Exercises:

Chapter 8 Infinite Series:

8.1 Sequences:

Exer. 2-14: The expression is the nth term a_n of a sequence $\{a_n\}$. Find the first four terms and $\lim_{n\to\infty} a_n$, if it exists.

2. $\frac{6n-5}{5n+1}$

7. $\frac{(2n-1)(3n+1)}{n^3+1}$

12. $(-1)^{n+1}\frac{\sqrt{n}}{n+1}$

14. $1-\frac{1}{2^n}$

Exer. 18-42: Determine whether the sequence converges or diverges, and if it converges, find the limit.

18. $\left\{8-\left(\frac{7}{8}\right)^n\right\}$

22. $\left\{\frac{(1.0001)^n}{1000}\right\}$

26. $\left\{\frac{\cos n}{n}\right\}$

28. $\{e^{-n}\ln n\}$

30. $\{(-1)^n n^3 3^{-n}\}$

31. $\{2^{-n}\sin n\}$

33. $\left\{\frac{n^2}{2n-1}-\frac{n^2}{2n+1}\right\}$

37. $\left\{n^{\frac{1}{n}}\right\}$

40. $\left\{(-1)^n\frac{n^2}{1+n^2}\right\}$

42. $\{\sqrt{n^2+n}-n\}$

8.2 Convergent or divergent series:

Exer. 2-6: Find (a) $S_1, S_2, and\ S_3$; (b) S_n; (c) the sum of the series, if it converges.

2. $\sum_{n=1}^{\infty}\frac{5}{(5n+2)(5n+7)}$

6. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}$ (Hint : Rationalize the denominator)

Exer. 7-16: Determine whether the geometric series converges or diverges; if it converges, find its sum.

7. $3 + \frac{3}{4} + \cdots + \frac{3}{4^{n-1}} + \cdots$

10. $1 + \left(\frac{e}{3}\right) + \cdots + \left(\frac{e}{3}\right)^{n-1} + \cdots$

12. $0.628 + 0.000628 + \cdots + \frac{628}{(100)^n} + \cdots$

13. $\sum_{n=1}^{\infty} 2^{-n}3^{n-1}$

16. $\sum_{n=1}^{\infty} \left(\sqrt{2}\right)^{n-1}$

Exer. 18-20: Find all values of x for which the series converges, and find the sum of the series.

18. $1 + x^2 + x^4 + \cdots + x^{2n} + \cdots$

20. $3 + (x-1) + \frac{(x-1)^2}{3} + \cdots + \frac{(x-1)^n}{3^{n-1}} + \cdots$

Exer. 26-30: Determine whether the series converges or diverges.

26. $\frac{1}{10\cdot 11} + \frac{1}{11\cdot 12} + \cdots + \frac{1}{(n+9)(n+10)} + \cdots$

30. $6^{-1} + 7^{-1} + \cdots + (n+5)^{-1} + \cdots$

Exer. 39-40: Use the nth-term test to determine whether the series diverges or needs further investigation.

39. $\sum_{n=1}^{\infty} \frac{n}{ln(n+1)}$

40. $\sum_{n=1}^{\infty} ln\left(\frac{2n}{7n-5}\right)$

Exer. 41-48: Use known convergent or divergent series to determine whether the series is convergent or divergent; if it converges, find its sum.

41. $\sum_{n=1}^{\infty}\left[\left(\frac{1}{4}\right)^n+\left(\frac{3}{4}\right)^n\right]$

42. $\sum_{n=1}^{\infty}\left[\left(\frac{3}{2}\right)^n+\left(\frac{2}{3}\right)^n\right]$

47. $\sum_{n=1}^{\infty}\left(\frac{5}{n+2}-\frac{5}{n+3}\right)$

48. $\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n}\right)$

8.3 Positive-term series:

Exer. 2-11: (a) Show that the function f determined by the nth term of the series satisfies the hypotheses of the integral test. (b) Use the integral test to determine whether the series converges or diverges.

2. $\sum_{n=1}^{\infty}\frac{1}{(4+n)^{\frac{3}{2}}}$

5. $\sum_{n=1}^{\infty}n^2e^{-n^3}$

7. $\sum_{n=3}^{\infty}\frac{ln\, n}{n}$

8. $\sum_{n=2}^{\infty}\frac{1}{n(ln\, n)^2}$

10. $\sum_{n=4}^{\infty}\left(\frac{1}{n-3}-\frac{1}{n}\right)$

11. $\sum_{n=l}^{\infty}\frac{tan^{-1} n}{1+n^2}$

Exer. 13-20: Use a basic comparison test to determine whether the series converges or diverges.

13. $\sum_{n=1}^{\infty}\frac{1}{n^4+n^2+1}$

17. $\sum_{n=1}^{\infty}\frac{tan^{-1} n}{n}$

18. $\sum_{n=1}^{\infty}\frac{sec^{-1} n}{(0.5)^n}$

20. $\sum_{n=1}^{\infty}\frac{1}{n!}$

Exer. 23-25: Use the limit comparison test to determine whether the series converges or diverges.

23. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{4n^3-5n}}$

24. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$

25. $\sum_{n=1}^{\infty} \frac{8n^2-7}{e^n(n+1)^2}$

Exer. 30-46: Determine whether the series converges or diverges.

30. $\sum_{n=1}^{\infty} \frac{n^5+4n^3+1}{2n^8+n^4+2}$

33. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{5n^2+1}}$

38. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$

43. $\sum_{n=1}^{\infty} \frac{n^2+2^n}{n+3^n}$

44. $\sum_{n=1}^{\infty} ln\left(1+\frac{1}{2^n}\right)$

45. $\sum_{n=1}^{\infty} \frac{ln\, n}{n^3}$

46. $\sum_{n=1}^{\infty} \frac{sin\, n+2^n}{n+5}$

8.4 Ratio and root tests:

Exer. 3-10: Find $\lim_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right)$, and use the ratio test to determine if the series converges or diverges or if the test is inconclusive.

3. $\sum_{n=1}^{\infty}\frac{5^n}{n(3^{n+1})}$

4. $\sum_{n=1}^{\infty}\frac{2^{n-1}}{5^n(n+1)}$

7. $\sum_{n=1}^{\infty}\frac{n+3}{n^2+2n+5}$

10. $\sum_{n=1}^{\infty}\frac{n!}{(n+1)^5}$

Exer. 12-18: Find $\lim_{n\to\infty}\sqrt[n]{a_n}$, and use the root test to determine if the series converges or diverges or if the test is conclusive.

12. $\sum_{n=1}^{\infty}\frac{(\ln n)^n}{n^{\frac{n}{2}}}$

14. $\sum_{n=2}^{\infty}\frac{5^{n+1}}{(\ln n)^n}$

17. $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$

18. $\sum_{n=2}^{\infty}\left(\frac{n}{\ln n}\right)^n$

Exer. 21-38: Determine whether the series converges or diverges.

21. $\sum_{n=1}^{\infty}\frac{99^n(n^5+2)}{n^2 10^{2n}}$

25. $\sum_{n=1}^{\infty}\left(\frac{2}{n}\right)^n n!$

30. $\sum_{n=1}^{\infty}\frac{(2n)!}{2^n}$

31. $\sum_{n=2}^{\infty}\frac{1}{n\sqrt[3]{\ln n}}$

36. $\sum_{n=1}^{\infty}\frac{\tan^{-1} n}{n^2}$

37. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n$

38. $\sum_{n=1}^{\infty}\frac{1}{(\ln n)^n}$

8.5 Alternating series and absolute convergence:

Exer. 1-4: Determine whether the series (a) satisfies the conditions of the alternating series test and (b) converges or diverges.

1. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^2+7}$

4. $\sum_{n=1}^{\infty}(-1)^{n}\frac{e^{2n}+1}{e^{2n}-1}$

Exer. 5-31: Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

5. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sqrt{2n+1}}$

6. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^{\frac{2}{3}}}$

8. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+4}$

11. $\sum_{n=1}^{\infty}(-1)^{n}\frac{5}{n^3+1}$

15. $\sum_{n=1}^{\infty}(-1)^{n}\frac{n^2+3}{(2n-5)^2}$

18. $\sum_{n=1}^{\infty}(-1)^{n}\frac{(n+1)^2}{n^5+1}$

21. $\sum_{n=1}^{\infty}(-1)^{n}n\,sin\frac{1}{n}$

22. $\sum_{n=1}^{\infty}(-1)^{n}\frac{tan^{-1}n}{n^2}$

26. $\sum_{n=1}^{\infty}\frac{(n^2+1)^n}{(-n)^n}$

28. $\sum_{n=1}^{\infty}(-1)^{n}\frac{n^4}{e^n}$

31. $\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{(n-4)^2+5}$

8.6 Power series:

Exer. 5-30: Find the interval of convergence of the power series.

5. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sqrt{n}}x^n$

6. $\sum_{n=1}^{\infty}\frac{1}{ln(n+1)}x^n$

9. $\sum_{n=2}^{\infty} \frac{\ln n}{n^3} x^n$

10. $\sum_{n=0}^{\infty} \frac{10^{n+1}}{3^{2n}} x^n$

12. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} (x-2)^n$

13. $\sum_{n=0}^{\infty} \frac{n!}{100^n} x^n$

16. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\sqrt[3]{n}3^n} x^n$

19. $\sum_{n=0}^{\infty} \frac{3^{2n}}{n+1} (x-2)^n$

22. $\sum_{n=0}^{\infty} \frac{1}{2n+1} (x+3)^n$

23. $\sum_{n=1}^{\infty} (-1)^n \frac{n^n}{n+1} (x-3)^n$

27. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n6^n} (2x-1)^n$

30. $\sum_{n=1}^{\infty} (-1)^n \frac{e^{n+1}}{n^n} (x-1)^n$

8.7 Power series representations of functions:

Exer. 2-4: (a) Find a power series representation for $f(x)$. (b) Find power series representation for $f'(x)$ and $\int_0^x f(t)\ dt$.

1. $f(x) = \frac{1}{1+5x}$; $|x| < \frac{1}{5}$

4. $f(x) = \frac{1}{3-2x}$; $|x| < \frac{3}{2}$

Exer. 7-10: Find a power series in x that has the given sum, and specify the radius of convergence.

