confusing, however, because they are often used more generally. In this more general usage, a position is long with respect to x if the value of the position goes up when x goes up, and it is short with respect to x if the value of the position goes down when x goes up.*
A purchased call is therefore long with respect to the stock, because the call becomes more valuable when the stock price goes up. Similarly, a purchased put is short with respect to the stock, because the put becomes more valuable when the stock price goes down. The case of a purchased put illustrates that a position can be simultaneously long with respect to one thing (its own price) and short with respect to something else (the price of the underlying asset). Finally, a written put is a "short put" because you lose money if the put price goes up. However, the written put is long with respect to the stock, because the put price goes down, and hence the written put makes money, when the stock price goes up. You may find that it takes a while to become comfortable with the long and short terminology. A position can be simultaneously long with respect to one thing and short with respect to something else, so you must always be clear about what you are long or short with respect to.

Profit diagrams for the three basic long positions: long forward, purchased call, and written put.



5.6.6 Positions Short with Respect to the Index

The following positions are short in the sense that there are circumstances in which they represent either a right or an obligation to sell the underlying asset:

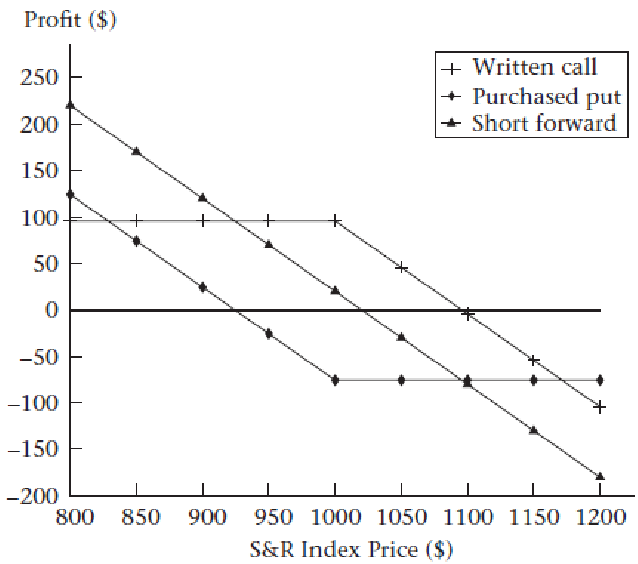
1. **Short forward**: An obligation to sell at a fixed price.

2. **Written call**: An obligation of the call writer to sell the underlying asset at a fixed price if it is advantageous to the option holder to buy at that price (recall that the option buyer decides whether to exercise).

3. **Purchased put**: The right to sell at a fixed price if it is advantageous to do so.

4. **Written put**: An obligation of the put writer to buy the underlying asset at a fixed price if it is advantageous to the option holder to sell at that price.

Profit diagrams for the three basic short positions: short forward, written call, and purchased put.



5.6.7 Option terms

Put: the right to sell at a certain price in the future

Call: the right to buy at a certain price in the future

Long: to purchase the option

Short: to sell or write the option

Bullish: feel the value will increase

Bearish: feel the value will decrease

Long a call. Investor buys the right (a contract) to buy an asset at a cretin price. They feel that the price in the future will exceed the strike price. This is a bullish position.

Short a Call. Investor sells the right (a contract) to someone that allows them to buy a asset at a certain price. The writer feels that the asset will devalue over the time period of the contract. This person is bearish on that asset.

Long a Put. Buy the right to sell an asset at a pre-determined price. You feel that the asset will devalue over the time of the contract. Therefore you can sell the asset at a higher price than is the current market value. This is a bearish position.

Short a Put. Sell the right to someone else. This will allow them to sell the asset at a specific price. They feel the price will go down and you do not. This is a bullish position.

Chapter 6

Hedging and investment strategies

In this chapter we continue to examine the link between forward prices and option prices, including the important concept of **call–put parity**.

We shall discuss some common option strategies, such as spreads, straddles, and collars. Among goals in this chapter you will be to become familiar with drawing and interpreting profit and loss diagrams for different option positions.

6.1 Insurance, collars, and other strategies

6.1.1 Options are insurance

In many investment strategies using options, we will see that options serve as insurance against a loss. In what sense are options the same as insurance? In this subsection we answer this question by considering homeowner's insurance. You will see that options are literally insurance, and insurance is an option.

A homeowner's insurance policy promises that in the event of damage to your house, the insurance company will compensate you for at least part of the damage. The greater the damage, the more the insurance company will pay. Your insurance policy thus derives its value from the value of your house: It is a **derivative**.

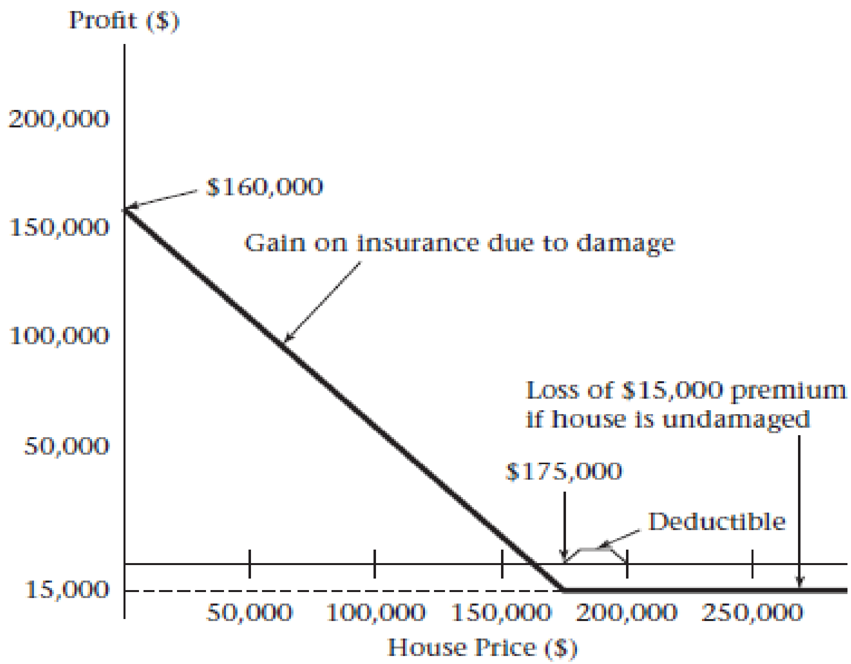
Homeowner's insurance is a put option Suppose that you own a house that costs \$200,000 to build. We assume that physical damage is the only thing that can affect the market value of the house.

Let us say you buy a \$15,000 insurance policy to compensate you for damage to the house. Like most policies, this has a deductible, meaning that there is an amount of damage for which you are obligated to pay before the insurance company pays anything. Suppose the deductible is \$25,000.

- If the house suffers \$4,000 damage from a storm, you pay for all repairs yourself.
- If the house suffers \$45,000 in damage from a storm, you pay \$25,000 and the insurance company pays the remaining \$20,000. Once damage occurs beyond the amount of the deductible, the insurance company pays for all further damage, up to \$175,000.

(Why \$175,000? Because the house can be rebuilt for \$200,000, and you pay \$25,000 of that the–deductible –yourself.)

Profit from insurance policy on a \$200,000 house.



Insurance companies are in the business of writing put options!

The \$15,000 insurance premium is like the premium of a put, and the \$175,000 level at which insurance begins to make payments is like the strike price on a put.

Call options are also insurance Call options can also be insurance. Whereas a put option is insurance for an asset we already own, a call option is insurance for an asset we plan to own in the future. Put differently, a put option is insurance for a long position while a call option is insurance for a short position.

Suppose that the current price of the ABC index is \$1000 and that we plan to buy the index in the future.

If we buy an ABC call option with a strike price of \$1000, this gives us the right to buy ABC for a maximum cost of \$1000 per share. By buying a call, we have bought insurance against an increase in the price.

There are many ways to combine options to create different payoffs. In this section we examine two important kinds of strategies in which the option is combined with a position in the underlying asset:

1. options can be used to insure long or short asset positions.
2. options can be written against an asset position, in which case the option writer is selling insurance.

In what follow we shall use the notation

$$x^+ = \max(x, 0) = \begin{cases} x \text{ if } x > 0 \\ 0 \text{ if } x \leq 0. \end{cases}$$

We consider four positions:

1. being long the asset coupled with a written call.
   The payoff of the combined two positions at expiration$T$ is
$$S_T - (S_T - K)^+ = \begin{cases} K \text{ if } S_T > K \\ S_T \text{ if } S_T \leq K. \end{cases} = \min(K; S_T)$$

2. being short the asset coupled with a purchased call.
   The payoff of the combined two positions at expiration$T$ is
$$-S_T + (S_T - K)^+ = \begin{cases} -K \text{ if } S_T > K \\ -S_T \text{ if } S_T \leq K. \end{cases} = -\min(K; S_T)$$

3. being long the asset coupled with a purchased put.
   The payoff of the combined two positions at expiration$T$ is
$$S_T + (K - S_T)^+ = \begin{cases} K \text{ if } K > S_T \\ S_T \text{ if } K \leq S_T. \end{cases} = \max(K; S_T)$$

4. being short the asset coupled with a written put.
   The payoff of the combined two positions at expiration$T$ is
$$-S_T - (K - S_T)^+ = \begin{cases} -K \text{ if } K > S_T \\ -S_T \text{ if } K \leq S_T. \end{cases} = -\max(K; S_T)$$

We assumed an ABC index price of \$1000, a 2% effective 6–month interest rate, and premiums of \$93.809 for the 1000–strike 6-month call and \$74.201 for the 1000–strike 6–month put.

