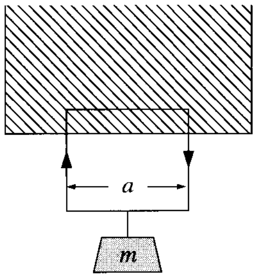
PHYS 507
HANDOUT 8 - *Questions on Magnetostatics*

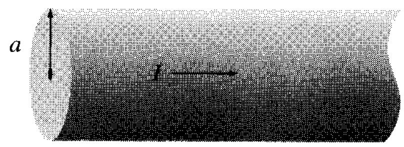
8.1 Study the motion of a charged particle, initially at rest, which is subjected to two fields: a uniform magnetic field **B** in the *x*-direction and a uniform electric field **E** in the *z*-direction.

8.2 Prove that a magnetic force does not produce any work.

8.3 A rectangular loop of wire, supporting a mass m, hangs vertically with one end in a uniform magnetic field **B**, which points into the page in the shaded region. For what current *I*, the loop, would the magnetic force upward exactly balance the gravitational force downward?



8.4 a) A current *I* is uniformly distributed over a wire of circular cross section, with radius *a*. Find the surface current density *J*. b) Suppose the current density in the wire is proportional to the distance from the axis $J = ks$ (for some constant *k*). Find the total current density.



8.5 Find the magnetic field a distance *s* from a long straight wire carrying a steady current *I*.

8.6 Prove the continuity equation.

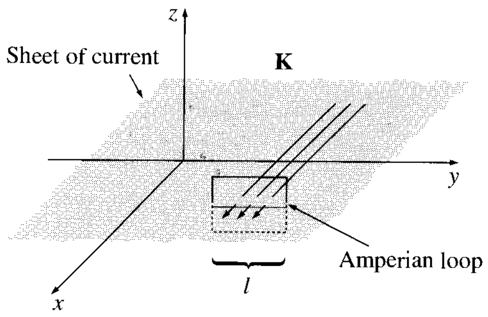
8.7 Find the force of attraction between two long, parallel wires a distance *d* apart, carrying currents I_1, I_2.

8.8 Find the magnetic field a distance *s* above the center of a circular loop of radius *R*. carrying a steady current *I*.

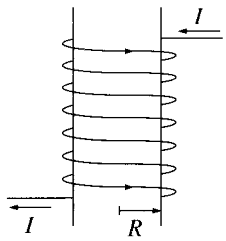
8.9 Prove that the divergence of the magnetic field is zero.

8.10 Prove that $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}(\mathbf{r})$.

8.11 Find the magnetic field of an infinite uniform surface current $\mathbf{K} = K\hat{\mathbf{x}}$ flowing over the xy plane as shown below.



8.12 Find the magnetic field of a very long solenoid, consisting of n closely wound turns per unit length on a cylinder of radius R and carrying a steady current I as shown below.

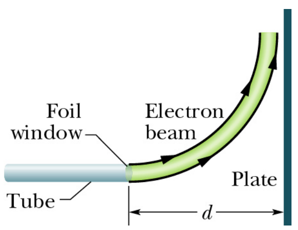


8.13 Show that Ampere's law may take the from $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$.

8.14 Prove the magnetostatic boundary conditions.

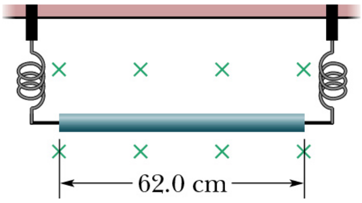
8.15 An electron that has a velocity $\vec{v} = (2\times10^6 m/s)\hat{i} + (2\times10^6 m/s)\hat{j}$ moves through the magnetic field $\vec{B} = (0.030T)\hat{i} - (0.15T)\hat{j}$. (a) Find the force on the electron. (b) Repeat the calculation for a proton having the same velocity.

8.16 A beam of electrons whose kinetic energy is K emerges from a thin-foil "window" at the end of an accelerator tube. There is a metal plate a distance d from the window and perpendicular to the direction of the emerging beam. Show that we can prevent the beam from hitting the plate if we apply a uniform magnetic field such that $B \geq \sqrt{\dfrac{2mK}{e^2 d^2}}$ in which m and e are the mass and charge of electron. How should be the magnetic field oriented?