7. $\frac{x}{2-3x}$

10. $\frac{x^2-3}{x-2}$

Exer. 16-21: Use power series representation obtained in this section to find a power series representation for $f(x)$.

16. $f(x) = x^2 e^{(x^2)}$

18. $f(x) = xe^{-3x}$

19. $f(x) = x^2 \ln(1+x^2)$; $|x| < 1$

21. $f(x) = \tan^{-1}\sqrt{x}$; $|x| < 1$ 25. $f(x) = x^2 \, cosh(x^3)$

Exer. 28-31: Use an infinite series to approximate the integral to four decimal places.

28. $\int_0^{\frac{1}{2}} tan^{-1} x^2 \, dx$

29. $\int_{0.1}^{0.2} \frac{tan^{-1} x}{x} dx$

31. $\int_0^1 e^{\frac{-x^2}{10}} dx$

33. Use the power series representation for $(1 - x^2)^{-1}$ to find a power representation for $2x(1 - x^2)^{-2}$.

8.8 Maclaurin and Taylor series:

Exer. 9-13: Use a MacLaurin series obtained in this section to obtain a MacLaurin series for $f(x)$.

9. $f(x) = x \, sin\, 3x$ 10. $f(x) = x^2 \, sin\, x$ 11. $f(x) = cos(-2x)$

13. $f(x) = cos^2 x$ (Hint : Use a half-angle formula.)

Exer. 16: Find a MacLaurin series for $f(x)$. (Do not verify that $\lim_{n\to\infty} R_n(x) = 0$.)

16. $f(x) = ln(3 + x)$

Exer. 26-27: Find the first three terms of the Taylor series for $f(x)$ at c.

26. $f(x) = tan^{-1} x; \quad c = 1$

27. $f(x) = xe^x; \qquad c = -1$

Exer. 35-37: Use the first two nonzero terms of a MacLaurin series to approximate the number, and estimate the error in the approximation.

35. $\int_0^1 e^{-x^2}\, dx$ 36. $\int_0^{\frac{1}{2}} x\, cos(x^3)\, dx$ 37. $\int_0^{0.5} cos(x^2)\, dx$

Exer. 39-42: Approximate the improper integral to four decimal places. (Assume that if the integrand is $f(x)$, then $f(0) = \lim_{x \to \infty} f(x)$.)

39. $\int_0^1 \frac{1-cos\, x}{x^2}\, dx$ 42. $\int_0^1 \frac{1-e^{-x}}{x}\, dx$

Chapter 10 Vectors and Surfaces:

10.1 Vectors and vectors algebra:

Exer. 6: Sketch the position vector of a and find $\|a\|$.

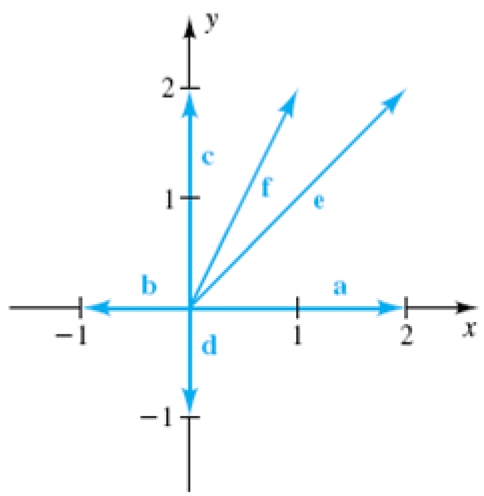
6. $a = 2i - 3j$

Exer. 9-10: Find $a + b$, $a - b$, $2a$, $-3b$, and $4a - 5b$.

9. $a = -\langle 7, -2 \rangle, \qquad b = 4\langle -2,1 \rangle$

10. $a = 2\langle 1,5 \rangle, \qquad b = -3\langle -1, -4 \rangle$

Exer. 13-14: Use components to express the sum or difference as a scalar multiple of one of the vectors a, b, c, d, e, or f shown in the figure.



13. a + b 14. c – d

Exer. 21-22: Find the vector a in V_2 that corresponds to $\overrightarrow{PQ}$. Sketch $\overrightarrow{PQ}$ and the position vector for a.

21. P(2,5), Q(-4,5)

22. P(-4,6), Q(-4,-2)

Exer. 25: Find a unit vector that has (a) the same direction as a and (b) the opposite direction of a.

25. $a = -8i + 15j$

29. Find a vector that has the same direction as $\langle -6,3\rangle$ and

(a) twice the magnitude.

(b) one-half the magnitude.

30. Find a vector that has the opposite direction of $8i - 5j$ and

(a) three times the magnitude.

(b) one-third the magnitude.

31. Find a vector of magnitude 4 that has the same direction as

$a = 4i - 7j$.

Exer. 34: Find all real numbers c such that

(a)$\|ca\| = 3$ (b) $\|ca\| = -3$ (c) $\|ca\| = 0$.

34. $a = \langle -5,12\rangle$

10.2 Vectors in three dimensions:

Exer. 5-6: Plot A and B and find (a) d(A,B), (b) the midpoint of AB, and (c) the vector in V_3 that corresponds to $\overrightarrow{AB}$.