6.1.2 Insuring a long position: Floors

We have seen that put options are insurance against a fall in the price of an asset. Thus, if we own the ABC index, we can insure the position by buying an ABC put option. The purchase of a put option is also called a **floor**, because we are guaranteeing a minimum sale price for the value of the index.

To examine this strategy, we want to look at the combined payoff of the index position and put. Now we add them together to see the net effect of holding both positions at the same time.

Payoff and profit at expiration from purchasing the ABC index and a 1000–strike put option.

The payoff of the global position is the sum of the first two columns. Cost plus interest for the position is
$$(\$1000 + \$74.201) \times 1.02 = \$1095.68.$$

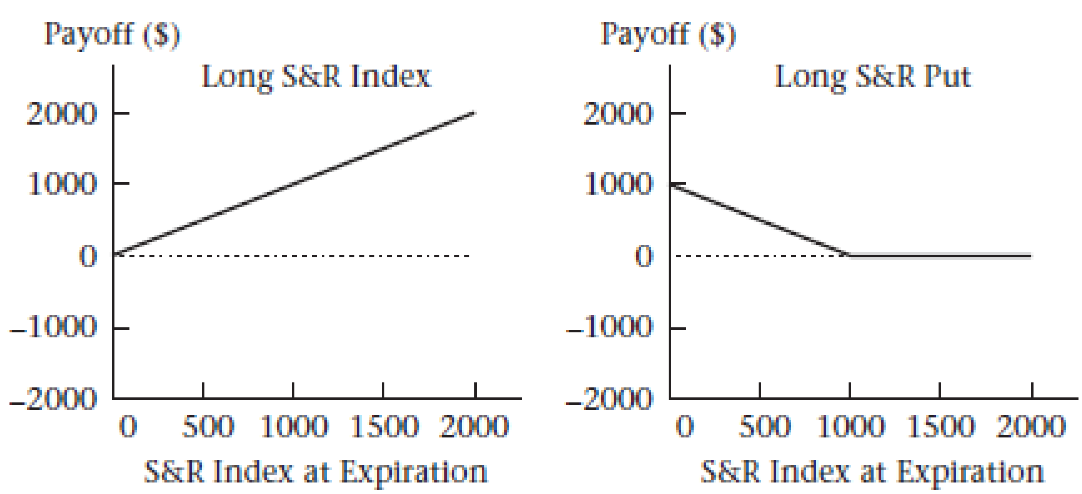
Profit is payoff less \$1095.68.

Payoff at Expiration				
S&R Index	S&R Put	Payoff	−(Cost + Interest)	Profit
\$900	\$100	\$1000	−\$1095.68	−\$95.68
950	50	1000	−1095.68	−95.68
1000	0	1000	−1095.68	−95.68
1050	0	1050	−1095.68	−45.68
1100	0	1100	−1095.68	4.32
1150	0	1150	−1095.68	54.32
1200	0	1200	−1095.68	104.32

This table summarizes the result of buying a 1000–strike put with 6 months to expiration, in conjunction with holding an index position with a current value of \$1000. The table computes the payoff for each position and sums them to obtain the total payoff. The final column takes account of financing cost by subtracting cost plus interest from the payoff to obtain profit. "Cost" here means the initial cash required to establish the position. This is positive when payment is required, and negative when cash is received.

We could also have computed profit separately for the put and index. For example, if the index is \$900 at expiration, we have

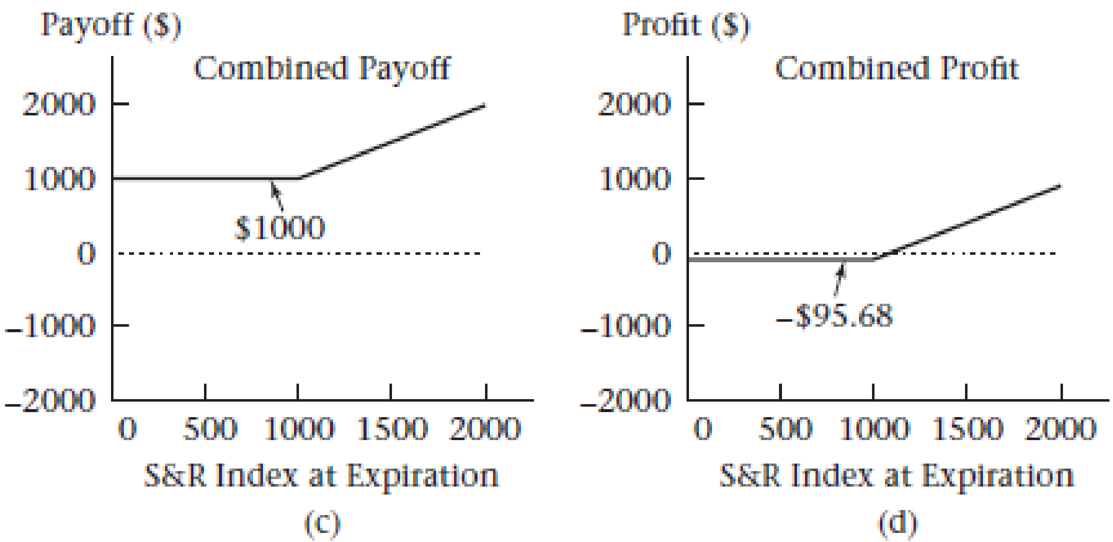
$$\overbrace{\$900 - (\$1000 \times 1.02)}^{\text{Profit on ABC Index}} + \overbrace{\$100 - (\$74.201 \times 1.02)}^{\text{Profit on Put}}$$
$$= -\$95.68$$



(a) (b)

Panel (a) shows the payoff diagram for a long position in the index (column 1 in the above table).

Panel (b) shows the payoff diagram for a purchased index put with a strike price of $1000 (column 2 in the table above).

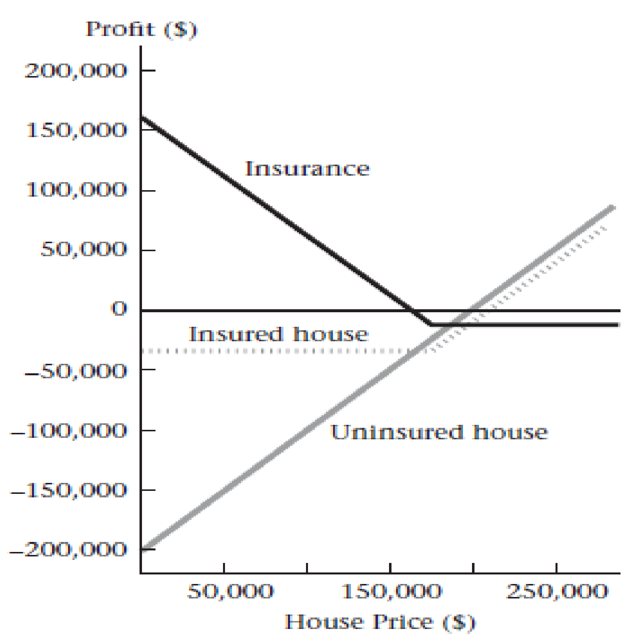


Panel (c) shows the combined payoff diagram for the index and put (column 3 in the table above).

Panel (d) shows the combined profit diagram for the index and put, obtained by subtracting $1095.68 from the payoff diagram in panel (c) (column 5 in the above table).

An insured house has a profit diagram that looks like a call option.

Payoff to owning a house and owning insurance. We assume a $25,000 deductible and a $200,000 house, with the policy costing $15,000.



6.1.3 Insuring a short position: Caps

If we have a short position in the ABC index, we experience a loss when the index rises.

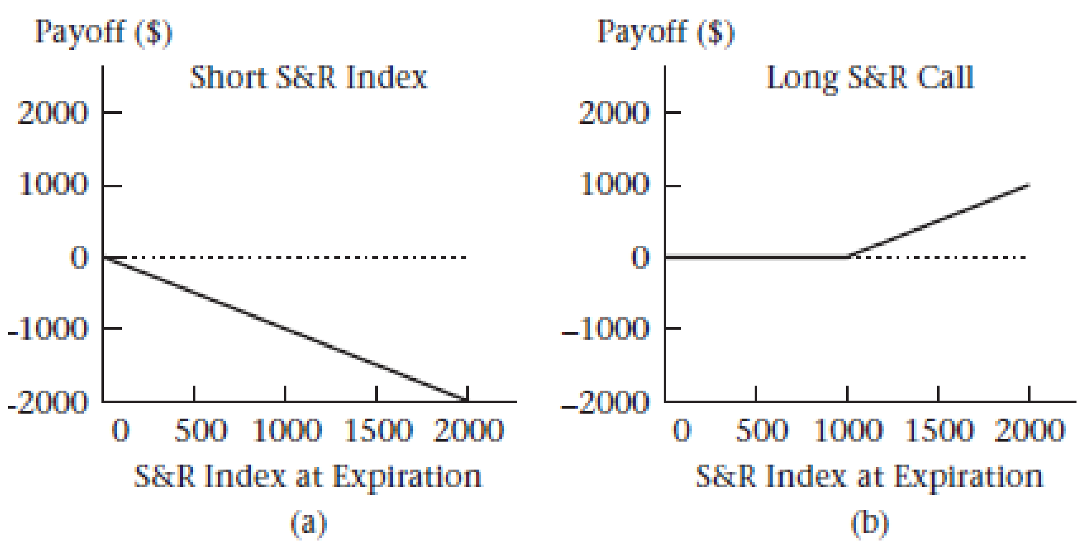
We can insure a short position by purchasing a call option to protect against a higher price of repurchasing the index. Buying a call option is also called a **cap**.

