Name : SOLUTION **Student ID :**

Question 1

A loan is amortized over five years with monthly payments at an annual nominal interest rate of 9% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 2% lower than the prior payment. Calculate the outstanding loan balance immediately after the 40th payment is made.

$$OB_{40} = (1000)(1-2\%)^{40}(DG_a)_{\overline{20}|\frac{9}{12}\%} = \frac{1-\left(\dfrac{1-2\%}{1+\frac{9}{12}\%}\right)^{20}}{\frac{9}{12}\% + 0.02} = 6889.114797$$

Alternatively ...

$$L = 29{,}452.83263$$

$$OB_{40} = L\left(1+\frac{9}{12}\%\right)^{40} - 1000(DG_S)_{\overline{40}|\frac{9}{12}\%}$$

$$= (29{,}452.83263)\left(1+\frac{9}{12}\%\right)^{40} - 1000\left(1+\frac{9}{12}\%\right)^{40}\frac{1-\left(\dfrac{1-2\%}{1+\frac{9}{12}\%}\right)^{40}}{\frac{9}{12}\% + 0.02} = 6889.114797$$

Question 2

Ron is repaying a loan with payments of 1 at the end of each year for n years. The annual effective interest rate on the loan is i. The amount of interest paid in year t plus the amount of principal repaid in year t + 1 equals X. Determine which of the following is equal to X.

(A) $1+\dfrac{v^{n-t}}{i}$

(B) $1+\dfrac{v^{n-t}}{d}$

(C) $1+v^{n-t}\cdot i$

(D) $1+v^{n-t}\cdot d$

(E) $1+v^{n-t}$

$$I_t + P_{t+1} = X$$

$$I_t = i\cdot BV_{t-1} = i\cdot a_{\overline{n-(t-1)}|i} = i\cdot\frac{1-v^{n-(t-1)}}{i} = 1-v^{n-t+1}$$

$$\text{Then } I_{t+1} = i\cdot BV_{t+1-1} = i\cdot a_{\overline{n-(t+1-1)}|i} = i\cdot\frac{1-v^{n-(t+1-1)}}{i} = 1-v^{n-t}$$

$$1 = P_{t+1} + I_{t+1}$$

$$P_{t+1} = 1 - I_{t+1}$$

$$P_{t+1} = 1-(1-v^{n-t}) = v^{n-t}$$

$$X = v^{n-t} + 1 - v^{n-t+1} = 1 + v^{n-t}(1-v)$$

$$\text{We know } 1-v = d$$

$$X = 1 + v^{n-t}\cdot d$$