5. A(1, 0, 0), B(0, 1, 1)

6. A(0, 0, 0), B(-8, -1, 4)

Exer. 8-12: Find (a) a + b, (b) a – b, (c) 5a – 4b, (d) $\|a\|$, and (e) $\|-3a\|$.

8. a = $\langle 1,2,-3\rangle$ b = $\langle -4,0,1\rangle$

10. a = $2i - j + 4k$ b = $i - k$

11. a = $i + j$ b = $-j + k$

12. a = $2i$ b = $3k$

Exer. 14: Sketch position vectors for a, b, 2a, -3b, a + b, and a – b.

14. a = $-i + 2j + 3k$, b = $-2j + k$

Exer. 16: Find the unit vector that has the same direction as a.

16. a = $3i - 7j + 2k$

Exer. 18: Find the vector that has (a) the same direction as a and twice the magnitude of a, (b) the opposite direction of a and one-third the magnitude of a, and (c) the same direction as a and magnitude 2.

18. a = $\langle -6, -3, 6 \rangle$

10.3 The dot product:

Exer. 5-10: Given a = $\langle -2,3,1 \rangle$, b = $\langle 7,4,5 \rangle$, and c = $\langle 1, -5, 2 \rangle$, find the number.

5. $(2a + b) \cdot 3c$ 6. $(a - b) \cdot (b + c)$ 10. $comp_c\ c$

Exer. 12-14: Find the angle between a and b.

12. a = $i - 7j + 4k$, b = $5i - k$

14. a = $\langle 3, -5, -1 \rangle$, b = $\langle 2,1,-3 \rangle$

Exer. 15: Show that a and b are orthogonal.

15. a = $3i - 2j + k,$ b = $4i + 5j - 2k$

Exer. 18: Find all values of c such that a and b are orthogonal.

18. a = $4i + 2j + ck,$ b = $i + 22j - 3ck$

Exer. 23-24: Given points P(3, -2, -1), Q(1, 5, 4), R(2, 0, -6), and S(-4, 1, 5), find the indicated quantity.

23. The component of $\overrightarrow{PS}$ along $\overrightarrow{QR}$

24. The component of $\overrightarrow{QR}$ along $\overrightarrow{PS}$

Exer. 26: If the vector a represents a constant force, find the work done when its point of application moves along the line segment from P to Q.

26. a = $\langle 8,0,-4\rangle$; P(-1, -2, 5), Q(4, 1, 0)

27. A constant force of magnitude 4 lb has same direction as the vector a = $i + j + k$. If distance is measured in feet, find the work done if the point of application moves along the y-axis from (0, 2, 0) to (0, -1, 0).

10.4 The cross product:

Exer. 7-9: Find a × b.

7. a = $-3i + j + 2k,$ b = $9i - 3j - 6k$

8. a = $3i - j + 8k,$ b = $5j$

9. a = $4i - 6j + 2k,$ b = $-2i + 3j - k$

Exer. 11-12: Use the vector product to show that a and b are parallel.

11. a = $\langle -6, -10, 4 \rangle$, b = $\langle 3, 5, -2 \rangle$

12. a = $2i - j + 4k$, b = $-6i + 3j - 12k$

Exer. 14: Let a = $\langle 2, 0, -1 \rangle$, b = $\langle -3, 1, 0 \rangle$, and c = $\langle 1, -2, 4 \rangle$.

14. Find $a \times (b - c)$ and $(a \times b) - (a \times c)$

Exer. 15-18: (a) Find a vector perpendicular to the plane determined by P, Q, and R. (b) Find the area of the triangle PQR.

15. P(1, -1, 2), Q(0, 3, -1), R(3, -4, 1)

16. P(-3, 0, 5), Q(2, -1, -3), R(4, 1, -1)

18. P(-1, 2, 0), Q(0, 2, -3), R(5, 0, 1)

Exer. 20: Find the distance from P to the line through Q and R.

20. P(-2, 5, 1), Q(3, -1, 4), R(1, 6, -3)

Exer. 23: Find the volume of the box having adjacent sides AB, AC, and AD.

23. A(2, 1, -1) B(3, 0, 2) C(4, -2, 1) D(5, -3, 0)

10.5 The lines and planes:

Exer. 2: Find parametric equations for the line through P parallel to a.

2. P(5, 0, -2); a = $\langle -1, -4, 1 \rangle$

Exer. 8: Find parametric equations for the line through P_1 and P_2. Determine (if possible) the points at which the line intersects each of the coordinate planes.

8. P_1(2, -2, 4), P_2(2, -2, -3)

9. If l has parametric equations x = 5 – 3t, y = -2 + t, z = 1 + 9t, find parametric equations for the line through P(-6, 4, -3) that is parallel to l.

Exer. 12: Determine whether the two lines intersect, and if so, find the point of intersection.

12. x = 1 – 6t, y = 3 + 2t, z = 1 – 2t

x = 2 + 2v, y = 6 + v, z = 2 + v

Exer. 15: Equations for two lines l_1 and l_2 are given. Find the angles between l_1 and l_2.