The following table shows the payoff and profit at expiration from short–selling the ABC index and buying a 1000–strike call option at a premium of \$93.809. The payoff is the sum of the first two columns. Cost plus interest for the position is

$$(-\$1000 + \$93.809) \times 1.02 = -\$924.32.$$

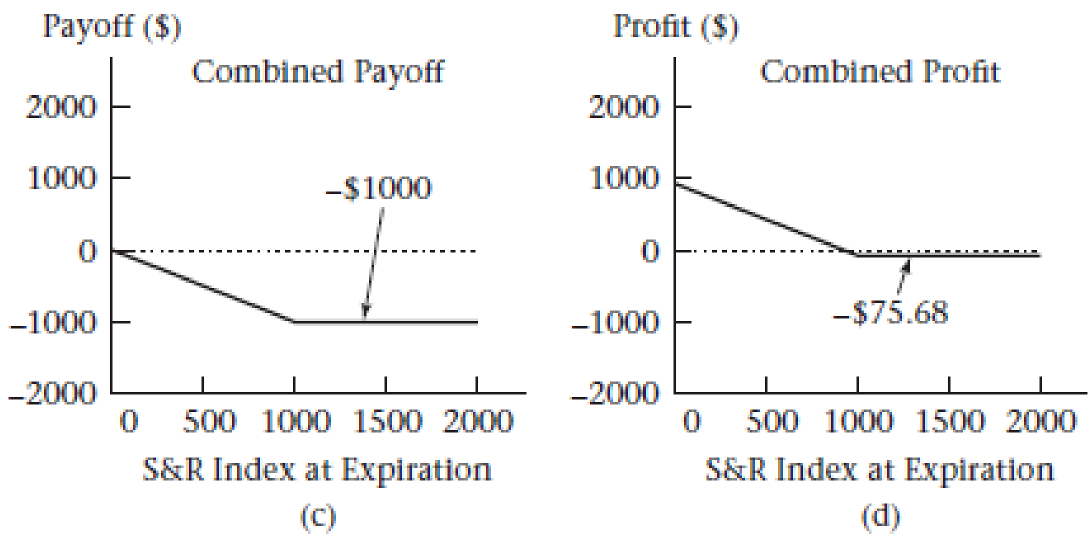
Profit is payoff plus \$924.32.

Payoff at Expiration				
Short S&R Index	S&R Call	Payoff	−(Cost + Interest)	Profit
−\$900	\$0	−\$900	\$924.32	\$24.32
−950	0	−950	924.32	−25.68
−1000	0	−1000	924.32	−75.68
−1050	50	−1000	924.32	−75.68
−1100	100	−1000	924.32	−75.68
−1150	150	−1000	924.32	−75.68
−1200	200	−1000	924.32	−75.68



Panel (a) shows the payoff diagram for a short position in the index (column 1 in the table above).

Panel (b) shows the payoff diagram for a purchased index call with a strike price of \$1000 (column 2 in the above table).



Panel (c) shows the combined payoff diagram for the short index and long call (column 3 in the table above.

Panel (d) shows the combined profit diagram for the short index and long call, obtained by adding $924.32 to the payoff diagram in panel (c) (column 5 in the table above).

Selling Insurance

We can expect that some investors want to purchase insurance. However, for every insurance buyer there must be an insurance seller. In this paragraph we examine strategies in which investors sell insurance.

It is possible, of course, for an investor to simply sell calls and puts. Often, however, investors also have a position in the asset when they sell insurance. Writing an option when there is a corresponding long position in the underlying asset is called **covered writing**, **option overwriting**, or selling a covered call. All three terms mean essentially the same thing. In contrast, **naked writing** occurs when the writer of an option does not have a position in the asset.

In addition to the covered writing strategies we will discuss here, there are other insurance–selling strategies, such as delta-hedging, which are less risky than naked writing and are used in practice by market-makers.

Covered Call Writing. If we own the ABC index and simultaneously sell a call option, we have written a covered call. A covered call will have limited profitability if the index increases, because an option writer is obligated to sell the index for the strike price. Should the index decrease, the loss on the index is offset by the premium earned from selling the call. A payoff with limited profit for price increases and potentially large losses for price decreases sounds like a written put.

The covered call looks exactly like a written put, and the maximum profit will be the same as with a written put. Suppose the index is \$1100 at expiration. The profit is

$$\overbrace{\$1100 - (\$1000 \times 1.02)}^{\text{Profit on ABC Index}} + \overbrace{(\$93.809 \times 1.02) - \$100}^{\text{Profit on written call}} = \$75.68$$

which is the future value of the premium received from writing a 1000–strike put.

Why would anyone write a covered call? Suppose you have the view that the index is unlikely to move either up or down. (This is sometimes called a "neutral" market view.)

If in fact the index does not move and you have written a call, then you keep the premium.

If you are wrong and the stock appreciates, you forgo gains you would have had if you did not write the call.

Covered Puts. A covered put is achieved by writing a put against a short position on the index. The written put obligates you to buy the index–for a loss–if it goes down in price. Thus, for index prices below the strike price, the loss on the written put offsets the short stock. For index prices above the strike price, you lose on the short stock.

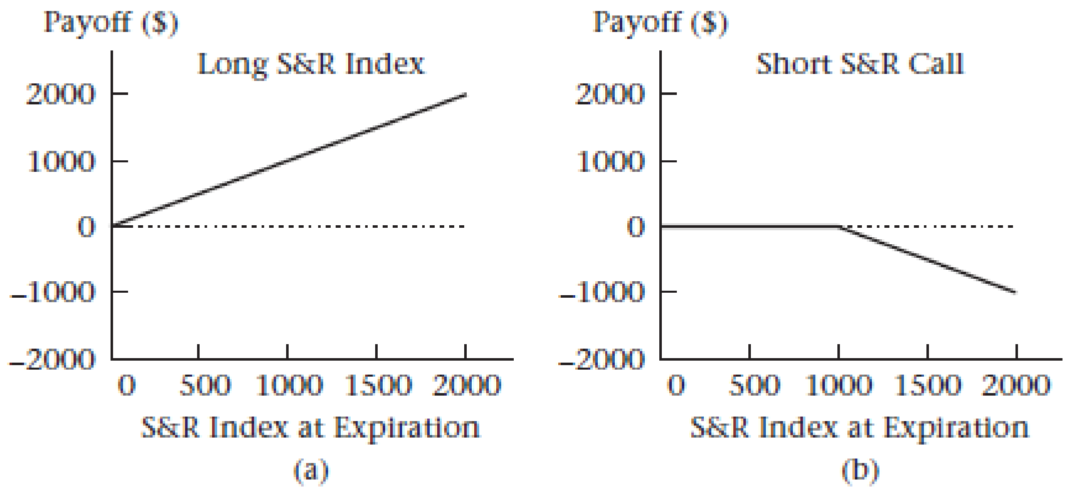
Payoff and profit at expiration from purchasing the ABC index and selling a 1000–strike call option. The payoff column is the sum of the first two columns. Cost plus interest for the position is

$$(\$1000 - \$93.809) \times 1.02 = \$924.32.$$

Profit is payoff less \$924.32.