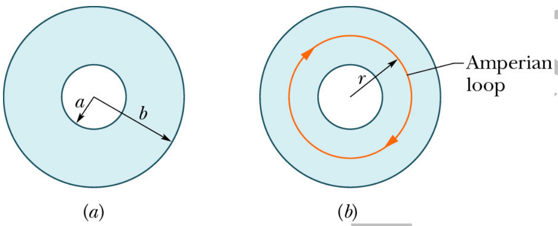


8.17 A neutral particle is at rest in a uniform magnetic field. At time $t = 0$ it decays into two charged particles, each of mass *m*. (a) If the charge of one of the particles is $+q$, what is the charge of the other? (b) The two particles move off in separate path, both of which lie in the plane perpendicular to the magnetic field. At a later time the particles collide. Express the time from decay until collision in terms of *m, B, q*.

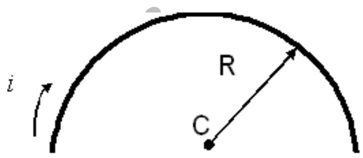
8.18 A wire $62cm$ length and $13g$ mass is suspended by a pair of flexible leads in a uniform magnetic field of magnitude $0.440T$. What are the magnitude and direction of the current required to remove the tension in the supporting leads?



8.19 The figure shows the cross section of a long conducting cylinder with inner radius $a = 2cm$ and an outer radius $b = 4cm$. The cylinder carries a current out of the page, and the current density in the cross section is given by $J = cr^2$ with $c = 3\times10^6\ A/m^4$ and *r* in meters. What is the magnetic field $\vec{B}$ at a point that is $3cm$ from the central axis of the cylinder?

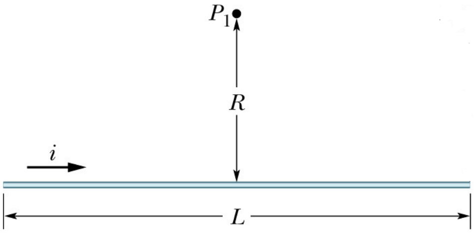


8.20 The wire shown in figure carries a current *i*. Calculate the magnetic field it produces at point C.



8.21 Show that at point P_1 the magnetic field is given by: $B=\dfrac{\mu_0 i}{2\pi R}\dfrac{L}{\left(L^2+4R^2\right)^{1/2}}$.

What happens if $L\to\infty$?

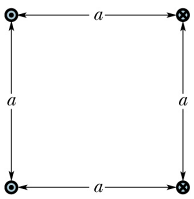


8.22 A square loop of wire of edge length *a* carries current *i.* Show that at the centre of the loop the magnetic field produced by the current is:

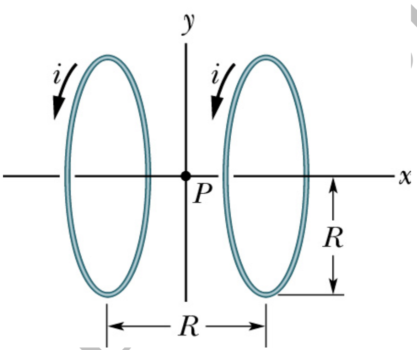
$$B=\frac{2\sqrt{2}\mu_0 i}{\pi\alpha}$$

(Hint: Use previous problem)

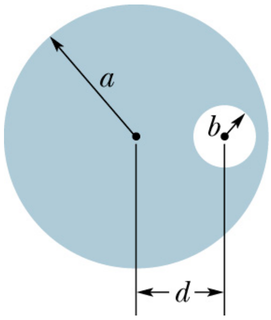
8.23 Four identical parallel currents *i* are arranged to form a square of edge length *a* as sown in figure find the magnetic field at the center of the square.



8.24 Find the magnetic field midway between the two coils which carry the same current.

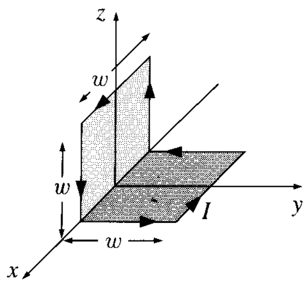


8.25 The wire in the figure has infinite length and has a hole as shown with infinite length as well. The wire has a current with a current density $\mathbf{J}=J_0\hat{\mathbf{z}}$ Find, by using superposition principle the magnetic field in the hole produced by this set up.



8.26 Find the vector potential at the point of an infinite solenoid with n turns per unit length, radius R, and current I.

8. 27 Find the magnetic dipole moment of the loop shown in figure below. All sides have length w, and it carries a current I.



8.28 Find the magnetic potential at point P_1 in question 8.21.

8.29 In a region there is a magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$. Show that the vector potential at position $\mathbf{r}$ is given by $\mathbf{A}(\mathbf{r})\frac{1}{2}\mathbf{B}\times\mathbf{r}$.