15. x = 7 – 2t, y = 4 + 3t, z = 5t

x = -1 + 4t, y = 3 + 4t, z = 1 + t

Exer. 20-21: Find an equation of the plane that satisfies the stated conditions.

20. Through P(-2, 5, -8) with normal vector

(a) i (b) j (c) k

21. Through P(-11, 4, -2) with normal vector a = $6i - 5j - k$

Exer. 28: Find an equation of the plane through P,Q, and R.

28. P(3, 2, 1), Q(-1, 1, -2), R(3, -4, 1)

Exer. 35: Sketch the graph of the equation in an xyz-coordinate system.

35. $2x - y + 5z + 10 = 0$

Exer. 42: Find an equation of the plane through P that is parallel to the given plane.

42. P(3, -2, 4); $-2x + 3y - z + 5 = 0$

Exer. 47: Find parametric equations for the line of intersection of the two planes.

47. $x + 2y - 9z = 7,$ $2x - 3y + 17z = 0$

Exer. 51: Find the distance from P to the plane.

51. P(1, -1, 2); $3x - 7y + z - 5 = 0$

10.6 Surfaces:

Exer. 2-5: Sketch the graph of the cylinder in an xyz-coordinate system.

2. $y^2 + z^2 = 16$ 3. $4y^2 + 9z^2 = 36$

5. $x^2 = 9z$

Exer. 21-30: Sketch the graph of the quadric surface.

Ellipsoids

21. $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$ 22. $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$

Hyperboloids of one sheet

24. (a) $z^2 + x^2 - y^2 = 1$ (b) $y^2 + \frac{z^2}{4} - x^2 = 1$

Hyperboloids of two sheets

25. (a) $x^2 - \frac{y^2}{4} - z^2 = 1$ (b) $\frac{z^2}{4} - y^2 - x^2 = 1$

Cones

28. (a) $\frac{x^2}{25} + \frac{y^2}{9} - z^2 = 0$ (b) $x^2 = 4y^2 + z^2$

Paraboloids

30. (a) $z = x^2 + \frac{y^2}{9}$ (b) $\frac{z^2}{25} + \frac{y^2}{9} - x = 0$

Exer. 33-40: Sketch the graph of the equation in an xyz-coordinate system, and identify the surface.

33. $16x^2 - 4y^2 - z^2 + 1 = 0$

36. $16x^2 + 100y^2 - 25z^2 = 400$

40. $16y = x^2 + 4z^2$

Chapter 11 Vector-valued functions:

11.1 Vector-valued functions:

Exer. 1-7: (a) Sketch the two vectors listed after the formula for r(t). (b) Sketch, on the same coordinate system, the curve C determined by r(t), and indicate the orientation for the given values of t.

1. r(t) = $3t\boldsymbol{i} + (1 - 9t^2)\boldsymbol{j}$, r(0), r(1); t in $\mathbb{R}$

7. r(t) = $t\boldsymbol{i} + 4\cos t\boldsymbol{j} + 9\sin t\boldsymbol{k}$, r(0), r($\pi$ / 2); t $\geq$ 0

Exer. 12: Sketch the curve C determined by r(t), and indicate the orientation.

12. r(t) = $t^3\boldsymbol{i} + t^2\boldsymbol{j} + t\boldsymbol{k}$; $0 \leq t \leq 4$

Exer. 21-22: Find the arc length of the parametrized curve. Estimate with numerical integration if needed, and express answers to four decimal places of accuracy.

21. x = 5t, y = $4t^2$, z = $3t^2$; $0 \leq t \leq 2$

22. x = t^2, y = t sin t, z = t cos t; $0 \leq t \leq 1$

11.2 Limits, derivatives and integrals:

Exer. 5-7: (a) Find the domain of r. (b) Find $r'(t)$ and $r''(t)$.

5. r(t) = $t^2\boldsymbol{i} + \tan t\,\boldsymbol{j} + 3\boldsymbol{k}$

7. r(t) = $\sqrt{t}\boldsymbol{i} + e^{2t}\boldsymbol{j} + t\boldsymbol{k}$

Exer. 18-20: A curve is given parametrically. Find parametric equations for the tangent line to C at P.

18. x = $4\sqrt{t}$, y = $t^2 - 10$, z = $4/t$; P(8, 6, 1)

20. x = $t\sin t$, y = $t\cos t$, z = t; P(π / 2,0, π / 2)

Exer. 18: Evaluate the integral.

28. $\int_{-1}^{1}(-5t\boldsymbol{i} + 8t^3\boldsymbol{j} - 3t^2\boldsymbol{k})\,dt$

Exer. 31: Find r(t) subject to the given conditions.

31. $r'(t) = t^2\boldsymbol{i} + (6t + 1)\boldsymbol{j} + 8t^3\boldsymbol{k}$, r(0) = $2i - 3j + k$

Exer. 35: If a curve C has a tangent vector a at a point P, then the normal plane to C at P is the plane through P with normal vector a. Find an equation of the normal plane to the given curve at P.

35. x = e^t,   y = te^t,   z = $t^2 + 4$;   P(1, 0, 4)

11.3 Velocity, speed and acceleration:

Exer. 9-16: If r(t) is the position vector of a moving point P, find its velocity, and speed at the given time t.