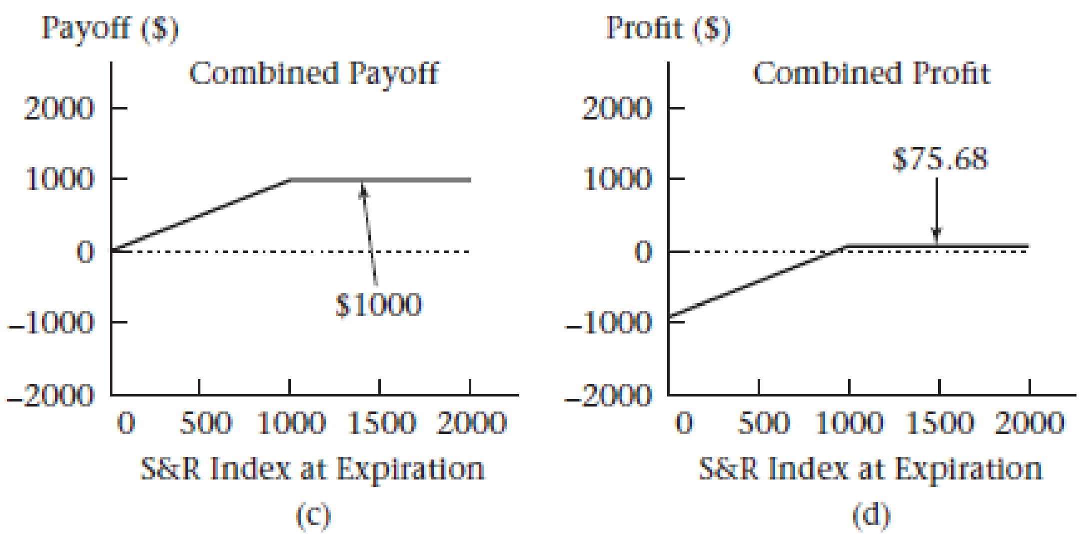
Payoff at Expiration				
S&R Index	Short S&R Call	Payoff	–(Cost + Interest)	Profit
\$900	\$0	\$900	–\$924.32	–\$24.32
950	0	950	–924.32	25.68
1000	0	1000	–924.32	75.68
1050	–50	1000	–924.32	75.68
1100	–100	1000	–924.32	75.68
1150	–150	1000	–924.32	75.68
1200	–200	1000	–924.32	75.68

Payoff and profit diagrams for writing a covered ABC call.



Panel (a) is the payoff to a long ABC position.

Panel (b) is the payoff to a short ABC call with strike price of 1000.



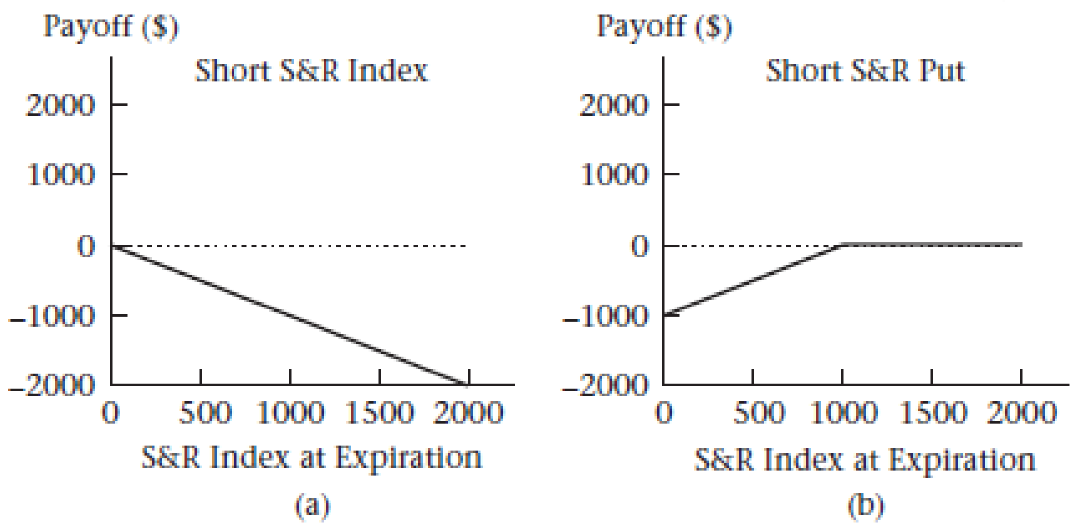
Panel (c) is the combined payoff for the ABC index and written call.

Panel (d) is the combined profit, obtained by subtracting

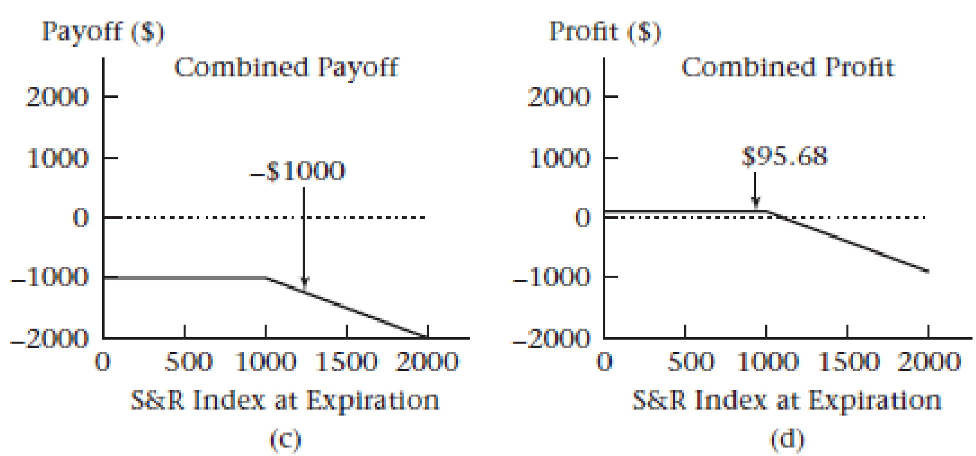
$$(\$1000 - \$93.809) \times 1.02 = \$924.32$$

from the payoff in panel (c).

Payoff and profit diagrams for writing a covered ABC put.



Panel (a) is the payoff to a short ABC position.
Panel (b) is the payoff to a short ABC put with a strike price of $1000.



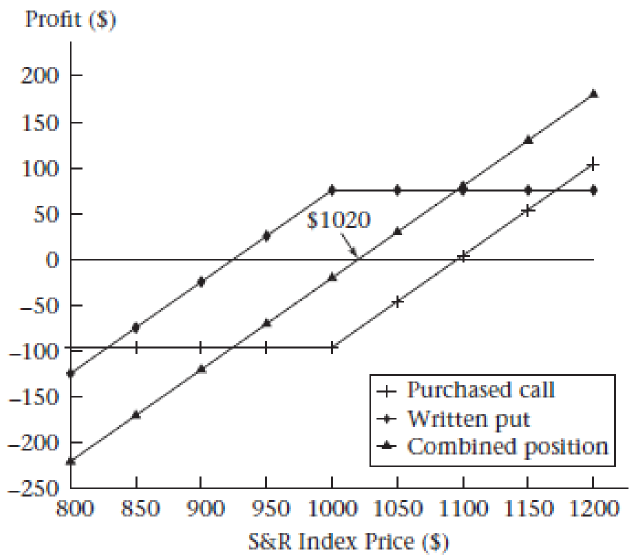
Panel (c) is the combined payoff for the short ABC index and written put.
Panel (d) is the combined profit, obtained by adding

$$(\$1000 + \$74.201) \times 1.02 = \$1095.68$$

to the payoff in panel (c).

A position where you have a constant payoff below the strike and increasing losses above the strike sounds like a written call. In fact, shorting the index and writing a put produces a profit diagram that is exactly the same as for a written call.

Purchase of a 1000-strike ABC call, sale of a 1000-strike ABC put, and the combined position. The combined position resembles the profit on a long forward contract.



6.1.4 Call–Put parity

We now discuss **call–put parity**, which is one of the most important relationships in the theory of options. The parity equation tells us the difference in the premiums of calls and puts, when the two options have the same strike price and time to expiration. In this section we will first discuss the use of options to create synthetic forward contracts. We then develop the **call–put** parity equation.

Synthetic Forwards

It is possible to mimic a long forward position on an asset by buying a call and selling a put, with each option having the same strike price and time to expiration. For example, we could buy the 6–month 1000–strike ABC call for \$93.81 and sell the 6–month 1000–strike ABC put for \$74.20. This position resembles a long forward contract: In 6 months we will pay \$1000 to buy the index. An important difference between the forward and the option position is that the forward contract has no premium, but the options have a net cost of $\$93.81 - \$74.20 = \$19.61$.

To better understand this position, suppose that the index in 6 months is at 900. We will not exercise the call, but we have written a put. The put buyer will exercise the right to sell the index for \$1000; therefore we are obligated to buy the index at \$1000. If instead the index is at \$1100, the put is not exercised, but we exercise the call, buying the index for \$1000. Thus, whether the index rises or falls, when the options expire we buy the index for the strike price of the options, \$1000.

The purchased call, written put, and combined positions are shown in the graph above. The purchase of a call and sale of a put creates a synthetic long forward contract, which has two minor differences from the actual forward:

1. The forward contract has a zero premium, while the synthetic forward requires that we pay the net option premium.

2. With the forward contract we pay the forward price, while with the synthetic forward we pay the strike price.

If you think about it, these two considerations must be related. If we set the strike price low, we are obligated to buy the index at a discount relative to the forward price. Buying at a lower price than the forward price is a benefit. In order to obtain this benefit we have to pay the positive net option premium, which stems from the call being more expensive than the put. In fact, in the previous figure, the implicit cost of the synthetic forward–the price at which the profit on the combined call–put position is zero–is \$1020, which is the ABC forward price.

Similarly, if we set the strike price high, we are obligated to buy the index at a high price relative to the forward price. To offset the extra cost of acquiring the index using the high strike options, it makes sense that we would receive payment initially. This would occur if the put that we sell is more expensive than the call we buy.

Finally, if we set the strike price equal to the forward price, then to mimic the forward the initial premium must equal zero. In this case, put and call premiums must be equal.

