Name : SOLUTION **Student ID :**

Question 1

Show that If a Bond is priced at par then

$$MaCD = \ddot{a}_{\overline{n|}i}$$

For any bond :

$$MaCD = 1 \cdot \frac{Fr \cdot v}{P} + 2 \cdot \frac{Fr \cdot v^2}{P} + 3 \cdot \frac{Fr \cdot v^3}{P} + 4 \cdot \frac{Fr \cdot v^4}{P} + 5 \cdot \frac{Fr \cdot v^5}{P} + \cdots + n \cdot \frac{Fr \cdot v^n}{P} + n \cdot \frac{Cv^n}{P}$$

$$MaCD = \frac{Fr \cdot (I_a)_{\overline{n|}} + n \cdot F \cdot v^n}{P}$$

for a bond priced at par

$$P = C \text{ and } i = r$$

Since we are not given the redemption value. Then, bond is redeemable at par.

$$F = C = P$$

$$MaCD = \frac{Pi \cdot (I_a)_{\overline{n|}} + n \cdot P \cdot v^n}{P}$$

$$MaCD = \frac{Pi \cdot [(I_a)_{\overline{n|}} + n \cdot v^n]}{P}$$

$$MaCD = i \cdot [(I_a)_{\overline{n|}} + nv^n]$$

$$MaCD = i \cdot \left[\frac{\ddot{a}_{\overline{n|}i} - nv^n}{i} + nv^n\right]$$

$$MaCD = \ddot{a}_{\overline{n|}i} - nv^n + nv^n$$

$$MaCD = \ddot{a}_{\overline{n|}i} \blacksquare$$

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Question 2

Calculate the **duration**, *the* **modified duration** *and the* **convexity** *of a level four year payments annuity– immediate if the current effective annual rate of interest is* 5%.

- ***Duration***

$$\bar{d} = \frac{(I_a)_4}{a_4} = \frac{8.648762604}{3.545950504} = 2.439053392$$

- ***Modified duration***

$$ModD = v\bar{d} = 1.05^{-1} \cdot 2.439053392 = 2.322907992$$

Or Alternatively

$$ModD = -\frac{P^{`}(i)}{P(i)}$$

$$P^{`}(i) = -(1+i)^{-2} - 2(1+i)^{-3} - 3(1+i)^{-4} - 4(1+i)^{-5}$$

$$P^{`}(0.05) = -(1+0.05)^{-2} - 2(1+0.05)^{-3} - 3(1+0.05)^{-4} - 4(1+0.05)^{-5} = -8.2369168$$

$$P(i) = (1+i)^{-1} + (1+i)^{-2} + (1+i)^{-3} + (1+i)^{-4}$$

$$P(0.05) = (1.05)^{-1} + (1.05)^{-2} + (1.05)^{-3} + (1.05)^{-4} = 3.54950504$$

$$ModD = -\frac{P^{`}(i)}{P(i)} = -\frac{-8.2369168}{3.54950504} = 2.322907992$$

- ***Convexity***

$$\bar{c} = \frac{P^{``}(i)}{P(i)}$$

$$P^{``}(i) = 2(1+i)^{-3} + 6(1+i)^{-4} + 12(1+i)^{-4} + 20(1+i)^{-5}$$

$$P^{``}(0.05) = 2(1+0.05)^{-3} + 6(1+0.05)^{-4} + 12(1+0.05)^{-5} + 20(1+0.05)^{-6} = 30.99051198$$

$$\bar{c} = \frac{P^{``}(i)}{P(i)} = \frac{30.99051198}{3.54950504} = 8.730938999$$