9. r(t) = $\frac{2}{t}\boldsymbol{i} + \frac{3}{t+1}\boldsymbol{j}$;   t = 2

12. r(t) = $2t\boldsymbol{i} + e^{-t^2}\boldsymbol{j}$;   t = 1

14. r(t) = $t(\cos t\boldsymbol{i} + \sin t\boldsymbol{j} + t\boldsymbol{k})$;   t = $\pi / 2$

16. r(t) = $2t\boldsymbol{i} + \boldsymbol{j} + 9t^2\boldsymbol{k}$;   t = 2

<u>Chapter 12 Partial Differentiation:</u>

12.1 Functions of several variables:

Exer. 1-6: Describe the domain of f, and find the indicated function values.

1. $f(x,y) = 2x - y^2;$ $\quad f(-2,5),\ \ f(5,-2),\ \ f(0,-2)$

2. $f(x,y) = \frac{y+2}{x};$ $\quad f(3,1),\ \ f(1,3),\ \ f(2,0)$

3. $f(u,v) = \frac{uv}{u-2v};$ $\quad f(2,3),\ \ f(-1,4),\ \ f(0,1)$

4. $f(r,s) = \sqrt{1-r} - e^{r/s};$ $f(1,1),\ \ f(0,4),\ \ f(-3,3)$

5. $f(x,y,z) = \sqrt{25 - x^2 - y^2 - z^2};$

$f(1,-2,2),\ \ f(-3,0,2)$

6. $f(x,y,z) = 2 + tan\, x + y\, sin\, z;$

$f\left(\frac{\pi}{4}, 4, \frac{\pi}{6}\right),\ \ f(0,0,0)$

12.2 Limits and continuity:

Exer. 1-7: Find the limit.

1. $\lim_{(x,y)\to(0,0)} \frac{x^2-2}{3+xy}$

4. $\lim_{(x,y)\to(-1,3)} \frac{y^2+x}{(x-1)(y+2)}$

5. $\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2}$

7. $\lim_{(x,y)\to(0,0)} \frac{3x^3-2x^2y+3y^2x-2y^3}{x^2+y^2}$

Exer. 11-15: Show that the limit does not exist.

11. $\lim_{(x,y)\to(0,0)} \frac{2x^2-y^2}{x^2+2y^2}$

12. $\lim_{(x,y)\to(0,0)} \frac{x^2-2xy+5y^2}{3x^2+4y^2}$

15. $\lim_{(x,y)\to(0,0)} \frac{4x^3y}{2x^4+3y^4}$

Exer. 22-23: Use polar coordinates to find the limit, if it exists.

22. $\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+y^2}$

23. $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{sin(x^2+y^2)}$

Exer. 26-27: Describe the set of all points in the xy-plane at which f is continuous.

26. $f(x,y) = \frac{xy}{x^2-y^2}$

27. $f(x,y) = \sqrt{x}e^{\sqrt{1-y^2}}$

Exer. 29-30: Describe the set of all points in an xyz-coordinate system at which f is continuous.

29. $f(x,y,z) = \frac{1}{\chi^2+y^2-z^2}$

30. $f(x,y,z) = \sqrt{xy}\, tan\, z$

12.3 Partial derivatives:

Exer. 1-17: Find the first partial derivatives of f.

1. $f(x,y) = 2x^4y^3 - xy^2 + 3y + 1$

4. $f(s,t) = \frac{t}{s} - \frac{s}{t}$

7. $f(t,v) = ln\sqrt{\frac{t+v}{t-v}}$

10. $f(x,y) = \sqrt{4x^2 - y^2}\, sec\, x$

13. $f(r,s,t) = r^2e^{2x} cos\, t$

17. $f(q,v,w) = sin^{-1}\sqrt{qv} + sin\, vw$

Exer. 24: Verify that $w_{xy} = w_{yx}$.

24. $w = \sqrt{x^2 + y^2 + z^2}$

26. If $w = u^4vt^2 - 3uv^2t^3$, find w_{tut}.

29. If $w = sin\,x\,yz$, find $\frac{\partial^3 w}{\partial z \partial y \partial x}$.

37. If $w = cos(x - y) + ln(x + y)$

Exer. 44: Show that the functions u and v satisfy the Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$.

44. $u(x, y) = \frac{y}{x^2+y^2}$; $v(x, y) = \frac{x}{x^2+y^2}$

51. Suppose the electrical potential V at the point (x, y, z) is given by

V = 100 / ($x^2 + y^2 + z^2$), where V is an volts and x, y, and z are in inches. Find the instantaneous rate of change of V with respect to distance at (2, -1, 1) in the direction of

(a) The x-axis
(b) The y-axis
(c) The z-axis

12.5 The Chain Rules:

Exer. 1: Find $\partial w / \partial x$ and $\partial w / \partial y$.

1. $w = u \sin v$; $u = x^2 + y^2$, $v = xy$

Exer. 3: Find ∂w / ∂r and ∂w / ∂s.