The Call–Put Parity Equation We can summarize this argument by saying that the net cost of buying the index at a future date using options must equal the net cost of buying the index using a forward contract. If at time 0 we enter into a long forward position expiring at time T, we obligate ourselves to buy the index at the forward price, $F_{0,T}$. The present value of buying the index in the future is just the present value of the forward price, $PV(F_{0,T})$.

If instead we buy a call and sell a put today to guarantee the purchase price for the index in the future, the present value of the cost is the net option premium for buying the call and selling the put, $Call(K,T) - Put(K,T)$, plus the present value of the strike price, $PV(K)$. (The notations "$Call(K,T)$" and "$Put(K,T)$" denote the premiums of options with strike price K and with T periods until expiration.)

Equating the costs of the alternative ways to buy the index at time T gives us

$$PV(F_{0,T}) = [Call(K,T) - Put(K,T)] + PV(K).$$

We can rewrite this as

$$Call(K,T) - Put(K,T) = PV(F_{0,T} - K). \tag{6.1}$$

In words, the present value of the bargain element from buying the index at the strike price [the right–hand side of equation (6.1)] must equal the initial net option premium [the left hand side of equation (6.1)]. Equation (6.1) is known as **call–put parity**.

Example 12. *As an example of equation (6.1), consider buying the 6–month 1000–strike ABC call for a premium of \$93.809 and selling the 6–month 1000–strike put for a premium of \$74.201. These transactions create a synthetic forward permitting us to buy the index in 6 months for \$1000. Because the actual forward price is \$1020, this synthetic forward permits us to buy the index at a bargain of \$20, the present value of which is* $\$20/1.02 = \$19.61$. *The difference in option premiums is also*

$$\$19.61 = \$93.809 - \$74.201 = \$19.61.$$

This result is exactly what we get with equation (6.1):

$$\$93.809 - \$74.201 = PV(\$1020 - \$1000)$$

A forward contract for which the premium is not zero is sometimes called an offmarket forward. This terminology arises since a true forward by definition has a zero premium. Therefore, a forward contract with a nonzero premium must have a forward price that is "off the market (forward) price." Unless the strike price equals the forward price, buying a call and selling a put creates an offmarket forward.

Equivalence of different positions. We have seen earlier that buying the index and buying a put generates the same profit as buying a call. Similarly, selling a covered call (buying the index and selling a call) generates the same profit as selling a put. Equation (6.1) explains why this happens.

Consider buying the index and buying a put. Recall that, in this example, we have the forward price equal to \$1020 and the index price equal to \$1000.

Thus, the present value of the forward price equals the index price. Rewriting equation (6.1) gives

$$\begin{aligned} PV(F_{0,T}) + Put(K,T) &= Call(K,T) + PV(K) \\ \$1000 + \$74.201 &= \$93.809 + \$980.39 \end{aligned}$$

Similarly, in the case of writing a covered call, we have

$$PV(F_{0,T}) - Call(K,T) = PV(K) - Put(K,T)$$

That is, writing a covered call has the same profit as lending $PV(K)$ and selling a put. Equation (6.1) provides a tool for constructing equivalent positions.

6.1.5 Spreads and collars

There are many well-known, commonly used strategies that combine two or more options.

In this subsection we discuss some of these strategies and explain the motivation for using them. The underlying theme in this section is that there are always trade-offs in designing a position: It is always possible to lower the cost of a position by reducing its payoff. Thus there are many variations on each particular strategy.

Bull and Bear Spreads

An option spread is a position consisting of only calls or only puts, in which some options are purchased and some written. Spreads are a common strategy. In this paragraph we define some typical spread strategies and explain why you might use a spread.

Suppose you believe a stock will appreciate. Let us compare two ways to speculate on this belief: entering into a long forward contract or buying a call option with the strike price equal to the forward price. The forward contract has a zero premium and the call has a positive premium. A difference in payoffs explains the difference in premiums. If the stock price at expiration is greater than the forward price, the forward contract and call have the same payoff. If the stock price is less than the forward price, however, the forward contract has a loss and the call is worth zero. call–put parity tells us that the call is equivalent to the forward contract plus a put option. Thus, the call premium equals the cost of the put, which is insurance against the stock price being less than the forward price.

A position in which you buy a call and sell an otherwise identical call with a higher strike price is an example of a **bull spread**.

Bull spreads can also be constructed using puts. Perhaps surprisingly, you can achieve the same result either by buying a low–strike call and selling a high–strike call, or by buying a low–strike put and selling a high–strike put.

See the file con combined options

Chapter 7

Swaps

7.1 Interest rate swap

So far we have talked about derivatives contracts that settle on a single date. A forward contract, for example, fixes a price for a transaction that will occur on a specific date in the future. However, many transactions occur repeatedly. Firms that issue bonds make periodic coupon payments. Multinational firms frequently exchange currencies. Firms that buy commodities as production inputs or that sell them make payments or receive income linked to commodity prices on an ongoing basis.

These situations raise the question: If a manager seeking to reduce risk confronts a risky payment stream–as opposed to a single risky payment–what is the easiest way to hedge this risk? One obvious answer is that we can enter into a separate forward contract for each payment we wish to hedge. However, it might be more convenient, and entail lower transaction costs, if we could hedge a stream of payments with a single transaction.

Definition 13. *A swap is a contract calling for an exchange of payments over time. One party makes a payment to the other depending upon whether a reference price turns out to be greater or less than a fixed price that is specified in the swap contract.*

Remark 14. *A swap thus provides a means to hedge a stream of risky payments.*
By entering into an Oil swap, for example, an Oil buyer confronting a stream of uncertain Oil payments can lock in a fixed price for Oil over a period of time.
The swap payments would be based on the difference between a fixed price for Oil and a market price that varies over time.

From this description, you can see that there is a relationship between swaps and forward contracts. In fact, a forward contract is a single–payment swap. It is possible to price a multi-date swap–determine the fixed price for Oil in the above example–by using information from the set of forward prices with different maturities (i.e., the strip). We will see that swaps are nothing more than forward contracts coupled with borrowing and lending money.

7.1.1 Illustration example

Consider a hypothetical 3–year swap initiated on March 5, 2007, between ABC and XYZ. We suppose ABC agrees to pay XYZ an interest rate of 5% per annum on a principal of

$100 million, and in return XYZ agrees to pay ABC the 6–month LIBOR rate on the same principal. ABC is the fixed–rate payer, XYZ is the floating–rate payer. We assume the agreement specifies that payments are to be exchanged every 6 months and that the 5% interest rate is **quoted with semiannual compounding**. This swap is represented the following diagram

Interest rate swap between ABC and XYZ

ABC	**5.00%** – – – – – – – – – – – – – > < – – – – – – – – – – – – – – **LIBOR**	XYZ

The first exchange of payments would take place on September 5, 2007, 6 months after the initiation of the agreement. ABC would pay XYZ

$$0.5 \times 0.05 \times \$100 \text{ million} = \$2.5 \text{ million}.$$

This is the interest on the $100 million principal for 6 months at 5%. XYZ would pay ABC interest on the $100 million principal at the 6–month LIBOR rate prevailing 6 months prior to September 5, 2007 that is, on March 5, 2007. Suppose that the 6–month LIBOR rate on March 5, 2007, is 4.2%, then XYZ pays ABC

$$0.5 \times 0.042 \times \$100 \text{ million} = \$2.1 \text{ million}$$

Note that there is no uncertainty about this first exchange of payments because it is determined by the LIBOR rate at the time the contract is entered into.

The second exchange of payments would take place on March 5, 2008, a year after the initiation of the agreement. ABC would pay $2.5 million to XYZ. XYZ would pay interest on the $100 million principal to ABC at the 6–month LIBOR rate prevailing 6 months prior to March 5, 2008 that is, on September 5, 2007. Suppose that the 6–month LIBOR rate on September 5, 2007, is 4.8%, then XYZ pays $0.5 \times 0.048 \times \$100 = \$2.4$ million to ABC.

In total, there are six exchanges of payment on the swap. The fixed payments are always $2.5 million. The floating–rate payments on a payment date are calculated using the 6–month LIBOR rate prevailing 6 months before the payment date. An interest rate swap is generally structured so that one side remits the difference between the two payments to the other side. In our example, ABC would pay XYZ $0.4 million (= $2.5 million −$2.1 million) on September 5, 2007, and $0.1 million (= $2.5 million −$2.4 million) on March 5, 2008.

The following table describes the Cash flows in (millions of dollars) paid to ABC in a $100 million 3–year interest rate swap when a fixed rate of 5% is paid and LIBOR is received.

Date	**Six–month LIBOR rate (%)**	**Floating cash flow received**	**Fixed cash flow paid**	**Net cash flow ABC**	**Net cash flow XYZ**
Mar. 5, 2007	4.20				
Sep. 5, 2007	4.80	+2.10	−2.50	−0.40	+0.40
Mar. 5, 2008	5.30	+2.40	−2.50	−0.10	+0.10
Sep. 5, 2008	5.50	+2.65	−2.50	+0.15	−0.15
Mar. 5, 2009	5.60	+2.75	−2.50	+0.25	−0.25
Mar. 5, 2009	5.90	+2.80	−2.50	+0.30	−0.30
Mar. 5, 2010		+2.95	−2.50	+0.45	−0.45

The above table provides a complete example of the payments made under the swap for one particular set of 6–month LIBOR rates. The table shows the swap cash flows from the perspective of ABC. Note that the $100 million principal is used only for the calculation of interest payments. The principal itself is not exchanged. For this reason it is termed the notional principal, or just the notional.

If the principal were exchanged at the end of the life of the swap, the nature of the deal would not be changed in any way. The principal is the same for both the fixed and floating payments. Exchanging $100 million for $100 million at the end of the life of the swap is a transaction that would have no financial value to either ABC or XYZ.

The following table shows the cash flows with a final exchange of principal added in.

Date	**Six–month LIBOR rate (%)**	**Floating cash flow received**	**Fixed cash flow paid**	**Net cash flow ABC**	**Net cash flow XYZ**
Mar. 5, 2007	4.20				
Sep. 5, 2007	4.80	+2.10	−2.50	−0.40	+0.40
Mar. 5, 2008	5.30	+2.40	−2.50	−0.10	+0.10
Sep. 5, 2008	5.50	+2.65	−2.50	+0.15	−0.15
Mar. 5, 2009	5.60	+2.75	−2.50	+0.25	−0.25
Mar. 5, 2009	5.90	+2.80	−2.50	+0.30	−0.30
Mar. 5, 2010		+2.95 + 100	−2.50 − 100	+0.45	−0.45

This provides an interesting way of viewing the swap. The cash flows in the third column of this table are the cash flows from a long position in a floating–rate bond. The cash flows in the fourth column of the table are the cash flows from a short position in a fixed–rate bond. The table shows that the swap can be regarded as the exchange-of a fixed–rate bond for a floating–rate bond. ABC, whose position is described by the previous table is long a floating–rate bond and short a fixed–rate bond. XYZ is long a fixed–rate bond and short a floating–rate bond.

This characterization of the cash flows in the swap helps to explain why the floating rate in the swap is set 6 months before it is paid. On a floating–rate bond, interest is generally set at the beginning of the period to which it will apply and is paid at the end of the period.

7.1.2 Using the swap to transform a liability

For ABC, the swap could be used to transform a floating–rate loan into a fixed–rate loan. Suppose that ABC has arranged to borrow $100 million at LIBOR plus 10 basis points. (**One basis point is one-hundredth of 1%**, so the rate is LIBOR plus 0.1%). After ABC has entered into the swap, it has the following three sets of cash flows:

1. It pays LIBOR plus 0.1% to its outside lenders.
2. It receives LIBOR under the terms of the swap.
3. It pays 5% under the terms of the swap.

These three sets of cash flows net out to an interest rate payment of 5.1%. Thus, for ABC, the swap could have the effect of transforming borrowings at a floating rate of LIBOR plus 10 basis points into borrowings at a fixed rate of 5.1%.

For XYZ, the swap could have the effect of transforming a fixed–rate loan into a floating–rate loan. Suppose that XYZ has a 3–year \$100 million loan outstanding on which it pays 5.2%. After it has entered into the swap, it has the following three sets of cash flows:

1. It pays 5.2% to its outside lenders.
2. It pays LIBOR under the terms of the swap.
3. It receives 5% under the terms of the swap.

These three sets of cash flows net out to an interest rate payment of LIBOR plus 0.2% (or LIBOR plus 20 basis points). Thus, for XYZ, the swap could have the effect of transforming borrowings at a fixed rate of 5.2% into borrowings at a floating rate of LIBOR plus 20 basis points. These potential uses of the swap by XYZ and ABC are illustrated in the following table.

ABC and XYZ use the swap to transform a liability

< – – – – – **LIBOR+0.1%**	ABC	**5.00%** – – – – – > < – – – – – **LIBOR**	XYZ	**5.20%** – – – – – >

7.1.3 Using the swap to transform an asset

Swaps can also be used to transform the nature of an asset. Consider ABC in our example. The swap could have the effect of transforming an asset earning a fixed rate of interest into an asset earning a floating rate of interest. Suppose that ABC owns \$100 million in bonds that will provide interest at 4.7% per annum over the next 3 years.

ABC and XYZ use the swap to transform asset

4.7% – – – – – >	ABC	**5.00%** – – – – – > < – – – – – **LIBOR**	XYZ	< – – – – – **LIBOR–0.2%**

After ABC has entered into the swap, it has the following three sets of cash flows:

1. It receives 4.7% on the bonds.
2. It receives LIBOR under the terms of the swap.
3. It pays 5% under the terms of the swap.

These three sets of cash flows net out to an interest rate inflow of LIBOR minus 30 basis points. Thus, one possible use of the swap for ABC is to transform an asset earning 4.7% into an asset earning LIBOR minus 30 basis points.

Next, consider XYZ. The swap could have the effect of transforming an asset earning a floating rate of interest into an asset earning a fixed rate of interest. Suppose that XYZ has an investment of \$100 million that yields LIBOR minus 20 basis points. After it has entered into the swap, it has the following three sets of cash flows:

1. It receives LIBOR minus 20 basis points on its investment.
2. It pays LIBOR under the terms of the swap.
3. It receives 5% under the terms of the swap.

These three sets of cash flows net out to an interest rate inflow of 4.8%. Thus, one possible use of the swap for XYZ is to transform an asset earning LIBOR minus 20 basis points into an asset earning 4.8%. These potential uses of the swap by XYZ and ABC are illustrated in the previous table.

7.1.4 Role of financial intermediary

Usually two nonfinancial companies such as XYZ and ABC do not get in touch directly to arrange a swap in the way indicated in previous tables. They each deal with a financial intermediary such as a bank or other financial institution. "**Plain vanilla**" fixed–for–floating swaps on US interest rates are usually structured so that the financial institution earns about 3 or 4 basis points (0.03% or 0.04%) on a pair of offsetting transactions.

ABC and XYZ use the swap to transform a liability

< – – – – – **LIBOR+0.1%**	ABC	**5.015%** – – – – – > < – – – – – **LIBOR**	Financial institution	**4.985%** – – – – – > < – – – – – **LIBOR**	XYZ	**5.20%** – – – – – >

This figure shows what the role of the financial institution. The financial institution enters into two offsetting swap transactions with XYZ and ABC. Assuming that both companies honor their obligations, the financial institution is certain to make a profit of 0.03% (3 basis points) per year multiplied by the notional principal of $100 million. This amounts to $30,000 per year for the 3–year period. ABC ends up borrowing at 5.115% (instead of 5.1%), and XYZ ends up borrowing at LIBOR plus 21.5 basis points (instead of at LIBOR plus 20 basis points).

ABC and XYZ use the swap to transform asset

4.7% – – – – – >	ABC	**5.015%** – – – – – > < – – – – – **LIBOR**	Financial institution	**4.985%** – – – – – > < – – – – – **LIBOR**	XYZ	< – – – – – **LIBOR–0.2%**

This figure illustrates the role of the financial institution. The swap is the same as before and the financial institution is certain to make a profit of 3 basis points if neither company defaults. ABC ends up earning LIBOR minus 31.5 basis points (instead of LIBOR minus 30 basis points), and XYZ ends up earning 4.785% (instead of 4.8%).

Note that in each case the financial institution has two separate contracts: one with XYZ and the other with ABC. In most instances, XYZ will not even know that, financial institution has entered into an offsetting swap with ABC, and vice versa.

If one of the companies defaults, the financial institution still has to honor its agreement with the other company. The 3–basis–point spread earned by the financial institution is partly to compensate it for the risk that one of the two companies will default on the swap payments.

7.1.5 Comparative advantage argument

An explanation commonly put forward to explain the popularity of swaps concerns comparative advantages. Consider the use of an interest rate swap to transform a liability. Some companies, it is argued, have a comparative advantage when borrowing in fixed–rate markets, whereas other companies have a comparative advantage in floating–rate markets. To obtain a new loan, it makes sense for a'-company to go to the market where it has a comparative advantage. As a result, the company may borrow fixed when it wants floating, or borrow floating when it wants fixed. The swap is used to transform a fixed–rate loan into a floating–rate loan, and vice versa.

Suppose that two companies, AAACorp and BBBCorp, both wish to borrow $10 million for 5 years and have been offered the rates shown below. AAACorp has a AAA credit rating; BBBCorp has a BBB credit rating. We assume that BBBCorp wants to borrow at a fixed rate of interest, whereas AAACorp wants to borrow at a floating rate of interest linked to 6–month LIBOR. Because it has a worse credit rating than AAACorp, BBBCorp pays a higher rate of interest than AAACorp in both fixed and floating markets.

A key feature of the rates offered to AAACorp and BBBCorp is that the difference between the two fixed rates is greater than the difference between the two floating rates.

BBBCorp pays 1.2% more than AAACorp in fixed–rate markets and only 0.7% more than AAACorp in floating–rate markets. BBBCorp appears to have a comparative advantage in floating–rate markets, whereas AAACorp appears to have a comparative advantage in fixed–rate markets. It is this apparent anomaly that can lead to a swap being negotiated. AAACorp borrows fixed–rate funds at 4% per annum. BBBCorp borrows floating–rate funds at LIBOR plus 0.6% per annum. They then enter into a swap agreement to ensure that AAACorp ends up with floating–rate funds and BBBCorp ends up with fixed–rate funds.