3. $w = u^2 + 2uv$; $u = r \ln s$, $v = 2r + s$

Exer. 6: Find $\partial z / \partial x$ and $\partial z / \partial y$.

6. $z = pq + qw$; $p = 2x - y$, $q = x - 2y$, $w = -2x + 2y$

9. If $p = u^2 + 3v^2 - 4w^2$, where $u = x - 3y + 2r - s$,
$v = 2x + y - r + 2s$, and $w = -x + 2y + r + s$, find $\partial p / \partial r$.

Exer. 12-13: Find dw/dt.

12. $w = \ln(u + v)$; $u = e^{-2t}$, $v = t^3 - t^2$

13. $w = r^2 - s \tan v$; $r = \sin^2 t$, $s = \cos t$, $v = 4t$

Exer. 16: Use partial derivatives to find $\frac{dy}{dx}$ if $y = f(x)$ is determined implicitly by the given equation.

`16. $x^4 + 2x^2y^2 - 3xy^3 + 2x = 0$

Exer. 19-20: Find $\partial z / \partial x$ and $\partial z / \partial y$ if $z = f(x, y)$ is determined implicitly by the given equation.

19. $2xz^3 - 3yz^2 + x^2y^2 + 4z = 0$

20. $xz^2 + 2x^2y - 4y^2z + 3y - 2 = 0$

12.6 Directional derivatives (Gradients):

Exer. 5: Find the gradient of f at P.

5. $f(x, y, z) = yz^3 - 2x^2$; P(2,-3,1)

Exer. 8-9: Estimate the directional derivative of f at P in the indicated direction with s = 0.02, 0.01, and 0.005.

8. $f(x,y) = x\ln(5x^2 + 4xy + y^2)$; P($\sqrt{5}, 3$), a = $-0.89i + 1.75j$

9. $f(x,y,z) = y^2 e^{z^3+5xy} + 6x^2yz$; P(0,1.2,-2.5),

a = $3.7i + 1.9j - 2.1k$

Exer. 12: Find the directional derivative of f at the point P in the indicated direction.

12. $f(x,y) = x^3 - 3x^2y - y^3$; P(1,-2), u = $\frac{1}{2}(-i + \sqrt{3}j)$

Exer. 27: (a) Find the directional derivative of f at P in the direction from P to Q. (b) Find a unit vector in the direction in which f increases most rapidly at P, and find the rate of change of f in that direction. (c) Find a unit vector in the direction in which f decreases most rapidly at P, and find the rate of change of f in that direction.

27. $f(x,y,z) = \sqrt{x^2 + y^2 + z^2}$; P(-2,3,1), Q(0,-5,4)

32. The temperature T at (x,y,z) is given by

$$T = 4x^2 - y^2 + 16z^2$$

(a) Find the rate of change of T at P(4,-2,1) in the direction of $2i + 6j - 3k$.

(b) In what direction does T increase most rapidly at P?

(c) What is the maximum rate of change?

(d) In which direction does T decrease most rapidly at P?

(e) What is this rate of change?

12.8 Extrema of functions of several variables:

Exer. 6-15: Find the extrema and saddle point of f.

6. $f(x, y) = -x^2 - 4x - y^2 + 2y - 1$

9. $f(x, y) = \frac{1}{2}x^2 + 2xy - \frac{1}{2}y^2 + x - 8y$

11. $f(x, y) = \frac{1}{3}x^3 - \frac{2}{3}y^3 + \frac{1}{2}x^2 - 6x + 32y + 4$

12. $f(x, y) = \frac{1}{3}x^3 + \frac{1}{3}y^3 - \frac{3}{2}x^2 - 4y$

13. $f(x, y) = \frac{1}{2}x^4 - 2x^3 + 4xy + y^2$

15. $f(x, y) = x^4 + y^3 + 32x - 9y$

12.9 Lagrange multipliers:

Exer. 2-8: Use lagrange multipliers to find the extrema of f subject to the stated constraints.

2. $f(x, y) = 2x^2 + xy - y^2 + y;$ $\quad 2x + 3y = 1$

3. $f(x, y, z) = x + y + z;$ $\quad x^2 + y^2 + z^2 = 25$

4. $f(x, y, z) = x^2 + y^2 + z^2;$ $\quad x + y + z = 25$

5. $f(x, y, z) = x^2 + y^2 + z^2;$ $\quad x - y + z = 1$

8. $f(x, y, z) = z - x^2 - y^2;$ $\quad x + y + z = 1, \; x^2 + y^2 = 4$

<u>Chapter 13 Multiple Integrals:</u>

13.1 Double integrals:

Exer. 13-19: Evaluate the iterated integral.

13. $\int_1^2 \int_{-1}^2 (12xy^2 - 8x^3)\, dy\, dx$

17. $\int_0^3 \int_{-2}^{-1} (4xy^3 + y)\, dx\, dy$

19. $\int_1^2 \int_{x^3}^{x} e^{\frac{y}{x}}\, dy\, dx$

Exer. 21-26: Sketch the region R bounded by the graphs of the given equations. If $f(x, y)$ is an arbitrary continuous function, express $\iint_R f(x, y)\, dA$ as an iterated integral.