Borrowing rates that provide a basis for the comparative–advantage argument		
	fixed	**floating**
AAACorp	4.0%	6−month LIBOR−0.1%
BBBCorp	5.2%	6−month LIBOR+0.6%

Swap agreement between AAACorp and BBBCorp when rates above apply.

< − − − − − **LIBOR+0.6%**	BBBCorp	**4.35%** − − − − − > < − − − − − **LIBOR**	AAACorp	**4%** − − − − − >

To understand how this swap might work, we first assume that AAACorp and BBBCorp get in touch with each other directly. AAACorp agrees to pay BBBCorp interest at 6–month LIBOR on $10 million. In return, BBBCorp agrees to pay AAACorp interest at a fixed rate of 4.35% per annum on $10 million.

AAACorp has three sets of interest rate cash flows:

1. It pays 4% per annum to outside lenders.

2. It receives 4.35% per annum from BBBCorp.

3. It pays LIBOR to BBBCorp.

The net effect of the three cash flows is that AAACorp pays LIBOR minus 0.35% per annum. This is 0.25% per annum less than it would pay if it went directly to floating–rate markets. BBBCorp also has three sets of interest rate cash flows:

1. It pays LIBOR +0.6% per annum to outside lenders.

2. It receives LIBOR from AAACorp.

3. It pays 4.35% per annum to AAACorp.

The net effect of the three cash flows is that BBBCorp pays 4.95% per annum. This is 0.25% per annum less than it would pay if it went directly to fixed–rate markets.

In this example, the swap has been structured so that the net gain to both sides is the same, 0.25%. This need not be the case. However, the total apparent gain from this type of interest rate swap arrangement is always $a - b$, where a is the difference between the interest rates facing the two companies in fixed–rate markets, and b is the difference between the interest rates facing the two companies in floating–rate markets. In this case, $a = 1.2\%$ and $b = 0.7\%$, so that the total gain is 0.5%.

If AAACorp and BBBCorp did not deal directly with each other and used a financial institution, an arrangement such as that shown in a figure above might result. In this case, AAACorp ends up borrowing at LIBOR minus 0.33%, BBBCorp ends up borrowing at 4.97%, and the financial institution earns a spread of 4 basis points per year. The gain to AAACorp is 0.23%; the gain to BBBCorp is 0.23%; and the gain to the financial institution is 0.04%. The total gain to all three parties is 0.50% as before.

Swap agreement between AAACorp and BBBCorp and a financial intermediary is involved.

Risk/Return Characteristics of a Swap

	Interest Rates Decrease	Interest Rates Increase
Floating-Rate Payer	Gain	Loss
Fixed-Rate Payer	Loss	Gain

7.2 Currency swap

Another popular type of swap is known as a currency swap. In its simplest form, this involves exchanging principal and interest payments in one currency for principal and interest payments in another.

A currency swap agreement requires the principal to be specified in each of the two currencies. The principal amounts in each currency are usually exchanged at the beginning and at the end of the life of the swap. Usually the principal amounts are chosen to be approximately equivalent using the exchange rate at the swap's initiation.

When they are exchanged at the end of the life of the swap, their values may be quite different.

A currency swap		
IBM	**dollars 6%** < − > **sterling 5%**	British petroleum

7.2.1 Illustration

Consider a hypothetical 5–year currency swap agreement between IBM and British Petroleum entered into on February 1, 2007. We suppose that IBM pays a fixed rate of interest of 5 % in sterling and receives a fixed rate of interest of 6% in dollars from British Petroleum. Interest rate payments are made once a year and the principal amounts are $18 million and £10 million. This is termed & fixed–for–fixed currency swap because the interest rate in both currencies is fixed. The swap is shown above. Initially, the principal amounts flow in the opposite direction to the arrows in the above diagram. The interest payments during the life of the swap and the final principal payment flow in the same direction as the arrows. Thus, at the outset of the swap, IBM pays $18 million and receives £10 million. Each year during the life of the swap contract, IBM receives $1.08 million (= 6% of $18 million) and pays £0.50 million (= 5% of £10 million). At the end of the life of the swap, it pays a principal of £10 million and receives a principal of $18 million. These cash flows are shown in the following table

Cash flows to IBM in currency swap

Date	Dollar cash flow (millions)	Sterling cash flow (millions)
February 1, 2007	−18.00	+10.00
February 1, 2008	+1.08	−0.50
February 1, 2009	+1.08	−0.50
February 1, 2010	+1.08	−0.50
February 1, 2011	+1.08	−0.50
February 1, 2012	+19.08	−10.50

7.2.2 Use of a Currency Swap to Transform Liabilities and Assets.

A swap such as the one just considered can be used to transform borrowings in one currency to borrowings in another. Suppose that IBM can issue $18 million of US–dollar–denominated bonds at 6% interest. The swap has the effect of transforming this transaction into one where IBM has borrowed £10 million at 5% interest. The initial exchange of principal converts the proceeds of the bond issue from US dollars to sterling. The subsequent exchanges in the swap have the effect of swapping the interest and principal payments from dollars to sterling.

The swap can also be used to transform the nature of assets. Suppose that IBM can invest £10 million in the UK to yield 5% per annum for the next 5 years, but feels that the US dollar will strengthen against sterling and prefers a US-dollar-denominated investment. The swap has the effect of transforming the UK investment into a $18 million investment in the US yielding 6%.

Comparative advantage

Currency swaps can be motivated by comparative advantage. To illustrate this, we consider another hypothetical example. Suppose the 5–year fixed–rate borrowing costs to General Electric and Qantas Airways in US dollars (USD) and Australian dollars (AUD) are shown below. The data in the table suggest that Australian rates are higher than USD interest rates, and also that General Electric is more creditworthy than Qantas Airways, because it is offered a more favorable rate of interest in both currencies. From the viewpoint of a swap trader, the interesting aspect is that the spreads between the rates paid by General Electric and Qantas Airways in the two markets are not the same. Qantas Airways pays 2% more than General Electric in the US dollar market and only 0.4% more than General Electric in the AUD market.

General Electric has a comparative advantage in the USD market, whereas Qantas Airways has a comparative advantage in the AUD market. In the table below, where a plain vanilla interest rate swap was considered, we argued that comparative advantages are largely illusory. Here we are comparing the rates offered in two different currencies, and it is more likely that the comparative advantages are genuine. One possible source of comparative advantage is tax. General Electric's position might be such that USD borrowings lead to lower taxes on its worldwide income than AUD borrowings. Qantas Airways' position might be the reverse.

We suppose that General Electric wants to borrow 20 million AUD and Qantas Airways wants to borrow 15 million USD and that the current exchange rate (USD per AUD) is 0.7500.

$$0.7500 \times \text{AUD } 20 \text{ Million } = \$15 \text{ Million}$$

This creates a perfect situation for a currency swap. General Electric and Qantas Airways each borrow in the market where they have a comparative advantage; that is, General Electric borrows USD whereas Qantas Airways borrows AUD. They then use a currency swap to transform General Electric's loan into an AUD loan and Qantas Airways' loan into a USD loan. As already mentioned, the difference between the USD interest rates is 2%, whereas the difference between the AUD interest rates is 0.4%. By analogy with the interest rate swap case, we expect the total gain to all parties to be $2.0 - 0.4 = 1.6\%$ per annum.

Borrowing rates providing basis for currency swap.

	USA	AUD
General Electric	5.0%	7.6%
Qantas Airways	7.0%	8%

Quoted rates have been adjusted to reflect the differential impact of taxes.

A currency swap motivated by comparative advantage

< − − − **USD 5.0%**	General Electric	**USD 5.0%** < − − − − − − > **AUD 6.9%**	Financial Institution	**USD 6.3%** < − − − − − − > **AUD 8.0%**	Qantas Airways	**AUD 8.0%** − − − >

There are many ways in which the swap can be arranged. This figure shows one way swaps might be entered into with a financial institution. General Electric borrows USD and

Qantas Airways borrows AUD. The effect of the swap is to transform the USD interest rate of 5% per annum to an AUD interest rate of 6.9% per annum for General Electric. As a result, General Electric is 0.7% per annum better off than it would be if it went directly to AUD markets. Similarly, Qantas exchanges an AUD loan at 8% per annum for a USD loan at 6.3% per annum and ends up 0.7% per annum better off than it would be if it went directly to USD markets. The financial institution gains 1.3% per annum on its USD cash flows and loses 1.1% per annum on its AUD flows. If we ignore the difference between the two currencies, the financial institution makes a net gain of 0.2% per annum. As predicted, the total gain to all parties is 1.6% per annum. Each year the financial institution makes a gain of USD 195,000 (= 1.3% of 15 million) and incurs a loss of AUD 220,000 (=1.1% of 20 million). The financial institution can avoid any foreign exchange risk by buying AUD 220,000 per annum in the forward market for each year of the life of the swap, thus locking in a net gain in USD.

It is possible to redesign the swap so that the financial institution makes a 0.2% spread in USD. These alternatives are unlikely to be used in practice because they do not lead to General Electric and Qantas being free of foreign exchange risk. Qantas bears some foreign exchange risk because it pays 1.1% per annum in AUD and pays 5.2% per annum in USD. General Electric bears some foreign exchange risk because it receives 1.1% per annum in USD and pays 8% per annum in AUD.

7.3 Commodity swap

7.3.1 An example of a commodity swap

We begin our study of swaps by presenting an example of a simple commodity swap. Our purpose here is to understand how a swap is related to forwards, why someone might use a swap, and how market–makers hedge the risk of swaps. In later sections we present swap price formulas and examine interest rate swaps, total return swaps, and more complicated commodity swap examples.

An industrial producer ABC is going to buy 100,000 barrels of Oil 1 year from today and 2 years from today. Suppose that the forward price for delivery in 1 year is \$110/barrel and in 2 years is \$111/barrel. Suppose that the 1– and 2–year annual zero–coupon bond yields are 6% and 6.5%.

ABC can use forward contracts to guarantee the cost of buying Oil for the next 2 years.

Specifically, ABC could enter into long forward contracts for 100,000 barrels in each of the next 2 years, committing to pay \$110/barrel in 1 year and \$111/barrel in 2 years.
The present value of this cost is

$$\frac{\$110}{1.06} + \frac{\$111}{(1.065)^2} = \$201.638$$

ABC could invest this amount today and ensure that it had the funds to buy Oil in 1 and 2 years. Alternatively, ABC could pay an Oil supplier \$201.638, and the supplier would commit to delivering one barrel in each of the next 2 years. A single payment today for a single delivery of Oil in the future is a prepaid forward. A single payment today to obtain multiple deliveries in the future is a **prepaid swap**.

Although it is possible to enter into a prepaid swap, buyers might worry about the resulting credit risk: They have fully paid for Oil that will not be delivered for up to 2 years.
(The prepaid forward has the same problem.) For the same reason, the swap counterparty would worry about a postpaid swap, where the Oil is delivered and full payment is made after 2 years. A more attractive solution for both parties is to defer payment until the Oil is delivered, while still fixing the total price.

Note that there are many feasible ways to have the buyer pay. Typically, however, a swap will call for equal payments in each year. The payment per year per barrel, x, will then have to be such that

$$\frac{x}{1.06}+\frac{x}{(1.065)^2}=\$201.638.$$

To satisfy this equation, the payments must be \$110.483 in each year. We then say that the 2–year swap price is \$110.483. However, any payments that have a present value of \$201.638 are acceptable.

7.3.2 Physical Versus Financial Settlement

Physical Settlement

We have described the swap as if the swap counterparty supplied physical Oil to the buyer. The following figure shows a swap that calls for physical settlement. In this case \$110.483 is the per–barrel cost of Oil.

Illustration of a swap where the Oil buyer pays \$110.483
per year and receives one barrel of Oil each year.

ABC: Oil buyer	**\$110.483** – – – – – – – – – – – – – – > < – – – – – – – – – – – – – – **Oil**	Swap counterparty

However, we could also arrange for financial settlement of the swap. With financial settlement, the Oil buyer, ABC, pays the swap counterparty the difference [\$110.483− spot price] (if the difference is negative, the counterparty pays the buyer), and the Oil buyer then buys Oil at the spot price.
For example, if the market price is \$115, the swap counterparty pays ABC

$$\text{Spot price - swap price} = \$115 - \$110.483 = \$4.517$$

If the market price is \$108, the spot price less the swap price is

$$\text{Spot price - swap price} = \$108 - \$110.483 = -\$2.483$$

In this case, the Oil buyer, ABC, makes a payment to the swap counterparty. Whatever the market price of Oil, the net cost to the buyer is the swap price, \$110.483:

$$\underbrace{\text{Spot price} - \text{swap price}}_{\textbf{Swap payment}} - \underbrace{\text{Spot price}}_{\textbf{Spot purchase of Oil}} = -\text{ Swap price}$$

Financial Settlement

The following figure depicts cash flows and transactions when the swap is settled financially.

Cash flows from a transaction where the Oil buyer enters into a financially settled 2–year swap. Each year the buyer pays the spot price for Oil and receives **spot price** – $**110.483**. The buyer's net cost of Oil is $110.483/barrel.

Oil buyer	**spot price – $110.483** < –	Swap counterpart
	spot price – >	Oil seller
	< – **Oil**	

The results for the buyer are the same whether the swap is settled physically or financially. In both cases, the net cost to the Oil buyer is $110.483. We have discussed the swap on a per–barrel basis. For a swap on 100,000 barrels, we simply multiply all cash flows by 100,000. In this example, 100,000 is the notional amount of the swap, meaning that 100,000 barrels is used to determine the magnitude of the payments when the swap is settled financially.

7.3.3 Why Is the swap price not $110.50?

The swap price, $110.483, is close to the average of the two Oil forward prices, $110.50.

However, it is not exactly the same. Why?

Suppose that the swap price were $110.50. The Oil buyer would then be committing to pay $0.50 more than the forward price the first year and would pay $0.50 less than the forward price the second year. Thus, relative to the forward curve, the buyer would have made an interest-free loan to the counterparty. There is implicit lending in the swap.

Now consider the actual swap price of $110.483/barrel. Relative to the forward curve prices of $110 in 1 year and $111 in 2 years, we are overpaying by $0.483 in the first year and we are underpaying by $0.517 in the second year. Therefore, the swap is equivalent to being long the two forward contracts, coupled with an agreement to lend $0.483 to the swap counterparty in 1 year, and receive $0.517 in 2 years. This loan has the effect of equalizing the net cash flow on the two dates.

The interest rate on this loan is 0.517/0.483 – 1 = 7%. Where does 7% come from? We assumed that 6% is the 1–year zero yield and 6.5% is the 2–year yield. Given these interest rates, 7% is the 1–year implied forward yield from year 1 to year 2. By entering into the swap, we are lending the counterparty money for one year beginning in one year. If the deal is priced fairly, the interest rate on this loan should be the implied forward interest rate.

Cash flows from a transaction where an Oil buyer and seller each enters into a financially settled 2–year swap. The buyer pays the spot price for Oil and receives the spot price −$110.483 each year as a swap payment. The Oil seller receives the spot price for Oil and receives $110.483− spot price as a swap payment.

Oil buyer	**spot price – $110.483** < – – – – – – – – – – –	Swap counterpart	**spot price – $110.483** < – – – – – – – – – – –	Oil seller
	spot price – – – – – – – – – – >	Spot Oil market	**spot price** – – – – – – – – – – >	
	< – – – – – – – – – – **Oil**		< – – – – – – – – – – **Oil**	

Positions and cash flows for a dealer who has an obligation to receive the fixed price in an Oil swap and who hedges the exposure by going long year 1 and year 2 Oil forwards.

Year	Payment from Oil buyer	Long forward	net
1	\$110.483− year 1 spot price	year 1 spot price −\$110	\$0.483
2	\$110.483− year 2 spot price	year 2 spot price −\$111	−0.517

This example shows that **hedging the Oil price risk in the swap**, with forwards only, does not fully hedge the position. The dealer also has interest rate exposure. If interest rates fall, the dealer will not be able to earn a sufficient return from investing \$0.483 in year 1 to repay \$0.517 in year 2. Thus, in addition to entering Oil forwards, it would make sense for the dealer to use Eurodollar contracts or forward rate agreements to hedge the resulting interest rate exposure.

7.3.4 A commodity swap example

A gold mining firm wants to fix the price it will receive for the gold. It will mine over the next 3 years.

A gold user wants to fix the price it will have to pay for the gold it needs for the next 3 years.

Notional = 10,000 ounce.
Fixed price = \$320 per ounce.

For physical settlement is semi–annual the cash payments are given by

Subsequently payment

Time	Producer delivers Gold	Investor pays	Producer receives
0.5	10.000 ounce	-3.2 Million	3.2 Million
1	10.000 ounce	-3.2 Million	3.2 Million
1.5	10.000 ounce	-3.2 Million	3.2 Million
2	10.000 ounce	-3.2 Million	3.2 Million
2.5	10.000 ounce	-3.2 Million	3.2 Million
3	10.000 ounce	-3.2 Million	3.2 Million

If the cash settlement is semi–annual, based on average price of gold during the past six months:

Subsequently payment			
Time	**Avg. gold price during past period**	**Producer pays (-) or receives (+)**	**Investor receives (+) or pays (-)**
0.5	\$305	$\$10,000(320-305) = +\$150,000$	$\$10,000(305-320) = -\$150,000$
1	\$330	$\$10,000(320-330) = -\$100,000$	$\$10,000(330-320) = +\$100,000$
1.5	\$368	$\$10,000(320-368) = -\$480,000$	$\$10,000(368-320) = +\$480,000$
2	\$402	$\$10,000(320-402) = -\$820,000$	$\$10,000(402-320) = +\$820,000$
2.5	\$348	$\$10,000(320-348) = -\$280,000$	$\$10,000(348-320) = +\$280,000$
3	\$300	$\$10,000(320-300) = +\$200,000$	$\$10,000(300-320) = -\$200,000$