21. $y = \sqrt{x}$, $\quad x = 4$, $\quad y = 0$

22. $y = \sqrt{x}$, $\quad x = 0$, $\quad y = 2$

26. $y = \sqrt{1 - x^2}$, $\quad y = 0$

Exer. 28-32: Express the double integral over the indicated region R as an iterated integral, and find its value.

28. $\iint_R (x - y)\, dA$; the triangular region with vertices (2,9), (2,1), (-2,1).

32. $\iint_R e^{\frac{x}{y}}\, dA$; the region bounded by the graphs of y = 2x, y = -x and y = 4.

Exer. 39-42: Sketch the region of integration for the iterated integral.

39. $\int_{-1}^{2} \int_{-\sqrt{4-x^2}}^{4-x^2} f(x, y)\, dy\, dx$

42. $\int_{-2}^{-1} \int_{3y}^{2y} f(x, y)\, dx\, dy$

Exer. 45-49: Reverse the order of integration, and evaluate the resulting integral.

45. $\int_0^1 \int_{2x}^{2} e^{y^2}\, dy\, dx$

49. $\int_0^8 \int_{\sqrt[3]{y}}^{2} \frac{y}{\sqrt{16+x^7}}\, dx\, dy$

13.2 Area and volume:

Exer. 6-10: Sketch the region bounded by the graphs of the equations, and find its area by using one or more double integrals.

6. $y = \sqrt{x}$, $y = -x$, $x = 1$, $x = 4$

7. $y^2 = -x$, $x - y = 4$, $y = -1$, $y = 2$

10. $x - y = -1$, $7x - y = 17$, $2x + y = -2$

Exer. 17-19: The iterated double integral represents the volume of a solid under a surface S and over a region R in the xy-plane. Describe S and sketch R.

17. $\int_0^4 \int_{-1}^2 3\, dy\, dx$

19. $\int_{-2}^1 \int_{x-1}^{1-x^2} (x^2 + y^2)\, dy\, dx$

Exer. 21: Find the volume of the solid that lies under the graph of the equation and over the region in the xy-plane bounded by the polygon with the given vertices.

21. $z = 4x^2 + y^2$; (0,0), (0,1), (2,0), (2,1)

Exer. 23-28: Sketch the solid in the first octant bounded by the graphs of the equations, and find its volume.

23. $x^2 + z^2 = 9$, $y = 2x$, $y = 0$, $z = 0$

25. $2x + y + z = 4$, $x = 0$; $y = 0$, $z = 0$

28. $z = y^3$, $y = x^3$, $x = 0$, $z = 0$, $y = 1$

13.3 Double integrals in polar coordinates:

Exer. 7-10: Use a double integral to find the area of the region that has the indicated shape.

7. One loop of $r = 4\sin 3\theta$

10. Bounded by $r = 3 + 2\sin\theta$

Exer. 14-24: Use polar coordinates to evaluate the integral.

14. $\iint_R x^2(x^2+y^2)^3\, dA$; R is bounded by the semicircle $y = \sqrt{1-x^2}$ and the x-axis.

15. $\iint_R \frac{x^2}{x^2+y^2}\, dA$; R is the annular region bounded by $x^2+y^2 = a^2$ and $x^2+y^2 = b^2$ with $0 < a < b$.

16. $\iint_R (x+y)\, dA$;R is bounded by the circle $x^2+y^2 = 2y$.

18. $\iint_R \sqrt{x^2+y^2}\, dA$; R is bounded by the semicircle $y = \sqrt{2x-x^2}$ and the line y = x.

19. $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)}\, dy\, dx$

24. $\int_{0}^{2}\int_{-\sqrt{2y-y^2}}^{\sqrt{2y-y^2}} x\, dx\, dy$

Exer. 25-28: Use polar coordinates to find the volume of the solid that has the shape of Q.

25. Q is the region inside the sphere $x^2 + y^2 + z^2 = 25$ and outside the cylinder $x^2 + y^2 = 9$

28. Q is bounded by the paraboloid $z = 4x^2 + 4y^2$, the cylinder $x^2 + y^2 = 3y$, and the plane $z = 0$.

13.4 Surface area:

Exer. 1-3: Set up an iterated double integral that can be used to find the surface area of the portion of the graph of the equation that lies over the region R in the xy-plane that has the given boundary. Use symmetry whenever possible.

1. $x^2 + y^2 + z^2 = 4$; the square with vertices (1,1), (1,-1), (-1,1), (-1,-1)

2. $x^2 - y^2 + z^2 = 1$; the square with vertices (0,1), (1,0), (-1,0), (0,-1)

3. $36z^2 = 16x^2 + 9y^2 + 144$; the circle with center at the origin and radius 3.

Exer. 5: Find the surface area of the portion of the graph of the equation that lies over the region R in the xy-plane that has the given boundary.

5. $z = y + \frac{1}{2}x^2$; the square with vertices (0,0), (1,0), (1,1), (0,1)

7. A portion of the plane $\left(\frac{x}{a}\right)+\left(\frac{y}{b}\right)+\left(\frac{z}{c}\right)=1$ is cut out by the cylinder $x^2+y^2=k^2$, where a, b, c and k are positive. Find area of that portion.